4.13.

(b) $F(j\omega) = \int_0^{\tau} \frac{1}{\tau} t e^{-j\omega t} dt$

$= \frac{1}{\tau}\int_0^{\tau} t e^{-j\omega t} dt = \frac{j}{\omega\tau}\int_0^{\tau} t\, de^{-j\omega t}$

$= \frac{j}{\omega\tau}\left(te^{-j\omega t} + \frac{e^{-j\omega t}}{j\omega}\right)\Big|_0^{\tau}$

$= \frac{j}{\omega\tau}\left(\tau e^{-j\omega\tau} + \frac{e^{-j\omega\tau}}{j\omega} - \frac{1}{j\omega}\right)$

$= \frac{j\omega\tau e^{-j\omega\tau} + e^{-j\omega\tau} - 1}{\omega^2\tau}$

(c) $F(j\omega) = \int_{-1}^{1} \cos(\frac{\pi}{2}t) e^{-j\omega t} dt$

$= \frac{1}{2}\int_{-1}^{1} (e^{\frac{\pi}{2}tj} + e^{-\frac{\pi}{2}tj}) e^{-j\omega t} dt$

$= \frac{1}{2}\int_{-1}^{1} e^{jt(\frac{\pi}{2}-\omega)} + e^{-jt(\frac{\pi}{2}+\omega)} dt$

$= \frac{1}{2}\left[\frac{e^{jt(\frac{\pi}{2}-\omega)}}{j(\frac{\pi}{2}-\omega)} - \frac{e^{-jt(\frac{\pi}{2}-\omega)}}{j(\frac{\pi}{2}+\omega)}\right]\Big|_{-1}^{1}$

$= \frac{1}{2}\left[\frac{e^{j(\frac{\pi}{2}-\omega)}}{j(\frac{\pi}{2}-\omega)} - \frac{e^{-j(\frac{\pi}{2}-\omega)}}{j(\frac{\pi}{2}-\omega)} + \frac{e^{j(\frac{\pi}{2}+\omega)}}{j(\frac{\pi}{2}+\omega)} - \frac{e^{-j(\frac{\pi}{2}+\omega)}}{j(\frac{\pi}{2}+\omega)}\right]$

$= \frac{1}{2}\left[\frac{2\sin(\frac{\pi}{2}-\omega)}{\frac{\pi}{2}-\omega} + \frac{2\sin(\frac{\pi}{2}+\omega)}{\frac{\pi}{2}+\omega}\right]$

$= \frac{\cos\omega}{\frac{\pi}{2}-\omega} + \frac{\cos\omega}{\frac{\pi}{2}+\omega}$

$= \frac{\pi\cos\omega}{\frac{\pi^2}{4}-\omega^2}$

4.18 (1) $f(t)=e^{-jt}\delta(t-2) = \int_{-\infty}^{\infty} e^{-jt}\delta(t-2)e^{-j\omega t}dt$

$= \int_{-\infty}^{\infty} e^{-2j} e^{-2j\omega}\delta(t-2)dt$

$= e^{-2j(1+\omega)}$

(5) $f(t)=\varepsilon(\frac{1}{2}t-1)$

$\because \varepsilon(t) \longleftrightarrow \pi\delta(\omega)+\frac{1}{j\omega}$

$\therefore \varepsilon(t-1) \longleftrightarrow e^{-j\omega}[\pi\delta(\omega)+\frac{1}{j\omega}] = \pi\delta(\omega)+e^{-j\omega}\frac{1}{j\omega}$

$\therefore \varepsilon(\frac{1}{2}t-1) \longleftrightarrow 2[\pi\delta(2\omega)+e^{-2j\omega}\frac{1}{2j\omega}]$

$= \pi\delta(\omega)+e^{-2j\omega}\frac{1}{j\omega}$

4.20 若已知 $f(t) \longleftrightarrow F(j\omega)$，试求下列函数的频谱.

(1) $tf(2t)$

$\because f(t) \longleftrightarrow F(j\omega)$ $\therefore f(2t) \longleftrightarrow \frac{1}{2}F(j\frac{\omega}{2})$

$\therefore -jtf(2t) \longleftrightarrow \frac{1}{2}\frac{dF(j\frac{\omega}{2})}{d\omega}$

$\therefore tf(2t) \longleftrightarrow \frac{j\,dF(j\frac{\omega}{2})}{2\,d\omega}$

(4) $f(1-t)$ $\because f(t) \longleftrightarrow F(j\omega)$ $\therefore f(t+1) \longleftrightarrow e^{j\omega}F(j\omega)$

$\therefore f(1-t) \longleftrightarrow e^{-j\omega}F(-j\omega)$

(5) $(1-t)f(1-t) = f(1-t) - tf(1-t)$

$\because f(1-t) \longleftrightarrow e^{-j\omega}F(-j\omega)$ $\therefore -jtf(1-t) \longleftrightarrow -je^{-j\omega}F(-j\omega)+e^{-j\omega}\frac{dF(-j\omega)}{d\omega}$

$\therefore -tf(1-t) \longleftrightarrow -e^{-j\omega}F(-j\omega) - j\cdot e^{-j\omega}\frac{dF(-j\omega)}{d\omega}$

$\therefore (1-t)f(1-t) = e^{-j\omega}F(-j\omega) - e^{-j\omega}F(-j\omega) - je^{-j\omega}\frac{dF(-j\omega)}{d\omega}$

$= -je^{-j\omega}\frac{dF(-j\omega)}{d\omega}$

4、24

(1) 利用傅里叶变换定义

$$F(j\omega)=\int_{-1}^{1}\cos(\frac{\pi}{2}t)e^{-j\omega t}dt=\frac{1}{2}\int_{-1}^{1}(e^{\frac{\pi}{2}tj}+e^{-\frac{\pi}{2}tj})e^{-j\omega t}dt$$

$$=\frac{1}{2}\int_{-1}^{1}e^{jt(\frac{\pi}{2}-\omega)}+e^{-jt(\frac{\pi}{2}+\omega)}dt$$

$$=\frac{1}{2}\left[\frac{e^{jt(\frac{\pi}{2}-\omega)}}{j(\frac{\pi}{2}-\omega)}-\frac{e^{-jt(\frac{\pi}{2}+\omega)}}{-j(\frac{\pi}{2}+\omega)}\right]\Big|_{-1}^{1}$$

$$=\frac{1}{2}\left[\frac{e^{j(\frac{\pi}{2}-\omega)}}{j(\frac{\pi}{2}-\omega)}-\frac{e^{-j(\frac{\pi}{2}+\omega)}}{j(\frac{\pi}{2}+\omega)}+\frac{e^{j(\frac{\pi}{2}+\omega)}}{j(\frac{\pi}{2}+\omega)}-\frac{e^{-j(\frac{\pi}{2}+\omega)}}{j(\frac{\pi}{2}+\omega)}\right]$$

$$=\frac{1}{2}\cos\left[\frac{2\sin(\frac{\pi}{2}-\omega)}{\frac{\pi}{2}-\omega}+\frac{2\sin(\frac{\pi}{2}+\omega)}{\frac{\pi}{2}+\omega}\right]$$

$$=\frac{\cos\omega}{\frac{\pi}{2}-\omega}+\frac{\cos\omega}{\frac{\pi}{2}+\omega}=\frac{\pi\cos\omega}{\frac{\pi^2}{4}-\omega^2}$$

(2) 利用微分、积分特性

$\because f(t)=\cos(\frac{\pi}{2}t)g_2(t)$

$\therefore f'(t)=-\frac{\pi}{2}\sin(\frac{\pi}{2}t)g_2(t)$

$\therefore f''(t)=-\frac{\pi^2}{4}\cos(\frac{\pi}{2}t)g_2(t)-\frac{\pi}{2}\sin(\frac{\pi}{2}t)[\delta(t+1)+\delta(t-1)]$

$=-\frac{\pi^2}{4}f(t)+\frac{\pi}{2}\delta(t+1)-\frac{\pi}{2}\delta(t-1)$

对此式两边作傅里叶变换

$\therefore (j\omega)^2F(j\omega)=-(\frac{\pi^2}{4})F(j\omega)+\frac{\pi}{2}e^{j\omega}-\frac{\pi}{2}e^{-j\omega}$

$\therefore F(j\omega)=\frac{\pi\cos\omega}{\frac{\pi^2}{4}-\omega^2}$

(3). $\because f(t) = g_2(t) \times \cos(\frac{\pi}{2}t)$

$\cos(\frac{\pi}{2}t) \longleftrightarrow \pi[\delta(w+\frac{\pi}{2}) + \delta(w-\frac{\pi}{2})]$

$g_2(t) \longleftrightarrow 2\,Sa(w)$

$\therefore\ F(jw) = \frac{1}{2\pi} 2Sa(w) * \pi[\delta(w+\frac{\pi}{2}) + \delta(w-\frac{\pi}{2})]$

$= Sa(w+\frac{\pi}{2}) + Sa(w-\frac{\pi}{2})$

$= \frac{\cos w}{w+\frac{\pi}{2}} - \frac{\cos w}{w-\frac{\pi}{2}} = \frac{\pi \cos w}{\frac{\pi^2}{4} - w^2}$