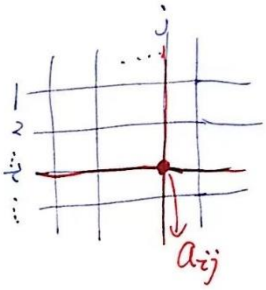
向量叉积

行列式 (3阶)

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 值$$

a_{ij} 第i行 第j列



$|\overrightarrow{AB}|$ 模

$|-2|$ 绝对值

解方程组 2元 (x_1, x_2)

$$\begin{cases} a_{11} x_1 + a_{12} x_2 = b_1 \cdots ① \\ a_{21} x_1 + a_{22} x_2 = b_2 \cdots ② \end{cases} \checkmark$$

$$\begin{array}{cc} x_1 \quad x_2 & X \quad B \\ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} & \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \end{array}$$

$A \cdot X = B$

$|A|$

(x_1, x_2)

消元法

① $\times a_{22}$ − ② $\times a_{12}$

$$\begin{cases} a_{11} \cdot a_{22} \cdot x_1 + \cancel{a_{12} \cdot a_{22}} \cdot x_2 = b_1 \cdot a_{22} \cdots ③ \\ a_{21} \cdot a_{12} \cdot x_1 + \cancel{a_{22} \cdot a_{12}} \cdot x_2 = b_2 \cdot a_{12} \cdots ④ \end{cases}$$

$$(a_{11} \cdot a_{22} - a_{21} \cdot a_{12})\, x_1 = b_1 \cdot a_{22} - b_2 \cdot a_{12}$$

$$\Rightarrow \quad x_1 = \frac{b_1 a_{22} - b_2 a_{12}}{\boxed{a_{11} a_{22} - a_{21} \cdot a_{12}}}, \quad x_2 =$$

$a_{11} a_{22} - a_{21} \cdot a_{12}$

① 右 − 左

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$$

二阶行列式

↓

行列式

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

三阶行列式

②

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \cdot a_{22} \cdot a_{33} + a_{12} \cdot a_{23} \cdot a_{33} + a_{21} \cdot a_{32} \cdot a_{13} - a_{13} \cdot a_{22} \cdot a_{31} - a_{23} \cdot a_{32} \cdot a_{11} - a_{12} a_{21} \cdot a_{33}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

例

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1 \times 5 \times 9 + 2 \times 6 \times 7 + 4 \times 8 \times 3 - 3 \times 5 \times 7 - 8 \times 6 \times 1 - 4 \times 2 \times 9$$

$$= 45 + 84 + 96 - \quad = \square$$

例

$$\begin{array}{r} \\ \vec{a} \leftarrow \\ \vec{b} \leftarrow \end{array}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & 2 \\ 1 & 2 & 4 \end{vmatrix} = \vec{c}$$

$(4, -6, 2)$

$$= \vec{i}\cdot 2\cdot 4 + \vec{j}\cdot 2\cdot 1 + 2\cdot 2\cdot \vec{k} - \vec{k}\cdot 2\cdot 1 - 2\cdot 2\cdot \vec{i} - \vec{j}\cdot 2\cdot 4$$

$$= 8\vec{i} + 2\vec{j} + 4\vec{k} - 2\vec{k} - 4\vec{i} - 8\vec{j}$$

$$= 4\vec{i} - 6\vec{j} + 2\vec{k}$$

$(2,2,2) = \vec{a}$

$(1,2,4) = \vec{b}$ ✓

$\vec{c} = \vec{a}\times\vec{b}$ 叉积结果

$\vec{c} = (4, -6, 2)$ ✓

$\vec{a}\cdot\vec{c} = 2\times 4 + 2\times(-6) + 2\times 2 = 0 \Rightarrow \vec{a}\perp\vec{c}$

① 向量叉积定义

$$\vec{c} = \vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$\vec{a} = (a_x, a_y, a_z)$

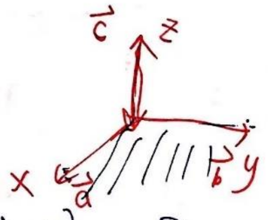
$\vec{b} = (b_x, b_y, b_z)$

② $\left.\begin{array}{l}\vec{a}\perp\vec{c}\\ \vec{b}\perp\vec{c}\end{array}\right. \Rightarrow \vec{c}\perp\begin{cases}\vec{a}\\ \vec{b}\end{cases}$

③ $\vec{c}$ 方向：右手法则

$\vec{b}\cdot\vec{c} = 1\times 4 + 2\times(-6) + 4\times 2 = 4 - 12 + 8 = 0$

$\Rightarrow \vec{b}\perp\vec{c}$



①

向量关系
- 垂直 ⇔ 内积为0
- 平行 ⇔ 对应坐标成比例
- 交角 ⇔ cosθ夹角公式

$\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

$\vec{a} \parallel \vec{a}$

$\vec{a} \times \vec{a}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ a_x & a_y & a_z \end{vmatrix}$$

$$= \vec{i} \cdot a_y \cdot a_z + \vec{j} \cdot a_z \cdot a_x + a_x a_y \vec{k} - a_x a_y \cdot \vec{k} - a_y \cdot a_z \cdot \vec{i} - a_x \cdot a_z \cdot \vec{j}$$

$$= 0 \cdot \vec{i} + 0 \cdot \vec{j} + 0 \cdot \vec{k}$$

$$= \vec{0}$$

$\vec{a} \times \vec{a} = \vec{0}$

$\vec{a} \parallel \vec{b} \Leftrightarrow \vec{b} = \lambda \vec{a}$

$(b_x \ b_y \ b_z) = \lambda (a_x \ a_y \ a_z)$

$\cancel{\vec{a} \times \vec{b}}$

$$\begin{cases} b_x = \lambda a_x \\ b_y = \lambda a_y \\ b_z = \lambda a_z \end{cases}$$

$\vec{a} \times \vec{b}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ \lambda a_x & \lambda a_y & \lambda a_z \end{vmatrix}$$

$$= \vec{i} \cdot a_y \cdot \lambda a_z + \vec{j} \cdot a_z \cdot \lambda a_x + a_x \cdot \lambda a_y \cdot \vec{k} - \lambda a_x \cdot a_y \cdot \vec{k} - a_x \cdot \lambda a_z \cdot \vec{j} - a_z \cdot \lambda a_y \cdot \vec{i}$$

$$= 0 \cdot \vec{i} + 0 \cdot \vec{j} + 0 \, \vec{k}$$

$$= \vec{0}$$

$\vec{a} \parallel \vec{b} \Leftrightarrow \vec{a} \times \vec{b} = \vec{0}$

$\vec{a}=(+2,2,2)$ $\quad\quad$ $\vec{a}\times\vec{b}=(4,-6,2)$

$\vec{b}=(1,2,4)$

$\vec{b}\times\vec{a}$

$$=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 4 \\ 2 & 2 & 2 \end{vmatrix}$$

$=\vec{i}\cdot 2\cdot 2+\vec{j}\cdot 4\cdot 2+1\cdot 2\cdot\vec{k}$

$-\vec{k}\cdot 2\cdot 2-\vec{j}\cdot 1\cdot 2-2\cdot 4\cdot\vec{i}$

$=4\vec{i}+8\vec{j}+2\vec{k}-4\vec{k}-2\vec{j}-8\vec{i}$

$=-4\vec{i}+6\vec{j}-2\vec{k}$

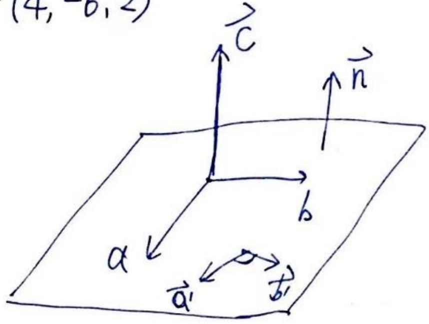
$=(-4,6,-2)$

$\therefore\ \vec{b}\times\vec{a}=(-4,6,-2)$

$\vec{a}\times\vec{b}=(4,-6,2)$

$$\boxed{\vec{b}\times\vec{a}=-\vec{a}\times\vec{b}}$$

向量叉积的结果是向量



$\vec{c}=\vec{a}\times\vec{b}$

求平面法向量 ✓

叉积应用之一

① $\vec{a}\times\vec{a}=\vec{0}$

② $\cancel{\vec{a}\times\vec{b}}$

$\because\ \vec{a}\parallel\vec{b}$ ✓

$\Rightarrow\vec{a}\times\vec{b}=\vec{0}$

✓ $\Rightarrow$ 叉积应用之二：

判断两向量是否平行

$\vec{a}\times\vec{b}=\vec{0}$

$\Leftrightarrow\ \vec{a}\parallel\vec{b}$