

$$5-1. \quad (1) \quad 1 \leftrightarrow \frac{1}{s}, \quad e^{-t} \leftrightarrow \frac{1}{s+1}$$

$$\therefore 1 - e^{-t} \leftrightarrow \frac{1}{s} - \frac{1}{s+1} = \frac{1}{s(s+1)}, \quad \text{Re}[s] > 0$$

$$(3) \quad \sin t \leftrightarrow \frac{1}{s^2+1}, \quad \cos t \leftrightarrow \frac{s}{s^2+1}$$

$$\therefore 3\sin t + 2\cos t \leftrightarrow \frac{3s+3}{s^2+1}, \quad \text{Re}[s] > 0$$

$$(8) \quad \delta(t) \leftrightarrow 1, \quad \text{Re}[s] > -\infty, \quad e^{-t} \leftrightarrow \frac{1}{s+1}, \quad \text{Re}[s] > -1$$

$$\therefore 2\delta(t) - e^{-t} \leftrightarrow \frac{2s+1}{s+1}, \quad \text{Re}[s] > -1$$

$$5.2 \quad (a) \quad \text{由图} \cdot f(t) = g_3(t - \frac{3}{2}) + g_1(t - \frac{3}{2})$$

$$\therefore g_1(t - \frac{1}{2}) \leftrightarrow \frac{1 - e^{-s}}{s}$$

$$\therefore f(t) \leftrightarrow \frac{1 - e^{-3s}}{s} + \frac{(1 - e^{-s})e^{-s}}{s} = \frac{1 - e^{-3s} - e^{-2s} - e^{-s}}{s}, \quad \text{Re}[s] > -\infty$$

$$(f) \quad \therefore \sin(\pi t) \varepsilon(t) \leftrightarrow \frac{\pi}{s^2 + \pi^2}$$

$$f(t) = \sin(\pi t) [\varepsilon(t) - \varepsilon(t-1)] = \sin(\pi t) \varepsilon(t) + \sin[\pi(t-1)] \varepsilon(t-1)$$

$$\therefore f(t) \leftrightarrow \frac{\pi}{s^2 + \pi^2} + \frac{\pi e^{-s}}{s^2 + \pi^2} = \frac{\pi + \pi e^{-s}}{s^2 + \pi^2}, \quad \text{Re}[s] > 0$$

5.3

$$(1) e^{-t} e^{ct} \leftrightarrow \frac{1}{s-c} \quad e^{-(t-2)} e^{c(t-2)} \leftrightarrow e^{-2s} \frac{1}{s-c}$$

$$\therefore e^{-t} e^{ct} - e^{-(t-2)} e^{c(t-2)} \leftrightarrow \frac{1}{s-c} (1 - e^{-2s})$$

$$(3) \sin(\pi t) e^{ct} \leftrightarrow \frac{\pi}{s^2 + \pi^2}$$

$$\sin(\pi t) [e^{ct} - e^{c(t-1)}] = \sin(\pi t) e^{ct} + \sin[\pi(t-1)] e^{c(t-1)}$$

$$\leftrightarrow \frac{\pi}{s^2 + \pi^2} + \frac{\pi}{s^2 + \pi^2} e^{-s} = \frac{\pi(1 + e^{-s})}{s^2 + \pi^2}$$

$$(5) \delta(4t-2) \leftrightarrow \frac{1}{4} e^{-\frac{1}{2}s}$$

$$(7) \sin(2t - \frac{\pi}{4}) e^{ct} = \frac{\sqrt{2}}{2} \sin(2t) e^{ct} - \frac{\sqrt{2}}{2} \cos(2t) e^{ct}$$

$$\therefore \sin(2t) e^{ct} \leftrightarrow \frac{2}{s^2 + 4}, \quad \cos(2t) e^{ct} \leftrightarrow \frac{s}{s^2 + 4}$$

$$\therefore \sin(2t - \frac{\pi}{4}) e^{ct} \leftrightarrow \frac{\sqrt{2}}{2} \left(\frac{2}{s^2 + 4} - \frac{s}{s^2 + 4} \right)$$

$$(9) \sin(\pi t) \leftrightarrow \frac{\pi}{s^2 + \pi^2} \quad \therefore \int_0^t \sin(\pi x) dx \leftrightarrow \frac{\pi}{s(s^2 + \pi^2)}$$

$$(11) \sin(\pi t) e^{ct} \leftrightarrow \frac{\pi}{s^2 + \pi^2} \quad \therefore \frac{d^2}{dt^2} [\sin(\pi t) e^{ct}] \leftrightarrow \frac{\pi s^2}{s^2 + \pi^2}$$

$$13. \because (-t)^2 \varepsilon(t) \leftrightarrow \frac{d^2}{ds^2} \left(\frac{1}{s} \right) = \frac{2}{s^3}$$

$$\therefore t^2 e^{-2t} \varepsilon(t) \leftrightarrow \frac{2}{(s+2)^3}$$

$$15. t e^{-(t-3)} \varepsilon(t-1) = e^2 t e^{-(t-1)} \varepsilon(t-1)$$

$$\text{For } e^{-(t-1)} \varepsilon(t-1) \leftrightarrow \frac{1}{s+1} e^{-s}$$

$$\text{Then } e^2 t e^{-(t-1)} \varepsilon(t-1) \leftrightarrow e^2 \left[\frac{d}{ds} \left(\frac{e^{-s}}{s+1} \right) \right] = \frac{st^2}{(s+1)^2} e^{-(s-2)}$$

$$5.4(2) f(t-1) \leftrightarrow F(s) e^{-s} = \frac{e^{-s}}{s^2 - s + 1}$$

$$f(2t-1) \leftrightarrow \frac{1}{2} \cdot \frac{e^{-\frac{s}{2}}}{\left(\frac{s}{2}\right)^2 - \frac{s}{2} + 1} = \frac{2e^{-\frac{s}{2}}}{s^2 - 2s + 4}$$

$$e^{-3t} f(2t-1) \leftrightarrow \frac{2e^{-\frac{s}{2}}}{s^2 + 4s + 7}$$

(4) 由(2)得:

$$f(2t-1) \leftrightarrow \frac{2e^{-\frac{s}{2}}}{s^2 - 2s + 4}$$

$$\text{Then } t f(2t-1) \leftrightarrow -\frac{d}{ds} \frac{2e^{-\frac{s}{2}}}{s^2 - 2s + 4} = \frac{s(s+2)e^{-\frac{s}{2}}}{(s^2 - 2s + 4)^2}$$