ERROR ANALYSIS and DATA PROCESS
误差分析和数据处理

吴先球
华南师范大学
物理学科基础课实验教学示范中心

Chapter 1 Preliminary Description of Error Analysis
误差分析基础

1.2 Inevitability of Uncertainty
误差的必然性

- 例子：门道（doorway）的高度(height)测量
  - 目测：210cm, 205~215cm
    - 粗 Rough/crude
  - 用软尺(tape)测量：211.3cm
    - 较精确 More precise
      - 误差原因
        - 灯, 尺的刻度(graduated)：
  - 激光干涉仪(laser interferometer)：仍有误差
    - 更精确 More precise : 0.5E-6 meters
  - 不同测量点，温度，湿度的影响

1.1 Error as Uncertainties

- 误差（ERROR)：
  - 测量（Measurement)中不确定度(Uncertainty)
  - 不可避免
  - 可以尽可能减小
  - Error = Uncertainty
  - Error != Uncertainty
- 误差分析（Error Analysis)：
  - 测量中不确定度的观察和估计
  - 实验误差在测量中无处不在

Conclusion

- No physics quantity can be measured with complete certainty

  没有物理量可以完全精确测定

- With care we may be able to reduce the uncertainties until they are extremely small, but to eliminate them entirely is impossible

  误差可以减小，但不能完全消除

- 何时需要关心误差？

1.3 Importance of Knowing the Uncertainties
研究误差的重要性

例子：王冠的密度测量

- 18K 金（18-karat gold)： 密度ρ=15.5gm/cm^{3}ρ
- 合金（alloy)： 密度ρ =13.8gm/cm^3

Measurement reported	Expert A	Expert B
Best estimate for ρ	15	13.9
Probable range for ρ	13.5 to 16.5	13.7 to 14.1

A &B有重叠部分
A的误差太大，测量结果没有用
B的结果确定crown由alloy 组成

Conclusion

- It is not necessary that the uncertainties be extremely small, but have to be reasonably small.

  误差无需太小，但要足够小

- Measurements would be useless if they had not included reliable statements of their uncertainties.

  如果没有包含误差的确实说明，测量结果是没有意义的.

1.4 More Examples

- The measurement of the bending of light when it passes near the Sun.

光线行进太阳附近时其弯度的测量

1.5 Estimating Uncertainties When Reading Scales
读数时估算误差

- 微波基本测量
  - 频率计　　9.464GHz(9.46~9.47GHz)
  - 驻波测量线

1.6 Estimating Uncertainties in Repeatable Measurements
在可重复测量中估算误差

2.3, 2.4, 2.5, 2.4 sec

Best estimate=average=2.4sec

Probable range: 2.3 to 2.5 sec

Chapter 2 How to Report and Use Uncertainties

记录和使用误差

2.1 Best Estimate ± Uncertainty
最佳估算值± 误差

- 2.3, 2.4, 2.5, 2.4 sec
- Best estimate=average=2.4sec
- Probable range: 2.3 to 2.5 sec

- Measured value of time=2.4±0.1 sec

$$(measured\ value\ of\ \ x) = x_{best} \pm \delta x$$

2.2 Significant Figures
有效数字

Rule for Stating Uncertainties
误差的表示

- Experimental uncertainties should usually rounded to one significant figure 实验误差通常舍入为1位有效数字
- But if the leading digit in the uncertainty δx is 1, then it may be better to keep to significant figures in δx 如果误差中最主要的数字为1，则可保留2位有效数字

Rule of Stating Answers
结果的表示

- The last significant figure in any stated answer should usually be of the same order of magnitude (in the same decimal position) as the uncertainty.结果中最后的有效数字应和误差大小有相同的数量级。
- Numbers to be used in calculations should generally be kept with one more signification figure than is finally justified. 用于计算的数通常比最终结果多保留一位有效数字。

Rules for Stating the units
单位的表示

- The uncertainty in any measured quantity has the same dimensions as the measure quantity itself. Write the units after both the answer and the uncertainty.

测量误差和被测量量具有相同的量纲，把单位写在测量结果和误差的后面。

2.3 Discrepancy 偏差

- discrepancy 偏差

  difference between two measured values of the same quantity

  对同一被测量量的两测量值之间的差异

  - significant or insignificant 显著或不显著

Accepted Value 公认值

- Quantities that have been accurately measured many times before. 通过多次精确测量的物理量
- The accepted value still has some uncertainty but is extremely small by the standards of most teaching laboratories. 仍然有误差但对于教学实验室来说足够小

$$(accept\ c) = 2999{,}792{,}458 \pm 1 m/\sec$$

True Value 真值

- 在某一时刻和某一位置或状态下，某量的效应体现出来的客观值或实际值 [肖]
- Measured value AND true value

2.4 Comparison of Measured and Accepted Values

- Take the vast majority of experiments lead to quantitative conclusions
- Compare if the accepted values INSIDE/OUTSIDE the measured range.
- What should be checked while it is OUTSIDE
  - Wrong accept value
  - Systematic error

2.5 Comparison of Two Measured Numbers

Measured momenta(all in kg.m/sec)	
Initial p (all ± .04)	Final p’ (all ± .06)
1.49 2.10 1.16 etc	1.56 2.12 1.05 etc

Measured momenta(all in kg.m/sec)		
Initial p (all ± .04)	Final p’ (all ± .06)	Difference p-p’ (all ± .1)
1.49	1.56	-0.07
2.10	2.12	-0.0
1.16	1.05	.11
etc	etc	etc

Uncertainty in a Difference
相减时的误差

- If the quantities x and y are measured with uncertainties δx and δy, and if the measured values of x and y are used to calculate the difference q=x-y, then the uncertainty in q is the sum of the uncertainties in x and y: δq≅δx+δy

$$q = x - y$$

$$\delta q \approx \delta x + \delta y$$

2.6 Checking Proportionality with a Graph用图表检查比例关系

2.7 Fractional Uncertainties
百分误差/相对误差

$$(measured x) = x_{best} \pm \delta x$$

$$fractional\ uncert = \frac{\delta x}{|x_{best}|}$$

2.8 Significant Figures and Fractional Uncertainties
有效数字和百分误差

2.9 Multiplying Two Measured Numbers 測量量相乘

Uncertainty in a Product

- If x and y have been measured with small fractional uncertainties $\delta x / |x_{best}|$ and and if the measured values of x and y are used to calculate the product q=xy, then the fractional uncertainty in q is the sum of the fractional uncertainties in x and y

$$\frac{\delta q}{|q_{best}|} = \frac{\delta x}{|x_{best}|} + \frac{\delta y}{|y_{best}|}$$

主要参考文献

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