第5章作业

5.1 求单边拉普拉斯变换，注明收敛域

(1) $1-e^{-t}$

$$\mathscr{L}[1-e^{-t}] = \int_0^\infty (1-e^{-t})e^{-st}dt$$
$$= \frac{1}{s} - \frac{1}{s+1} = \frac{1}{s(s+1)}, \mathrm{Re}[s]>0.$$

(3) $3\sin t+2\cos t$

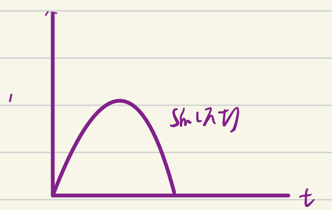
$$\mathscr{L}[3\sin t+2\cos t]$$
$$= 3\int_0^\infty \sin t\, e^{-st}dt + 2\int_0^\infty \cos t\, e^{-st}dt$$
$$= \frac{3}{2j}\left(\frac{1}{s-j} - \frac{1}{s+j}\right) + \left(\frac{1}{s-j}+\frac{1}{s+j}\right)$$
$$= \frac{3}{2j}\cdot\frac{2j}{s+1} + \frac{2s}{s^2+1} = \frac{2s+3}{s^2+1}, \mathrm{Re}[s]>0$$

(5) $e^{t}+e^{-t}$

$$\mathscr{L}[e^{t}+e^{-t}]$$
$$= \int_0^\infty e^{t} e^{sL} dt + \int_0^\infty e^{-t}e^{-st}dt$$
$$= \frac{1}{s-1} + \frac{1}{s+1}$$
$$= \frac{2s}{s-1}, \quad \mathrm{Re}[s]>1$$

5.2 求图各信号拉普拉斯变换，并注明收敛域

(a)



(f)

解：(a) $f(t) = \varepsilon(t) + \varepsilon(t-1) - \varepsilon(t-2) - \varepsilon(t-3)$

$\mathscr{L}[\varepsilon(t)] = \frac{1}{s}, \ \mathrm{Re}[s]>0$;

得 $\mathscr{L}[f(t)] = F(s) = \frac{1}{s} + \frac{1}{s}e^{-s} - \frac{1}{s}e^{-2s} - \frac{1}{s}e^{-3s}$

$$= \frac{(1+e^{-s})(1-e^{-2s})}{s}, \mathrm{Re}[s]>0$$

(7) $f(t) = \sin(\lambda t)[\varepsilon(t) - \varepsilon(t-1)]$

$= \sin(\lambda t)\varepsilon(t) + \sin[\lambda(t-1)]\varepsilon(t-1)$

$\mathcal{L}[\sin(\lambda t)\varepsilon(t)] = \frac{\lambda}{s^2+\lambda^2}$

由时移特性可得

$\mathcal{L}[f(t)] = \frac{\lambda}{s^2+\lambda^2} + \frac{\lambda}{s^2+\lambda} e^{-s} - \frac{\lambda(1-e^{-s})}{s^2+\lambda^2}$, $Re[s]>0$

5.3 求下列函数拉普拉斯变换

(1) $e^{-t}\varepsilon(t) - e^{-(t-2)}\varepsilon(t-2)$

$F(s) = \frac{1}{s+1} - \frac{1}{s+1}e^{-2s} = \frac{1-e^{-2s}}{s+1}$

(3). $\sin(\lambda t)[\varepsilon(t) - \varepsilon(t-1)]$

$f(t) = \sin(\lambda t)\varepsilon(t) + \sin[\lambda(t-1)]\varepsilon(t-1)$

$F(s) = \frac{\lambda}{s^2+\lambda^2}(1+e^{-s}) = \frac{\lambda(1+e^{-s})}{s^2+\lambda^2}$

(5) $\delta(4t-2)$

$f(t) = \frac{1}{4}\delta(t-\frac{1}{2})$

$F(s) = \frac{1}{4}e^{-\frac{s}{2}}$

(7) $\sin(2t-\frac{\pi}{4})\varepsilon(t)$

$f(t) = \sin(2t)\cos\frac{\pi}{4}\varepsilon(t) - \cos(2t)\sin\frac{\pi}{4}\varepsilon(t)$

$F(s) = \frac{2}{s^2+4}\cdot\frac{\sqrt{2}}{2} - \frac{s}{s^2+4}\cdot\frac{\sqrt{2}}{2} = \frac{2-s}{\sqrt{2}(s^2+4)}$

(9) $\int_0^t \sin(\lambda x)\,dx$

$\mathcal{L}[\sin(\lambda t)\varepsilon(t)] = \frac{\lambda}{s^2+\lambda^2}$

由时域积分 $\mathcal{L}[\int_0^t \sin(\lambda x)\varepsilon(x)\,dx] = \frac{\lambda}{s(s^2+\lambda^2)}$

(11) $\frac{d^2}{dt^2}[\sin(\lambda t)\varepsilon(t)]$

$F(s) = s^2\cdot\frac{\lambda}{s^2+\lambda^2} = \frac{s^2\lambda}{s^2+\lambda^2}$

(13) $t^2 e^{-2t}\varepsilon(t)$

$\mathcal{L}[e^{-2t}\varepsilon(t)] = \frac{1}{s+2}$ 由s域微分特性

$\mathcal{L}[(-t)^2 e^{-2t}\varepsilon(t)] = \frac{d^2}{ds^2}(\frac{1}{s+2})$

$\mathcal{L}[t^2 e^{-2t}\varepsilon(t)] = \frac{2}{(s+2)^3}$

(15) $te^{-(t-3)}\varepsilon(t-1)$

$f(t) = te^{-(t-3)}\varepsilon(t-1) = te^{2}\cdot e^{-(t-1)}\varepsilon(t-1)$

$\mathcal{L}[e^{-t}\varepsilon(t)] = \frac{1}{s+1}$ 时移特性

$\mathcal{L}[e^{-(t-1)}\varepsilon(t-1)] = \frac{e^{-s}}{s+1}$ s域微分特性

$\mathcal{L}[te^{-(t-1)}\varepsilon(t-1)] = -\frac{d}{ds}(\frac{e^{-s}}{s+1}) = \frac{(s+2)e^{-s}}{(s+1)^2}$

$F(s) = \frac{(s+2)e^{-s}}{(s+1)^2} e^{2} = \frac{s+2}{(s+1)^2} e^{-(s-2)}$

5.4 $f(t)$的象函数 $F(s) = \frac{1}{s^2-s+1}$，求 $Y(s)$

(2) $e^{-3t}f(2t-1)$

$f(2t-1) \leftrightarrow \frac{1}{2}F(\frac{s}{2})e^{-\frac{s}{2}}$

$e^{-3t}f(2t-1) \leftrightarrow \frac{1}{2}F(\frac{1}{2}(s+3))e^{-\frac{1}{2}(s+3)}$

$Y(s) = \frac{1}{2}\frac{1}{[\frac{1}{2}(s+3)]^2-\frac{1}{2}(s+3)+1}e^{-\frac{1}{2}(s+3)} = \frac{2e^{-\frac{1}{2}(s+3)}}{s^2+4s+7}$

(4) $tf(2t-1)$

$tf(2t-1) \leftrightarrow -\frac{1}{2}\frac{d}{ds}[F(\frac{s}{2})e^{-\frac{s}{2}}]$

$Y(s) = -\frac{1}{2}\frac{d}{ds}[\frac{1}{(\frac{s}{2})^2-\frac{s}{2}+1}e^{-\frac{s}{2}}] = -\frac{1}{2}\frac{d}{ds}[\frac{4}{s^2-2s+4}e^{-\frac{s}{2}}]$

$= -\frac{1}{2}[\frac{(-8s+8)e^{-\frac{s}{2}}}{(s^2-2s+4)^2} + \frac{-2e^{-\frac{s}{2}}}{s^2-2s+4}]$

$= \frac{s^2+2s}{(s^2-2s+4)^2}e^{-\frac{s}{2}}$