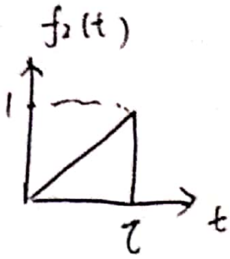
4.13

(b)



$$f_2(t)=\begin{cases}\frac{1}{\tau}t, & 0<t<\tau \\ 0, & 其它\end{cases}$$

根据傅立叶变换定义:

$$F(jw)=\int_{-\infty}^{\infty} f(t)e^{-jwt}dt$$

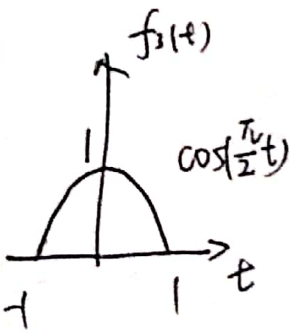
$$=\int_0^{\tau}\frac{1}{\tau}t\,e^{-jwt}dt$$

$$=\frac{1}{\tau}\int_0^{\tau} te^{-jwt}dt=\frac{1}{\tau}\left[\int_0^{\tau} t\cdot de^{-jwt}\right]\cdot\left(\frac{1}{-jw}\right)$$

$$=-\frac{1}{jw\tau}\left[e^{-jwt}t-\int_0^{\tau}\frac{1}{-jw}e^{-jwt}\Big|_0^{\tau}\right]$$

$$=\frac{e^{-jwt}}{-jw}-\frac{1}{jw\tau}\cdot\frac{e^{-jw\tau}-1}{jw}$$

(C)



$$f_3(t)=\begin{cases}\cos\frac{\pi}{2}t, & -1<t<1 \\ 0, & 其它\end{cases}$$

根据傅立叶变换定义:

$$F(jw)=\int_{-\infty}^{\infty} f(t)e^{-jwt}dt=\int_{-1}^{1}\cos\frac{\pi}{2}t\,e^{-jwt}dt$$

$$=\int_{-1}^{1}\frac{1}{2}\left(e^{j\frac{\pi}{2}t}+e^{j(-\frac{\pi}{2}t)}\right)\cdot e^{-jwt}dt$$

$$=\int_{-1}^{1}\frac{1}{2}\left[e^{j(\frac{\pi}{2}-w)t}+e^{-j(\frac{\pi}{2}+w)t}\right]dt$$

$$=\frac{\sin(\frac{\pi}{2}-w)}{\frac{\pi}{2}-w}+\frac{\sin(\frac{\pi}{2}+w)}{\frac{\pi}{2}+w}=\frac{\pi\cos w}{(\frac{\pi}{2})^2-w^2}$$

~~4.18.~~

~~(1) $f(t)=e^{-jt}\delta(t-2)$~~

~~由傅里叶变换定义:~~

~~由基本变换对: $\delta(t)\leftrightarrow 1$ 可得~~

4.18

(1) $f(t)=e^{-jt}\delta(t-2)$

变换对 $\delta(t)\leftrightarrow 1$

由时移性质 $\delta(t-2)\leftrightarrow e^{-j2w}$

由频移性质: $f(t)=e^{-jt}\delta(t-2)\leftrightarrow e^{-j2(w+1)}$

(5) $f(t)=\varepsilon(\frac{1}{2}t-1)$

变换对: $\varepsilon(t)\leftrightarrow \pi\delta(w)+\frac{1}{jw}$

由时移性质: $\varepsilon(t-1)\leftrightarrow [\pi\delta(w)+\frac{1}{jw}]e^{-jw}=\pi\delta(w)+\frac{e^{-jw}}{jw}$

由尺度变换性质可得: $f(t)=\varepsilon(\frac{1}{2}t-1)\leftrightarrow 2e^{-j2w}[\pi\delta(2w)+\frac{1}{j2w}]$

$$=\frac{e^{-j2w}}{jw}+\pi\delta(w)$$

4.20

(1) $tf(2t)$

① 尺度变换 $f(2t)\leftrightarrow \frac{1}{2}F(j\frac{w}{2})$

② 频域微分特性: $(-jt)f(2t)\leftrightarrow \frac{1}{2}\frac{d}{dw}F(j\frac{w}{2})$

$\therefore\ tf(2t)\leftrightarrow j\frac{1}{2}\frac{d}{dw}F(j\frac{w}{2})$

(4) $f(1-t)$

① 根据时移特性 $f(t+1) \leftrightarrow e^{jw}F(jw)$

② 根据反转特性，$f(-t+1) \leftrightarrow e^{-jw}F(-jw)$

(5) $(1-t)f(1-t)$

① 由频域微分性质：$tf(t) \leftrightarrow j\frac{d}{dw}F(jw)$

② 反转特性：$-tf(-t) \leftrightarrow -j\frac{d}{dw}F(-jw)$

时移特性，$(1-t)f(1-t) \leftrightarrow -je^{-jw}\frac{d}{dw}F(-jw)$

~~4.20~~

4.24.

(1) 由傅里叶变换定义

$$F(jw) = \int_{-\infty}^{\infty} f(t)e^{-jwt}dt, \quad f(t) = \begin{cases} \cos\frac{\pi}{2}t, & -1\le t\le 1 \\ 0, & 其它 \end{cases}$$

$$= \int_{-1}^{1}\cos\frac{\pi}{2}t\, e^{-jwt}dt$$

$$= \frac{1}{2}\int_{-1}^{1}\left[e^{j(\frac{\pi}{2}-w)t}+e^{-j(\frac{\pi}{2}+w)t}\right]dt$$

$$= \frac{1}{2}\left[\frac{e^{j(\frac{\pi}{2}-w)t}}{j(\frac{\pi}{2}-w)}+\frac{e^{-j(\frac{\pi}{2}+w)t}}{-j(\frac{\pi}{2}+w)}\right]\Bigg|_{-1}^{1}$$

$$= \cancel{\sin}\,\frac{\pi\cos w}{\frac{\pi^2}{4}-w^2}$$

(2) ~~对f(t)求导~~

$$f'(t) = -\frac{\pi}{2}\sin(\frac{\pi}{2}t)g_2(t)$$

$$f''(t) = -\frac{\pi}{2}\sin(\frac{\pi}{2}t)[\delta(t+1)+\delta(t-1)] - (\frac{\pi}{2})^2\cos(\frac{\pi}{2}t)g_2(t)$$

$$= \frac{\pi}{2}\delta(t+1) - \frac{\pi}{2}\delta(t-1) - (\frac{\pi}{2})^2 f(t)$$

根据时域微分特性：

$$(jw)^2F(jw) = \frac{\pi}{2}e^{jw} - \frac{\pi}{2}e^{-jw} - (\frac{\pi}{2})^2F(jw)$$

$$\therefore F(jw) = \frac{\pi\cos w}{(\frac{\pi}{2})^2-w^2}$$

(3) 由 $f(t) = \cos(\frac{\pi}{2}t) g_2(t)$

由: $g_2(t) \leftrightarrow 2Sa(w)$.

$\cos(\frac{\pi}{2}t) \leftrightarrow \pi[\delta(w+\frac{\pi}{2})+\delta(w-\frac{\pi}{2})]$

频域卷积定理: $F(jw) = Sa(w-\frac{\pi}{2}) + Sa(w+\frac{\pi}{2}) = \frac{\pi \cos w}{(\frac{\pi}{2})^2 - w^2}$