5-1

(1) $\because 1 \longleftrightarrow \frac{1}{s}$; $e^{-t} \longleftrightarrow \frac{1}{s+1}$

$\therefore 1-e^{-t} \longleftrightarrow \frac{1}{s}-\frac{1}{s+1}=\frac{1}{s(s+1)}$, $Re[s]>0$

(3) $\because 3\sin t \longleftrightarrow \frac{3}{s^2+1}$; $2\cos t \longleftrightarrow \frac{2s}{s^2+1}$

$\therefore 3\sin t+2\cos t \longleftrightarrow \frac{2s+3}{s^2+1}$. $Re[s]>0$

(8) $\because \delta(t) \underset{Re[s]>-\infty}{\longleftrightarrow} 1$ $e^{-t} \longleftrightarrow \frac{1}{s+1}$; $Re[s]>-1$

$\therefore 2\delta(t)-e^{-t} \longleftrightarrow \frac{2s+1}{s+1}$ $Re[s]>-1$

5.2

(a) 由图知 $f(t)=g_3(t-\frac{3}{2})+g_1(t-\frac{1}{2}-1)$

$\because g_\tau(t-\frac{\tau}{2}) \longleftrightarrow \frac{1-e^{-s\tau}}{s}$

$\therefore g_3(t-\frac{3}{2}) \longleftrightarrow \frac{1-e^{-3s}}{s}$ $g_1(t-\frac{1}{2}) \longleftrightarrow \frac{1-e^{-s}}{s}$

由时移特性得 $g_1(t-\frac{1}{2}-1) \longleftrightarrow \frac{(1-e^{-s})e^{-s}}{s}$

$\therefore f(t) \longleftrightarrow \frac{1-e^{-3s}}{s}+\frac{(1-e^{-s})e^{-s}}{s}=\frac{1-e^{-3s}-e^{-2s}-e^{-s}}{s}$ $Re[s]>-\infty$

(f) $\because \sin(\pi t)\varepsilon(t) \longleftrightarrow \frac{\pi}{s^2+\pi^2}$

$f(t)=\sin(\pi t)[\varepsilon(t)-\varepsilon(t-1)]=\sin(\pi t)\varepsilon(t)+\sin[\pi(t-1)]\varepsilon(t-1)$

$\therefore \sin[\pi(t-1)]\varepsilon(t-1) \longleftrightarrow \frac{\pi e^{-s}}{s^2+\pi^2}$

$\therefore f(t) \longleftrightarrow \frac{\pi}{s^2+\pi^2}+\frac{\pi e^{-s}}{s^2+\pi^2}=\frac{\pi+\pi e^{-s}}{s^2+\pi^2}$ $Re[s]>0$

5.3

(1) $\because e^{-t}\varepsilon(t) \longleftrightarrow \frac{1}{s+1}$; $e^{-(t-2)}\varepsilon(t-2) \longleftrightarrow \frac{e^{-2s}}{s+1}$

$\therefore e^{-t}\varepsilon(t) - e^{-(t-2)}\varepsilon(t-2) \longleftrightarrow \frac{1}{s+1} - \frac{e^{-2s}}{s+1} = \frac{1-e^{-2s}}{s+1}$

(3) $\because \sin(\pi t)\varepsilon(t) \longleftrightarrow \frac{\pi}{s^2+\pi^2}$

$\sin(\pi t)\varepsilon(t-1) = -\sin[\pi(t-1)]\varepsilon(t-1) \longleftrightarrow \frac{-\pi e^{-s}}{s^2+\pi^2}$

$\therefore \sin(\pi t)[\varepsilon(t) - \varepsilon(t-1)] \longleftrightarrow \frac{\pi}{s^2+\pi^2} - (\frac{-\pi e^{-s}}{s^2+\pi^2}) = \frac{\pi + \pi e^{-s}}{s^2+\pi^2}$

(5) $\because \delta(t) \longleftrightarrow 1$ $\therefore \delta(t-2) \longleftrightarrow e^{-2s}$

$\therefore \delta(4t-2) \longleftrightarrow \frac{1}{4}e^{-\frac{s}{2}}$

(7) $\sin(2t-\frac{\pi}{4})\varepsilon(t) = (\frac{\sqrt{2}}{2}\sin 2t - \frac{\sqrt{2}}{2}\cos 2t)\varepsilon(t) \longleftrightarrow \frac{\sqrt{2}}{2}\frac{2}{s^2+4} - \frac{\sqrt{2}}{2}\frac{s}{s^2+4} = \frac{2-s}{\sqrt{2}(s^2+4)}$

(9) $\because \int_0^t \sin(\pi x)dx \longleftrightarrow \frac{1}{s}\frac{\pi}{s^2+\pi^2} \cancel{+ \frac{1}{s}} = \frac{\pi}{s(s^2+\pi^2)}$

(11) $\frac{d^2[\sin(\pi t)\varepsilon(t)]}{dt^2} \longleftrightarrow s^2 \cdot \frac{\pi}{s^2+\pi^2} = \frac{s^2\pi}{s^2+\pi^2}$

(13) $\because \varepsilon(t) \longleftrightarrow \frac{1}{s}$ $\therefore e^{-2t}\varepsilon(t) \longleftrightarrow \frac{1}{s+2}$

$\therefore t^2e^{-2t}\varepsilon(t) = (-t)^2e^{-2t}\varepsilon(t) \longleftrightarrow \frac{d^2(\frac{1}{s+2})}{ds^2} = \frac{2}{(s+2)^3}$

(15) $\because e^{-t}\varepsilon(t) \longleftrightarrow \frac{1}{s+1}$ $\therefore e^{-(t-1)}\varepsilon(t-1) \longleftrightarrow \frac{e^{-s}}{s+1}$ $\therefore e^{-(t-3)}\varepsilon(t-1) \longleftrightarrow \frac{e^{-s+2}}{s+1}$

$\therefore -te^{-(t-3)}\varepsilon(t-1) \longleftrightarrow \frac{d(\frac{e^{-s+2}}{s+1})}{ds} = \frac{-e^{-s+2}(s+2)}{(s+1)^2}$

$\therefore te^{-(t-3)}\varepsilon(t-1) \longleftrightarrow \frac{e^{-s+2}(s+2)}{(s+1)^2}$

5.4 (2) $\because f(t) \longleftrightarrow \frac{1}{s^2-s+1}$

$\therefore f(t-1) \longleftrightarrow \frac{e^{-s}}{s^2-s+1}$ $\therefore f(2t-1) \longleftrightarrow \frac{1}{2}\frac{e^{-\frac{s}{2}}}{\frac{s^2}{4}-\frac{s}{2}+1} = \frac{2e^{-\frac{s}{2}}}{s^2-2s+4}$

$\therefore e^{-3t} \longleftrightarrow \frac{2e^{-\frac{(s+3)}{2}}}{(s+3)^2-2(s+3)+4} = \frac{2e^{-\frac{(s+3)}{2}}}{s^2+4s+7}$

(4) $f(2t-1) \longleftrightarrow \frac{1}{2}\frac{e^{-\frac{s}{2}}}{\frac{s^2}{4}-\frac{s}{2}+1} = \frac{2e^{-\frac{s}{2}}}{s^2-2s+4}$

$\therefore -tf(2t-1) \longleftrightarrow \frac{-(s^2+2s)e^{-\frac{s}{2}}}{(s^2-2s+4)^2}$

$\therefore tf(2t-1) \longleftrightarrow \frac{(s^2+2s)e^{-\frac{s}{2}}}{(s^2-2s+4)^2}$