

4.13.

$$\begin{aligned} \text{(b) } F(j\omega) &= \int_0^T \frac{1}{t} t e^{-j\omega t} dt \\ &= \int_0^T t e^{-j\omega t} dt = \frac{j}{\omega} \int_0^T t e^{-j\omega t} dt \\ &= \frac{j}{\omega} (t e^{-j\omega t} + \frac{e^{-j\omega t}}{\omega}) \Big|_0^T \\ &= \frac{j}{\omega} (T e^{-j\omega T} + \frac{e^{-j\omega T}}{\omega} - \frac{1}{\omega}) \\ &= \frac{j\omega T e^{-j\omega T} + e^{-j\omega T} - 1}{\omega^2} \end{aligned}$$

$$\begin{aligned} \text{(c) } f F(j\omega) &= \int_{-1}^1 \cos\left(\frac{\pi}{2}t\right) e^{-j\omega t} dt \\ &= \frac{1}{2} \int_{-1}^1 (e^{j\frac{\pi}{2}t} + e^{-j\frac{\pi}{2}t}) e^{-j\omega t} dt \\ &= \frac{1}{2} \int_{-1}^1 e^{jt(\frac{\pi}{2}-\omega)} + e^{-jt(\frac{\pi}{2}+\omega)} dt \\ &= \frac{1}{2} \left[\frac{e^{jt(\frac{\pi}{2}-\omega)}}{j(\frac{\pi}{2}-\omega)} - \frac{e^{-jt(\frac{\pi}{2}+\omega)}}{j(\frac{\pi}{2}+\omega)} \right] \Big|_{-1}^1 \\ &= \frac{1}{2} \left[\frac{e^{j(\frac{\pi}{2}-\omega)}}{j(\frac{\pi}{2}-\omega)} - \frac{e^{-j(\frac{\pi}{2}-\omega)}}{j(\frac{\pi}{2}-\omega)} + \frac{e^{j(\frac{\pi}{2}+\omega)}}{j(\frac{\pi}{2}+\omega)} - \frac{e^{-j(\frac{\pi}{2}+\omega)}}{j(\frac{\pi}{2}+\omega)} \right] \\ &= \frac{1}{2} \left[\frac{2 \sin(\frac{\pi}{2}-\omega)}{\frac{\pi}{2}-\omega} + \frac{2 \sin(\frac{\pi}{2}+\omega)}{\frac{\pi}{2}+\omega} \right] \\ &= \frac{\cos \omega}{\frac{\pi}{2}-\omega} + \frac{\cos \omega}{\frac{\pi}{2}+\omega} \\ &= \frac{\pi \cos \omega}{\frac{\pi^2}{4} - \omega^2} \end{aligned}$$

$$4.18 \text{ 1) } f(t) = e^{-jt} \delta(t-2) = \int_{-\infty}^{\infty} e^{-jt} f(t-2) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-2j} e^{-j\omega} f(t-2) dt$$

$$= e^{-2j} e^{-j\omega}$$

$$(5) f(t) = \varepsilon(\frac{1}{2}t-1)$$

$$\because \varepsilon(t) \leftrightarrow \pi \delta(\omega) + \frac{1}{j\omega}$$

$$\therefore \varepsilon(t-1) \leftrightarrow e^{-j\omega} [\pi \delta(\omega) + \frac{1}{j\omega}] = \pi \delta(\omega) + e^{-j\omega} \frac{1}{j\omega}$$

$$\therefore \varepsilon(\frac{1}{2}t-1) \leftrightarrow 2[\pi \delta(2\omega) + e^{-2j\omega} \frac{1}{2j\omega}]$$

$$= \pi \delta(\omega) + e^{-2j\omega} \frac{1}{j\omega}$$

4.20 若已知 $f(t) \leftrightarrow F(j\omega)$, 试求下列函数的频谱.

$$1) t f(2t)$$

$$\because f(t) \leftrightarrow F(j\omega) \quad \therefore f(2t) \leftrightarrow \frac{1}{2} F(j\frac{\omega}{2})$$

$$\therefore -jt f(2t) \leftrightarrow \frac{1}{2} \frac{dF(j\frac{\omega}{2})}{d\omega}$$

$$\therefore t f(2t) \leftrightarrow \frac{j dF(j\frac{\omega}{2})}{2 d\omega}$$

$$4) f(1-t) \quad \because f(t) \leftrightarrow F(j\omega) \quad \therefore f(t+1) \leftrightarrow e^{j\omega} F(j\omega)$$

$$\therefore f(1-t) \leftrightarrow e^{-j\omega} F(-j\omega)$$

$$5) (1-t)f(1-t) = f(1-t) - t f(1-t)$$

$$\because f(1-t) \leftrightarrow e^{-j\omega} F(-j\omega) \quad \therefore -jt f(1-t) \leftrightarrow -j e^{-j\omega} F(-j\omega) + e^{-j\omega} \frac{dF(-j\omega)}{d\omega}$$

$$\therefore -t f(1-t) \leftrightarrow -e^{-j\omega} F(-j\omega) - j e^{-j\omega} \frac{dF(-j\omega)}{d\omega}$$

$$\therefore (1-t)f(1-t) = e^{-j\omega} F(-j\omega) - e^{-j\omega} F(-j\omega) - j e^{-j\omega} \frac{dF(-j\omega)}{d\omega}$$

$$= -j e^{-j\omega} \frac{dF(-j\omega)}{d\omega}$$

4.24

1) 利用傅里叶变换定义

$$\begin{aligned} F(j\omega) &= \int_{-\infty}^{\infty} \cos\left(\frac{\pi}{4}t\right) e^{-j\omega t} dt = \frac{1}{2} \int_{-\infty}^{\infty} (e^{j\frac{\pi}{4}t} + e^{-j\frac{\pi}{4}t}) e^{-j\omega t} dt \\ &= \frac{1}{2} \int_{-\infty}^{\infty} e^{jt(\frac{\pi}{4}-\omega)} + e^{-jt(\frac{\pi}{4}+\omega)} dt \\ &= \frac{1}{2} \left[\frac{e^{jt(\frac{\pi}{4}-\omega)}}{j(\frac{\pi}{4}-\omega)} - \frac{e^{-jt(\frac{\pi}{4}+\omega)}}{j(\frac{\pi}{4}+\omega)} \right] \Big|_{-\infty}^{\infty} \\ &= \frac{1}{2} \left[\frac{e^{j(\frac{\pi}{4}-\omega)} - e^{-j(\frac{\pi}{4}+\omega)}}{j(\frac{\pi}{4}-\omega)} + \frac{e^{j(\frac{\pi}{4}+\omega)} - e^{-j(\frac{\pi}{4}-\omega)}}{j(\frac{\pi}{4}+\omega)} \right] \\ &= \frac{1}{2} \cos \left[\frac{2\sin(\frac{\pi}{4}-\omega)}{\frac{\pi}{4}-\omega} + \frac{2\sin(\frac{\pi}{4}+\omega)}{\frac{\pi}{4}+\omega} \right] \\ &= \frac{\cos\omega}{\frac{\pi}{4}-\omega} + \frac{\cos\omega}{\frac{\pi}{4}+\omega} = \frac{\pi \cos\omega}{\frac{\pi^2}{4} - \omega^2} \end{aligned}$$

2) 利用微分. 积分特性

$$\because f_1(t) = \cos\left(\frac{\pi}{4}t\right) g_2(t)$$

$$\therefore f'(t) = -\frac{\pi}{2} \sin\left(\frac{\pi}{4}t\right) g_2(t)$$

$$\begin{aligned} \therefore f''(t) &= -\frac{\pi^2}{4} \cos\left(\frac{\pi}{4}t\right) g_2(t) - \frac{\pi}{2} \sin\left(\frac{\pi}{4}t\right) [g_2(t+1) + g_2(t-1)] \\ &= -\frac{\pi^2}{4} f(t) + \frac{\pi}{2} g_2(t+1) - \frac{\pi}{2} g_2(t-1) \end{aligned}$$

对式(2)两边作傅里叶变换

$$\therefore (j\omega)^2 F(j\omega) = -\left(\frac{\pi^2}{4}\right) F(j\omega) + \frac{\pi}{2} e^{j\omega} - \frac{\pi}{2} e^{-j\omega}$$

$$\therefore F(j\omega) = \frac{\pi \cos\omega}{\frac{\pi^2}{4} - \omega^2}$$

$$B). \because f(t) = g_2(t) \times \cos\left(\frac{\pi}{2}t\right)$$

$$\cos\left(\frac{\pi}{2}t\right) \leftrightarrow \pi[f(\omega + \frac{\pi}{2}) + f(\omega - \frac{\pi}{2})]$$

$$g_2(t) \leftrightarrow 2\text{Sa}(\omega)$$

$$\therefore F(\omega) = \frac{1}{2\pi} 2\text{Sa}(\omega) * \pi[f(\omega + \frac{\pi}{2}) + f(\omega - \frac{\pi}{2})]$$

$$= \text{Sa}(\omega + \frac{\pi}{2}) + \text{Sa}(\omega - \frac{\pi}{2})$$

$$= \frac{\cos \omega}{\omega + \frac{\pi}{2}} - \frac{\cos \omega}{\omega - \frac{\pi}{2}} = \frac{\pi \cos \omega}{\frac{\pi^2}{4} - \omega^2}$$