5.1

(1) $F(s)=\int_{0_-}^{\infty} f(t)e^{-st}dt=\int_{0_-}^{\infty}(1-e^{-t})e^{-st}dt$

$=\int_{0_-}^{\infty}e^{-st}dt-\int_{0_-}^{\infty}e^{-(1+s)t}dt$

$=\frac{1}{s}-\frac{1}{s+1}=\frac{1}{s(s+1)}$, $Re[s]>0$

(3) $F(s)=\int_{0_-}^{\infty} f(t)e^{-st}dt=\int_{0_-}^{\infty}(3\sin t+2\cos t)e^{-st}dt$

$=\int_{0_-}^{\infty}\frac{(3e^{jt}-3e^{-jt})e^{-st}}{2j}dt+\int_{0_-}^{\infty}(e^{jt}+e^{-jt})e^{-st}dt$

$=\frac{3}{2j}\int_{0_-}^{\infty}e^{(j-s)t}dt-\frac{3}{2j}\int_{0_-}^{\infty}e^{-(j+s)t}dt$

$+\int_{0_-}^{\infty}e^{(j-s)t}dt+\int_{0_-}^{\infty}e^{-(j+s)t}dt$

$=\frac{6j}{2j(s^2+1)}+\frac{2s}{s^2+1}=\frac{2s+3}{s^2+1}$, $Re[S]>0$

(8) $F(s)=2-\frac{1}{s+1}=\frac{2s+1}{s+1}$, $Re[s]>\cancel{0}-1$

5.2

(a) 由图知 $f(t)=\cancel{g_3(t)+g_1(t-1)}\ g_3(t-1.5)+g_1(t-1.5)$

$=\varepsilon(t)-\varepsilon(t-3)+\varepsilon(t-1)-\varepsilon(t-2)$

$\because \varepsilon(t) \longleftrightarrow \frac{1}{s}, Re[s]>0$

$\therefore$ 由时移特性得

$$F(s)=\frac{1}{s}-e^{-3s}\frac{1}{s}+e^{-s}\frac{1}{s}-e^{-2s}\frac{1}{s}$$

$$=\frac{1}{s}(1+e^{-s})(1-e^{-2s}), Re[s]>0$$

(f) 由图知 $f(t)=\sin(\pi t)g_1(t-0.5)=\sin(\pi t)(\varepsilon(t)-\varepsilon(t-1))$

$$=\varepsilon(t)\sin(\pi t)-\varepsilon(t-1)\sin(\pi t)$$

$\because \varepsilon(t)\sin(\pi t) \longleftrightarrow e^{-0s}\frac{\pi}{s^2+\pi^2}=\frac{\pi}{s^2+\pi^2}$

$\therefore$ 由时移特性得

$$F(s)=\frac{\pi}{s^2+\pi^2}-e^{-s}\frac{\pi}{s^2+\pi^2}=\frac{\pi(1-e^{-s})}{s^2+\pi^2}, Re[s]>0$$

5.3

(1) $\because e^{-t}\varepsilon(t) \longleftrightarrow \frac{1}{s+1}$

$\therefore e^{-(t-2)}\varepsilon(t-2) \longleftrightarrow e^{-2s}\frac{1}{s+1}$

$\therefore e^{-t}\varepsilon(t)-e^{-(t-2)}\varepsilon(t-2) \longleftrightarrow \frac{1-e^{-2s}}{s+1}$

(3) $\because \sin(\pi t)[\varepsilon(t)-\varepsilon(t-1)]=\sin(\pi t)\varepsilon(t)-\sin(\pi t)\varepsilon(t-1)$

$$=\sin(\pi t)\varepsilon(t)-\sin[\pi(t-1)+\pi]\varepsilon(t-1)$$

$$=\sin(\pi t)\varepsilon(t)+\sin[\pi(t-1)]\varepsilon(t-1)$$

$\therefore \sin(\pi t)[\varepsilon(t)-\varepsilon(t-1)] \longleftrightarrow \dfrac{\pi}{s^2+\pi^2}+e^{-s}\dfrac{\pi}{s^2+\pi^2}$

即 $\sin(\pi t)[\varepsilon(t)-\varepsilon(t-1)] \longleftrightarrow \dfrac{\pi(1+e^{-s})}{s^2+\pi^2}$

(5) $\because \delta(t) \longleftrightarrow 1$

$\therefore \delta(4t-2) \longleftrightarrow \dfrac{1}{4}e^{-\frac{2}{4}s} = \dfrac{1}{4}e^{-\frac{1}{2}s}$

(7) $\because \sin(2t-\frac{\pi}{4})\varepsilon(t)$

$= \sin(2t)\varepsilon(t)\times\frac{\sqrt{2}}{2} - \cos(2t)\varepsilon(t)\times\frac{\sqrt{2}}{2}$

且 $\sin(2t)\varepsilon(t) \longleftrightarrow \dfrac{2}{s^2+4}$

$\cos(2t)\varepsilon(t) \longleftrightarrow \dfrac{s}{s^2+4}$

$\therefore \cancel{\sin(2t)}\ \sin(2t-\frac{\pi}{4})\varepsilon(t) \longleftrightarrow \dfrac{\sqrt{2}(2-s)}{2(s^2+4)}$

(9) $\because \cancel{\sin(\pi 0)}\ \sin(\pi t)\varepsilon(t) \longleftrightarrow \dfrac{\pi}{s^2+\pi^2}$

∴ 由积分性质得

$\int_0^t \sin(\pi x)dx \longleftrightarrow \dfrac{1}{s}\dfrac{\pi}{s^2+\pi^2}$

(11) $\because \sin(\pi t)\varepsilon(t) \longleftrightarrow \dfrac{\pi}{s^2+\pi^2}$

∴ 由时域微分特性得：

$\dfrac{d^2}{dt^2}[\sin(\pi t)\varepsilon(t)] \longleftrightarrow \dfrac{s^2\pi}{s^2+\pi^2}$

(13) $\because t^2e^{-2t}\varepsilon(t)=(-t)^2\cdot e^{-2t}\varepsilon(t)$

且 $e^{-2t}\varepsilon(t)\longleftrightarrow\frac{1}{s+2}$

$\therefore$ s域微分特性得

$$t^2e^{-2t}\varepsilon(t)=(-t)^2e^{-2t}\varepsilon(t)\longleftrightarrow 2(s+2)^{-3}=\frac{2}{(s+2)^3}$$

(15) $\because te^{-(t-3)}\varepsilon(t-1)=te^{2}e^{-(t-1)}\varepsilon(t-1)$

且 $e^{-t}\varepsilon(t)\longleftrightarrow\frac{1}{s+1}$

$\therefore$ 由时移特性得 $e^{-(t-1)}\varepsilon(t-1)\longleftrightarrow\frac{e^{-s}}{s+1}$

$\therefore$ 由 s域微分得

$$-te^{-(t-1)}\varepsilon(t-1)\longleftrightarrow e^{-s}\frac{-(s+2)}{(s+1)^2}$$

即 $te^{-(t-1)}\varepsilon(t-1)\longleftrightarrow\frac{(s+2)e^{-s}}{(s+1)^2}$

$\therefore$ ~~由复频移特性得~~

$\cancel{e^{-2}t}$

$\therefore te^{-(t-3)}\varepsilon(t-1)\longleftrightarrow\frac{(s+2)e^{-(s+2)}}{(s+1)^2}$

5.4

(2) $\because f(t)\longleftrightarrow F(s)=\frac{1}{s^2-s+1}$

$$\therefore f(2t-1)\longleftrightarrow\frac{1}{2}e^{-\frac{s}{2}}F\left(\frac{s}{2}\right)=\frac{2e^{-\frac{s}{2}}}{s^2-2s+4}$$

$$\therefore e^{-3t}f(2t-1) \longleftrightarrow \frac{2e^{-\frac{1}{2}(s+3)}}{(s+3)^2-2(s+3)+4} = \frac{2e^{-\frac{1}{2}(s+3)}}{s^2+4s+7}$$

(4) 由(2)知 $f(2t-1) \longleftrightarrow \frac{2e^{-\frac{s}{2}}}{s^2-2s+4}$

$\therefore$ 由s域微分得 $-tf(2t-1) \longleftrightarrow \frac{-e^{-\frac{s}{2}}2(2s-2)-(s^2-2s+4)}{(s^2-2s+4)^2}$

$$\therefore tf(2t-1) \longleftrightarrow \frac{(s^2+2s)e^{-\frac{s}{2}}}{(s^2+4-2s)^2}$$