

4.1 傅里叶变换 $f(t) \leftrightarrow 1$

时移特性: $f(t-2) \leftrightarrow e^{-j2\omega}$

频移特性: $f(t) = e^{-jt} g(t-1) \leftrightarrow e^{-j2(\omega+1)}$

(5) $\varepsilon(t) \leftrightarrow \pi \delta(\omega) + \frac{1}{j\omega}$

时移特性: $\varepsilon(t-1) \leftrightarrow [\pi \delta(\omega) + \frac{1}{j\omega}] e^{-j\omega} = \pi \delta(\omega) \frac{e^{-j\omega}}{j\omega}$

尺度变换: $f(t) = \varepsilon(\frac{1}{2}t-1) \leftrightarrow 2 [\pi \delta(\omega) + \frac{1}{j\omega} e^{-j2\omega}] = \pi \delta(\omega) + \frac{e^{-2j\omega}}{j\omega}$

4.20 (1) 由尺度变换 $f(2t) \leftrightarrow \frac{1}{2} F(j\frac{\omega}{2})$

时域微分特性: $(-jt) f(2t) \leftrightarrow \frac{1}{2} \frac{d}{d\omega} F(j\frac{\omega}{2})$

$\therefore t f(2t) \leftrightarrow j \frac{1}{2} \frac{d}{d\omega} F(j\frac{\omega}{2})$

(4) 根据傅里叶变换的时移性质 $f(t+1) \leftrightarrow F(j\omega) e^{j\omega}$
由反转特性 $f(-t) \leftrightarrow F(-j\omega) e^{-j\omega}$

(5) 由微分特性: $t f(t) \leftrightarrow j \frac{d}{d\omega} F(j\omega)$

微分特性: $t f(-t) \leftrightarrow -j \frac{d}{d\omega} F(-j\omega)$

由时移特性 $(1-t) f(1-t) \leftrightarrow -j e^{-j\omega} \frac{d}{d\omega} F(-j\omega)$

4.24 (1) $F(j\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt = \frac{\pi \cos \omega}{\frac{\pi^2}{4} - \omega^2}$

(2) $f'(t) = -\frac{\pi}{2} \sin(\frac{\pi}{2}t) g_2(t)$, 则有

$f''(t) = \frac{\pi}{2} \delta(t+1) - \frac{\pi}{2} \delta(t-1) - (\frac{\pi}{2})^2 f(t)$

由微分特性 $(j\omega)^2 F(j\omega) = \frac{\pi}{2} e^{j\omega} - \frac{\pi}{2} e^{-j\omega} - (\frac{\pi}{2})^2 F(j\omega)$

$\therefore F(j\omega) = \frac{\pi \cos \omega}{(\frac{\pi}{2})^2 - \omega^2}$

(3) 由 $f(t)$ 的级数展开: $f(t) = \cos(\frac{\pi}{2}t) g_2(t)$

由 $g_2(t) \leftrightarrow 2 \text{Sa}(\omega)$, $\cos(\frac{\pi}{2}t) \leftrightarrow \pi [\delta(\omega + \frac{\pi}{2}) + \delta(\omega - \frac{\pi}{2})]$

$\therefore F(j\omega) = \text{Sa}(\omega - \frac{\pi}{2}) + \text{Sa}(\omega + \frac{\pi}{2}) = \frac{\pi \cos \omega}{(\frac{\pi}{2})^2 - \omega^2}$

