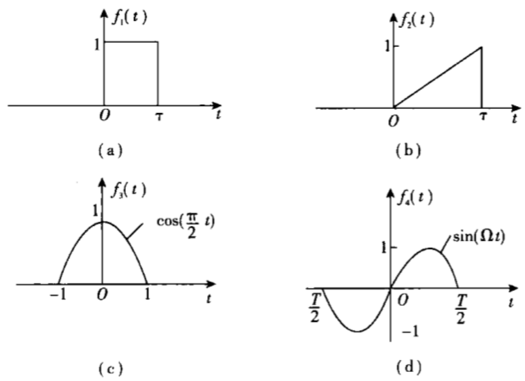
4.13 求题 4.13 图所示各信号的傅里叶变换。



解：(b) $f_2(t)=\begin{cases}\frac{1}{\tau}t, & 0\le t\le \tau\\ 0, & 其他\end{cases}$

$$\therefore F_2(jw)=\int_{-\infty}^{+\infty} f_2(t)e^{-jwt}dt=\int_0^{\tau}\frac{1}{\tau}te^{-jwt}dt=\frac{1-e^{-jw\tau}-jw\tau e^{-jw\tau}}{-w^2\tau}$$

(c) $f_3(t)=\begin{cases}\cos(\frac{\pi}{2}t), & -1\le t\le 1\\ 0, & 其他\end{cases}$

$$\therefore F_3(jw)=\int_{-\infty}^{+\infty} f_3(t)e^{-jwt}dt=\int_{-1}^{1}\cos(\tfrac{\pi}{2}t)e^{-jwt}dt=\int_{-1}^{1}\frac{1}{2}(e^{j\frac{\pi}{2}t}+e^{-j\frac{\pi}{2}t})e^{-jwt}dt$$

$$=\int_{-1}^{1}\frac{1}{2}[e^{j(\frac{\pi}{2}-w)t}+e^{-j(\frac{\pi}{2}+w)t}]dt=\frac{\sin(\frac{\pi}{2}-w)}{\frac{\pi}{2}-w}+\frac{\sin(\frac{\pi}{2}+w)}{\frac{\pi}{2}+w}=\frac{\pi\cos w}{(\frac{\pi}{2})^2-w^2}$$

4.18 求下列信号的傅里叶变换：

(1) $f(t)=e^{-jt}\delta(t-2)$

(2) $f(t)=e^{-3(t-1)}\delta'(t-1)$

(3) $f(t)=\mathrm{sgn}(t^2-9)$

(4) $f(t)=e^{-2t}\varepsilon(t+1)$

(5) $f(t)=\varepsilon\left(\frac{t}{2}-1\right)$

解：(1)

$\delta(t) \xrightarrow{右2} \delta(t-2) \xrightarrow{频移} e^{-jt}\delta(t-2)$

$1 \longrightarrow e^{-jw2} \longrightarrow e^{-j(w+1)2}$

$\therefore F(jw) = e^{-j(w+1)2}$

(5)

$\varepsilon(t) \longrightarrow \varepsilon(t-1) \longrightarrow \varepsilon(\frac{t}{2}-1)$

$\pi\delta(w) + \frac{1}{jw} \longrightarrow [\pi\delta(w) + \frac{1}{jw}]e^{-jw} \longrightarrow 2[\pi\delta(2w) + \frac{1}{j2w}e^{-2jw}]$

$= \pi\delta(w) + \frac{1}{jw}e^{-jw}$ $\qquad = \pi\delta(w) + \frac{1}{j2w}e^{-2jw}$

4.20 若已知 $\mathscr{F}[f(t)] = F(j\omega)$，试求下列函数的频谱：

(1) $tf(2t)$ (2) $(t-2)f(t)$ (3) $t\frac{\mathrm{d}f(t)}{\mathrm{d}t}$

(4) $f(1-t)$ (5) $(1-t)f(1-t)$ (6) $f(2t-5)$

(7) $\int_{-\infty}^{1-\frac{1}{2}t} f(\tau)\mathrm{d}\tau$ (8) $e^{jt}f(3-2t)$ (9) $\frac{\mathrm{d}f(t)}{\mathrm{d}t} * \frac{1}{\pi t}$

解：(1) $f(t) \longrightarrow tf(t) \longrightarrow 2tf(2t)$

$F(jw) \longrightarrow j\frac{dF(jw)}{dw} \longrightarrow \frac{j}{2}\frac{dF(j\frac{w}{2})}{dw}$

$\therefore \mathcal{F}[tf(2t)] = \frac{j}{4}\frac{dF(j\frac{w}{2})}{dw}$

(4) $f(t) \longrightarrow f(-t) \longrightarrow f(1-t)$

$F(jw) \longrightarrow F(-jw) \longrightarrow e^{-jw}F(-jw)$

$\therefore \mathcal{F}[f(1-t)] = e^{-jw}F(-jw)$

(5) $f(t) \to tf(t) \longrightarrow (t+1)f(t+1) \longrightarrow (-t+1)f(-t+1)$

$F(jw) \to j\dfrac{dF(jw)}{dw} \longrightarrow je^{jw}\dfrac{dF(jw)}{dw} \longrightarrow je^{-jw}\dfrac{d(-jw)}{d(-w)}$

$= -je^{-jw}\dfrac{d(-jw)}{dw}$

$\therefore g[(1-t)f(1-t)] = -je^{-jw}\dfrac{dF(-jw)}{dw}$

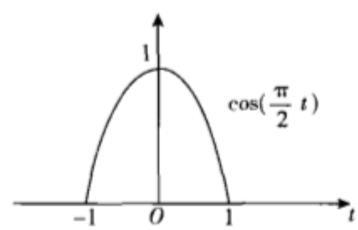
$\cos w_0 t = \frac{1}{2}[e^{jw_0t} + e^{-jw_0t}]$

4.24 试用下列方法求题 4.24 图所示余弦脉冲的频谱函数。

（1）利用傅里叶变换定义。

（2）利用微分、积分特性。

（3）将它看做门函数 $g_2(t)$ 与周期余弦函数 $\cos\left(\frac{\pi t}{2}\right)$ 的乘积。



题 4.24 图

解：$f(t) = \cos(\frac{\pi}{2}t)$

$F(jw) = \int_{-\infty}^{+\infty} f(t)e^{-jwt}dt = \int_{-1}^{1}\cos(\frac{\pi}{2}t)e^{-jwt}dt$

$= \int_{-1}^{1}(e^{j\frac{\pi}{2}t} + e^{-j\frac{\pi}{2}t})e^{-jwt}\,dt$

$= \dfrac{2\sin(\frac{\pi}{2}-w)}{\frac{\pi}{2}-w} + \dfrac{2\sin(\frac{\pi}{2}+w)}{\frac{\pi}{2}+w}$

$= \dfrac{\pi\cos w}{(\frac{\pi}{2})^2 - w^2}$

(2) $f(t)=\cos(\frac{\pi}{2}t)[\varepsilon(t+1)-\varepsilon(t-1)]$

$\therefore f'(t)=-\frac{\pi}{2}\sin(\frac{\pi}{2}t)[\varepsilon(t+1)-\varepsilon(t-1)]+\cos(\frac{\pi}{2}t)[\delta(t+1)-\delta(t-1)]$

$=-\frac{\pi}{2}\sin(\frac{\pi}{2}t)[\varepsilon(t+1)-\varepsilon(t-1)]$

$\because \varepsilon(t+1)-\varepsilon(t-1)=g_2(t)\longleftrightarrow 2Sa(w)=\frac{2\sin w}{w}$

$\sin(\frac{\pi}{2}t)\longleftrightarrow j\pi[\delta(w+\frac{\pi}{2})-\delta(w-\frac{\pi}{2})]$

$\therefore f'(t)$的频谱函数为:

$F_1(jw)=\frac{1}{2\pi}[-\frac{\pi}{2}jw[\delta(w+\frac{\pi}{2})-\delta(w-\frac{\pi}{2})]*\frac{2\sin w}{w}]$

$=-j\frac{\pi}{2}[\frac{\sin(w+\frac{\pi}{2})}{w+\frac{\pi}{2}}-\frac{\sin(w-\frac{\pi}{2})}{w-\frac{\pi}{2}}]$

$F(jw)=\frac{F_1(jw)}{jw}=\frac{1}{jw}\cdot(-j)\frac{\pi}{2}[\frac{\sin(w+\frac{\pi}{2})}{w+\frac{\pi}{2}}-\frac{\sin(\frac{\pi}{2}-w)}{\frac{\pi}{2}-w}]=\frac{\pi\cos w}{(\frac{\pi}{2})^2-w^2}$

(3) 由$f(t)$的波形可知 $f(t)=\cos(\frac{\pi}{2}t)g_2(t)$

$\because \cos(\frac{\pi}{2}t)\longleftrightarrow\pi[\delta(w+\frac{\pi}{2})+\delta(w-\frac{\pi}{2})]$

$g_2(t)\longleftrightarrow 2Sa(w)$

$F(jw)=\frac{1}{2\pi}\{\pi[\delta(w+\frac{\pi}{2})+\delta(w-\frac{\pi}{2})]*2Sa(w)\}=Sa(w+\frac{\pi}{2})+Sa(w-\frac{\pi}{2})$