习题五

5.1、求单边拉普拉斯变换，注明收敛域

(1) $1-e^{-t}$.

$$F(s) = \int_{0_-}^{+\infty} (1-e^{-t}) e^{-st}\, dt$$

$$= \int_{0_-}^{+\infty} e^{-st} - e^{-(1+s)t}\, dt = \left[-\frac{1}{s}e^{-st} + \frac{1}{1+s}e^{-(1+s)t}\right]\Big|_{0}^{+\infty}$$

$$= \frac{1}{s} - \frac{1}{1+s}, \quad Re[s] > 0.$$

(3) $3\sin t + 2\cos t$

$$F(s) = \int_{0_-}^{+\infty} (3\sin t + 2\cos t) e^{-st}\, dt$$

$$= \int_{0_-}^{+\infty} \left[\frac{3}{2j}(e^{jt} - e^{-jt}) + (e^{jt} + e^{-jt})\right] e^{-st}\, dt$$

$$\cancel{= \frac{5}{2}\cdot\frac{1}{j-s} e^{(j-s)t} + \frac{1}{2}\cdot\frac{1}{j+s} e^{-(j+s)t}\Big|_{0}^{+\infty}}$$

$$\cancel{= -\frac{1}{2}\left(\frac{5}{j-s} + \frac{1}{j+s}\right)}$$

$$\cancel{= \frac{6j+4s}{2(s^2-1)} \quad Re[s] > 0}$$

$$= \frac{3}{2j}\int_{0_-}^{+\infty} e^{(j-s)t} - e^{-(j+s)t}\, dt + \int_{0_-}^{+\infty} e^{(j-s)t} + e^{-(j+s)t}\, dt.$$

$$= \frac{3}{s^2+1} + \frac{2s}{s^2+1} = \frac{3+2s}{s^2+1} \quad Re[s] > 0$$

(8) $2\delta(t)-e^{-t}$

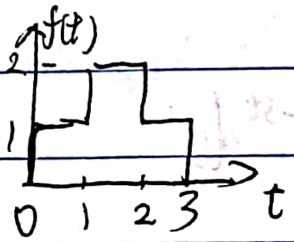
$$F(s)=\int_{0_-}^{+\infty}\left(2\delta(t)-e^{-t}\right)e^{-st}\,dt$$

$$=\int_{0_-}^{+\infty} 2-e^{-(1+s)t}\,dt$$

$$=2+\frac{1}{1+s}e^{-(1+s)t}\Big|_{0_-}^{+\infty}$$

$$=2-\frac{1}{s+1}=\cancel{\frac{s+2}{s(s+1)}}\ \frac{2s+1}{s+1}\quad Re[s]>\cancel{0}\,-1$$

六.2. 求拉普拉斯变换，注明收敛域

(a) 

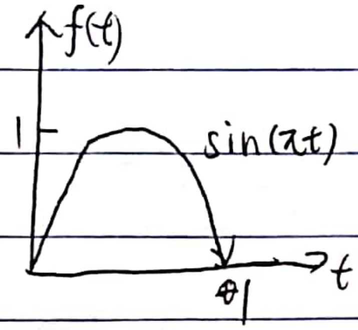
由图得 $f(t)=g_3(t-\frac{3}{2})+g_1(t-\frac{3}{2})$

则 $F(s)=\int_{0_-}^{+\infty}f(t)e^{-st}dt=\int_0^1 e^{-st}+\int_1^2 2e^{-st}+\int_2^3 e^{-st}\,dt$

$$=-\frac{1}{s}e^{-st}\Big|_0^1-\frac{2}{s}e^{-st}\Big|_1^2-\frac{1}{s}e^{-st}\Big|_2^3$$

$$=\frac{1}{s}\left(e^{-s}-e^{-2s}-e^{-3s}+1\right),\quad Re[s]>0.$$

(f)



$\cancel{F(s)} = f(t) = \sin(\pi t)\cdot[\varepsilon(t) - \varepsilon(t-1)]$

$F(s) = \mathcal{L}[\sin(\pi t)\,\varepsilon(t)] + \mathcal{L}\,[\sin[\pi(t-1)]\cdot\varepsilon(t-1)]$

$= \dfrac{\pi}{s^2+\pi^2} + \dfrac{\pi}{s^2+\pi^2}e^{-s} = \dfrac{\pi(1+e^{-s})}{s^2+\pi^2}$

5.3. 求下列 $f(t)$ 的拉普拉斯变换 $F(s)$

(1) $e^{-t}\varepsilon(t) - e^{-(t-2)}\varepsilon(t-2)$

由 $\mathcal{L}[e^{-t}\varepsilon(t)] = \frac{1}{s+1}$

由时移特性可知 $e^{-(t-2)}\varepsilon(t-2) = \frac{1}{s+1}e^{-2s}$

由线性特性可得 ~~$e^{-t}\varepsilon(t) - e^{-(t-2)}\varepsilon(t-2) =$~~

$F(s) = \frac{1}{s+1} - \frac{1}{s+1}e^{-2s}$ √

(3) $\sin(\pi t)[\varepsilon(t) - \varepsilon(t-1)]$

$\sin(\pi t) = \frac{1}{2j}(e^{j\pi t} - e^{-j\pi t})$

所以 $\sin(\pi t)[\varepsilon(t) - \varepsilon(t-1)] = \frac{1}{2j}[e^{j\pi t}\varepsilon(t) - e^{j\pi t}\varepsilon(t-1) - e^{-j\pi t}\varepsilon(t) + e^{-j\pi t}\varepsilon(t-1)]$

$\varepsilon(t) \leftrightarrow \frac{1}{s}$ 则 $\varepsilon(t-1) \leftrightarrow \frac{e^{-s}}{s}$

所以 $F(s) = \mathcal{L}[\sin(\pi t)[\varepsilon(t) - \varepsilon(t-1)]] = \frac{1}{2j}\left[\frac{1}{s-j\pi} - \frac{e^{-(s-j\pi)}}{s-j\pi} - \frac{1}{s+j\pi} + \frac{e^{-(s+j\pi)}}{s+j\pi}\right]$

$= \frac{1}{2j} \cdot \frac{(s-j\pi)e^{-(s+j\pi)} - (s+j\pi)e^{-(s-j\pi)}}{s^2+\pi^2}$

$= \frac{\pi(1+e^{-s})}{s^2+\pi^2}$

(5) $\delta(4t-2)$.

由 $\mathcal{L}_0[\delta(t)] = 1$

则由时移特性得 $\mathcal{L}_0[\delta(t-2)] = e^{-2s}$

由尺度变换特性得：

$$F(s) = \mathcal{L}_0[\delta(4t-2)] = \frac{1}{4}e^{-\frac{s}{2}}$$

(7) $\sin(2t-\frac{\pi}{4})\varepsilon(t)$

$$\sin(2t-\frac{\pi}{4})\varepsilon(t) = \frac{\sqrt{2}}{2}(\sin 2t - \cos 2t)\varepsilon(t)$$

$$= -\frac{\sqrt{2}}{4}[(j+1)e^{j2t} + (1-j)e^{-j2t}]\varepsilon(t).$$

所以由复频移特性得

$$F(s) = \mathcal{L}_0[\sin(2t-\frac{\pi}{4})\varepsilon(t)] = -\frac{\sqrt{2}}{4}[(1+j)\frac{1}{s-2j} + (1-j)\frac{1}{s+2j}]$$

$$= -\frac{\sqrt{2}}{2}\cdot\frac{s-2}{s^2+4}.$$

(9) $\int_0^t \sin(\pi x)\,dx$.

~~$\sin(\pi x) = \frac{1}{2j}(e^{j\pi t} - e^{-j\pi t})$~~

$\sin(\pi x) \longleftrightarrow \frac{\pi}{s^2+\pi^2}$

所以 $F(s) = \mathcal{L}_0\left[\int_0^t \sin(\pi x)\,dx\right] = \frac{1}{s} \cdot \frac{\pi}{s^2+\pi^2} = \frac{\pi}{s(s^2+\pi^2)}$

(11) $\frac{d^2}{dt^2}[\sin(\pi t)\varepsilon(t)]$

已知 $\sin(\pi t)\varepsilon(t) \longleftrightarrow \frac{\pi}{s^2+\pi^2}$

所以 $F(s) = \mathcal{L}_0\left[\frac{d^2}{dt^2}[\sin(\pi t)\varepsilon(t)]\right] = s^2 \cdot \frac{\pi}{s^2+\pi^2}$ ~~$s f(0$~~

$= \frac{\pi s^2}{s^2+\pi^2}$

(13) $t^2 e^{-2t}\varepsilon(t)$

已知 $e^{-2t}\varepsilon(t) \longleftrightarrow \frac{1}{s+2}$

则 $F(s) = \mathcal{L}[t^2 e^{-2t}\varepsilon(t)] = \cancel{\left(\frac{1}{s+2}\right)^{(2)}} \quad \frac{d^3}{ds^2}\cdot\left(\frac{1}{s+2}\right)$

$$= \frac{2}{(s+2)^3}$$

(15) $te^{-(t-3)}\varepsilon(t-1)$

$\varepsilon(t) \longleftrightarrow \frac{1}{s}$ 所以 $\varepsilon(t-1) \longleftrightarrow \frac{1}{s}e^{-s}$

所以 $e^{-(t-3)}\varepsilon(t-1) \longleftrightarrow \cancel{\frac{1}{s-2}e^{-(s-3)}} \quad \frac{1}{s+1}e^{\frac{-(s-2)}{\cancel{-(s+1)}}}$

所以 $F(s) = \mathcal{L}[te^{-(t-3)}\varepsilon(t-1)] = -\frac{d}{ds}\left[\frac{1}{s+1}e^{-(s-2)}\right]$

$$= \frac{s+2}{(s+1)^2}e^{-(s-2)}$$

5.4 已知因果函数 $f(t)$ 的像函数 $F(s)=\frac{1}{s^2-s+1}$，求 $y(t)$ 的像函数 $Y(s)$

(2) $e^{-3t}f(2t-1)$

$\because f(t) \longleftrightarrow \frac{1}{s^2-s+1}$ 则 $f(t-1) \longleftrightarrow \frac{1}{s^2-s+1}\cdot e^{-s}$

$f(2t-1) \longleftrightarrow \frac{1}{2}\cdot\frac{1}{(\frac{1}{2}s)^2-\frac{1}{2}s+1}\cdot e^{-\frac{1}{2}s}$

所以 $Y(s)=\mathcal{L}[e^{-3t}f(2t-1)]=\frac{1}{2}\cdot\frac{1}{[\frac{1}{2}(s+3)]^2-\frac{1}{2}(s+3)+1}\cdot e^{-\frac{1}{2}(s+3)}$

复频域平移特性

$=\frac{2}{s^2+4s+7}\cdot e^{-\frac{1}{2}(s+3)}$

(4) $tf(2t-1)$

$f(t) \longleftrightarrow \frac{1}{s^2-s+1}$ $\quad f(2t-1) \longleftrightarrow \frac{1}{2}\cdot\frac{1}{(\frac{1}{2}s)^2-\frac{1}{2}s+1}\cdot e^{-\frac{1}{2}s}$

由s域微分特性

所以 ~~$tf(2t-1) \longleftrightarrow$~~

$Y(s)=\mathcal{L}[tf(2t-1)]=-\frac{d}{ds}\cdot\mathcal{L}[f(2t-1)]=\frac{s^2+2s \; \cancel{2s^2+4}}{(s^2-2s+4)^2}\cdot e^{-\frac{1}{2}s}$