5.1

(1) $\mathcal{L}[1-e^{-t}] = \int_{0-}^{\infty}(1-e^{-t})e^{-st}dt = \int_{0-}^{\infty}e^{-st} - \int_{0-}^{\infty}e^{-(s+1)t}dt$

$= \frac{1}{s} - \frac{1}{s+1} = \frac{1}{s(s+1)}$, $Re[s]>0$

(3) $\mathcal{L}[3\sin t+2\cos t]$ ~~$=3\int_{0-}$~~ $=3\int_{0-}^{\infty}\sin t\, e^{-st}dt + 2\int_{0-}^{\infty}\cos t\, e^{-st}dt$

$= \frac{3}{2j}\int_{0-}^{\infty}e^{jt}e^{-st}dt - \frac{3}{2j}\int_{0-}^{\infty}e^{-jt}e^{-st}dt +$

$\int_{0-}^{\infty}e^{jt}e^{-st}dt + \int_{0-}^{\infty}e^{-jt}e^{-st}dt$

$= \frac{3}{2j}(\frac{1}{s-j} - \frac{1}{s+j}) + \frac{1}{s-j} + \frac{1}{s+j})$

$= \frac{3}{2j}\cdot\frac{2j}{s^2+1} + \frac{2s}{s^2+1} = \frac{2s+3}{s^2+1}$, $Re[s]>0$

(8) $\mathcal{L}[2\delta(t)-e^{-t}] = 2-\frac{1}{s+1} = \frac{2s+1}{s+1}$; $Re[s]>-1$

5.2

(a) 由图可得 $f(t)=\varepsilon(t)+\varepsilon(t-1)-\varepsilon(t-2)-\varepsilon(t-3)$

由 $\mathcal{L}[\varepsilon(t)]=\frac{1}{s}$, $Re[s]>0$

~~由~~ 则 $\mathcal{L}[f(t)]=F(s)=\frac{1}{s}+\frac{1}{s}e^{-s}-\frac{1}{s}e^{-2s}-\frac{1}{s}e^{-3s}$

$= \frac{(1+e^{-s})(1-e^{-2s})}{s}$, $Re[s]>0$

(f) $f(t)=\sin(\pi t)[\varepsilon(t)-\varepsilon(t-1)]$

$=\sin(\pi t)\varepsilon(t)+\sin[\pi(t-1)]\varepsilon(t-1)$

~~由~~ $\mathcal{L}[\sin(\pi t)\varepsilon(t)]=\frac{\pi}{s^2+\pi^2}$

$\mathcal{L}[f(t)]=\frac{\pi}{s^2+\pi^2}+\frac{\pi}{s^2+\pi^2}e^{-s} = \frac{\pi(1+e^{-s})}{s^2+\pi^2}$, $Re[s]>0$

5.3

(1) $F(s)=\frac{1}{s+1}-\frac{1}{s+1}e^{-2s}=\frac{1-e^{-2s}}{s+1}$

~~(2) 由 $e^{-t}\varepsilon(t-2)=e^{-2}e^{-(t-2)}\varepsilon(t-2)$~~

(3) 由 $\sin(\pi t)[\varepsilon(t)-\varepsilon(t-1)]=\sin(\pi t)\varepsilon(t)+\sin[\pi(t-1)]\varepsilon(t-1)$

得 $F(s)=\frac{\pi}{s^2+\pi^2}(1+e^{-s})=\frac{\pi(1+e^{-s})}{s^2+\pi^2}$

(5) 由 $\delta(4t-2)=\frac{1}{4}\delta(t-\frac{1}{2})$

得 $F(s)=\frac{1}{4}e^{-\frac{s}{2}}$

(7) 由于 $\sin(2t-\frac{\pi}{4})\varepsilon(t)=\sin(2t)\cos\frac{\pi}{4}\varepsilon(t)-\cos(2t)\sin\frac{\pi}{4}\varepsilon(t)$

得 $F(s)=\frac{2}{s^2+4}\cdot\frac{\sqrt{2}}{2}-\frac{s}{s^2+4}\cdot\frac{\sqrt{2}}{2}=\frac{2-s}{\sqrt{2}(s^2+4)}$

(9) 已知 $\mathcal{L}[\sin(\pi t)\varepsilon(t)]=\frac{\pi}{s^2+\pi^2}$

则 $\mathcal{L}[\int_{0_-}^{t}\sin(\pi x)\varepsilon(x)dx]=\mathcal{L}[\int_{0_-}^{t}\sin(\pi x)dx]=\frac{1}{s}\cdot\frac{\pi}{s^2+\pi^2}$

$=\frac{\pi}{s(s^2+\pi^2)}$

(11) $\mathcal{L}[\frac{d^2}{dt^2}[\sin(\pi t)\varepsilon(t)]]=s^2\cdot\frac{\pi}{s^2+\pi^2}=\frac{s^2\pi}{s^2+\pi^2}$

(13) 已知 $\mathcal{L}[e^{-2t}\varepsilon(t)]=\frac{1}{s+2}$

$\mathcal{L}[(-t)^2e^{-2t}\varepsilon(t)]=\frac{d^2}{dt^2}(\frac{1}{s+2})$

即 $\mathcal{L}[t^2e^{-2t}\varepsilon(t)]=\frac{2}{(s+2)^3}$

…函数的初值 $f(0_+)$ 和终值 $f(\infty)$。

(15) $f(t) = te^{-(t-3)}\varepsilon(t-1) = te^{2}\cdot e^{-(t-1)}\varepsilon(t-1)$

由 $\mathcal{L}[e^{-t}\varepsilon(t)] = \frac{1}{s+1}$ 得 $\mathcal{L}[\varepsilon^{-(t-1)}\varepsilon(t-1)] = \frac{e^{-s}}{s+1}$

则 $\mathcal{L}[te^{-(t-1)}\varepsilon(t-1)] = -\frac{d}{ds}\left(\frac{e^{-s}}{s+1}\right) = \frac{(s+2)e^{-s}}{(s+1)^2}$

则 $F(s) = \frac{(s+2)e^{-s}}{(s+1)^2}e^{2} = \frac{s+2}{(s+1)^2}e^{-(s-2)}$

5.4

(2) 由 $f(t) \longleftrightarrow F(s)$ 得，$f(t-1) \longleftrightarrow F(s)e^{-s}$

得 $f(2t-1) \longleftrightarrow \frac{1}{2}F(\frac{s}{2})e^{-\frac{s}{2}}$

故 $e^{-3t}f(2t-1) \longleftrightarrow \frac{1}{2}F[\frac{1}{2}(s+3)]e^{-\frac{1}{2}(s+3)}$

象函数 $Y(s) = \frac{1}{2}\cdot\frac{1}{[\frac{1}{2}(s+3)]^2 - \frac{1}{2}(s+3)+1}e^{-\frac{1}{2}(s+3)} = \frac{2e^{-\frac{1}{2}(s+3)}}{s^2+4s+7}$

(4) 已知 $f(t) \longleftrightarrow F(s)$

则 $f(2t) \longleftrightarrow \frac{1}{2}F(\frac{s}{2})$，又 $f[2(t-\frac{1}{2})] \longleftrightarrow \frac{1}{2}F(\frac{s}{2})e^{-\frac{s}{2}}$

s域中，有 $tf(2t-1) \longleftrightarrow -\frac{1}{2}\frac{d}{ds}[F(\frac{s}{2})e^{-\frac{s}{2}}]$

则

$$Y(s) = -\frac{1}{2}\frac{d}{ds}\left\{\frac{1}{(\frac{s}{2})^2-\frac{s}{2}+1}e^{-\frac{s}{2}}\right\} = -\frac{1}{2}\frac{d}{ds}\left\{\frac{4}{s^2-2s+4}e^{-\frac{s}{2}}\right\}$$

$$= \cancel{\frac{s^2+2s}{s^2+}}\ \frac{s^2+2s}{(s^2-2s+4)^2}e^{-\frac{s}{2}}$$