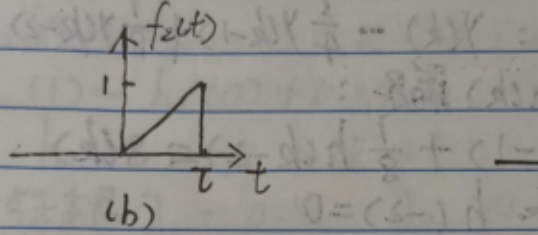
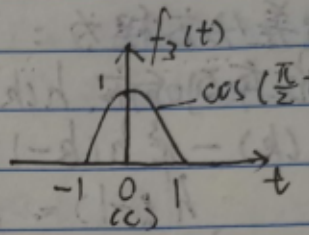
4.13



(b)



(c)

b：如图：

$$f_2(t)=\begin{cases}\frac{1}{\tau}t, & 0<t<\tau\\ 0, & \text{其他}\end{cases}$$

则 $F_2(j\omega)=\int_{-\infty}^{\infty} f_2(t)e^{-j\omega t}dt=\int_0^{\tau}\frac{1}{\tau}te^{-j\omega t}$

$$=\frac{e^{-j\omega\tau}}{-j\omega}-\frac{e^{-j\omega\tau}}{j\omega}-\frac{1}{j\omega\tau}\cdot\frac{e^{-j\omega\tau}-1}{j\omega}=\frac{1-e^{-j\omega\tau}-j\omega\tau e^{-j\omega\tau}}{-\omega^2\tau}$$

c：如图：

$$f_3(t)=\begin{cases}\cos(\frac{\pi}{2}t), & -1<t<1\\ 0, & \text{其他}\end{cases}$$

则 $F_3(j\omega)=\int_{-\infty}^{\infty} f_3(t)e^{-j\omega t}dt=\int_{-1}^{1}\cos(\frac{\pi}{2}t)e^{-j\omega t}dt$

$$= \int_{-1}^{1} \frac{1}{2}\left(e^{j\frac{\pi}{2}t} + e^{-j\frac{\pi}{2}t}\right)e^{-j\omega t}\,dt = \int_{-1}^{1}\frac{1}{2}\left[e^{j(\frac{\pi}{2}-\omega)t} + e^{-j(\frac{\pi}{2}+\omega)t}\right]dt$$

$$= \frac{\sin\left(\frac{\pi}{2}-\omega\right)}{\frac{\pi}{2}-\omega} + \frac{\sin\left(\frac{\pi}{2}+\omega\right)}{\frac{\pi}{2}+\omega} = \frac{\pi\cos\omega}{\left(\frac{\pi}{2}\right)^2-\omega^2}$$

4-18. (1) $\delta(t) \longleftrightarrow 1$

$\delta(t-2) \longleftrightarrow e^{-j2\omega}$

则 $f(t) = e^{-jt}\delta(t-2) \longleftrightarrow e^{-j2(\omega+1)}$

(2) $\varepsilon(t) \longleftrightarrow \pi\delta(\omega) + \frac{1}{j\omega}$

$\varepsilon(t-1) \longleftrightarrow \left[\pi\delta(\omega) + \frac{1}{j\omega}\right]e^{-j\omega} = \pi\delta(\omega) + \frac{e^{-j\omega}}{j\omega}$

则 $f(t) = \varepsilon\left(\frac{1}{2}t-1\right) \longleftrightarrow 2\left[\pi\delta(2\omega) + \frac{1}{j2\omega}e^{-j2\omega}\right]$

$= \pi\delta(\omega) + \frac{1}{j\omega}e^{-j2\omega}$

4.20 (1) $f(2t) \longleftrightarrow \frac{1}{2}\cdot F\left(j\frac{\omega}{2}\right)$

$(-jt)f(2t) \longleftrightarrow \frac{1}{2}\frac{d}{d\omega}F\left(j\frac{\omega}{2}\right)$

则有：$tf(t) \longleftrightarrow j\frac{1}{2}\frac{d}{d\omega}F\left(j\frac{\omega}{2}\right)$

(4) $f(t) \longleftrightarrow F(j\omega)$

$f(t+1) \longleftrightarrow e^{j\omega} F(j\omega)$

$\therefore f(1-t) \longleftrightarrow e^{-j\omega} F(-j\omega)$

(5) $tf(t) \longleftrightarrow j\frac{d}{d\omega} F(j\omega)$

$(t+1)f(t+1) \longleftrightarrow je^{j\omega}\frac{d}{d\omega} F(j\omega)$

$(-t+1)f(-t+1) \longleftrightarrow je^{-j\omega}\frac{d}{d(-\omega)} F(-j\omega)$

即 $(1-t)f(1-t) \longleftrightarrow -je^{-j\omega}\frac{d}{d\omega} F(-j\omega)$

4.24 (1) $F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt = \int_{-1}^{1} \cos(\frac{\pi}{2}t)e^{-j\omega t}dt$

$= \frac{1}{2}\int_{-1}^{1}(e^{j\frac{\pi}{2}t} + e^{-j\frac{\pi}{2}t})e^{-j\omega t}dt$

$= \frac{\pi\cos\omega}{(\frac{\pi}{2})^2 - \omega^2}$

(2) $f'(t) = -\frac{\pi}{2}\sin(\frac{\pi}{2}t)g_2(t)$

$f''(t) = -\frac{\pi}{2}\sin(\frac{\pi}{2}t)[\delta(t+1) + \delta(t-1)] - (\frac{\pi}{2})^2\cos(\frac{\pi}{2}t)g_2(t)$

故得: $f''(t) = \frac{\pi}{2}\delta(t+1) - \frac{\pi}{2}\delta(t-1) - (\frac{\pi}{2})^2 f(t)$

傅里叶变换: $(j\omega)^2 F(j\omega) = \frac{\pi}{2}e^{j\omega} - \frac{\pi}{2}e^{-j\omega} - (\frac{\pi}{2})^2 F(j\omega)$

解得: $F(j\omega) = \frac{\pi\cos\omega}{(\frac{\pi}{2})^2 - \omega^2}$

(3) 由 $f(t)$ 波形，得：

$$f(t)=\cos\left(\frac{\pi}{2}t\right)g_2(t)=0.5\left(e^{j\frac{\pi}{2}t}+e^{-j\frac{\pi}{2}t}\right)g_2(t)$$

又 $g_2(t)\longleftrightarrow 2Sa(\omega)$.

得：$f(t)\longleftrightarrow Sa\left(\omega-\frac{\pi}{2}\right)+Sa\left(\omega+\frac{\pi}{2}\right)=\frac{\pi\cos\omega}{\left(\frac{\pi}{2}\right)^2-\omega^2}$