4.13 (b) 由图可知，$f_2(t) = \begin{cases} \frac{t}{\tau}, & 0<t<\tau \\ 0, & 其他 \end{cases}$

$$F_2(j\omega) = \int_{-\infty}^{\infty} f_2(t) e^{-j\omega t} dt = \int_0^{\tau} \frac{1}{\tau} t e^{-j\omega t} dt$$

$$= -\frac{e^{-j\omega\tau}}{j\omega} - \frac{1}{j\omega\tau} \cdot \frac{e^{-j\omega\tau}-1}{j\omega}$$

$$= \frac{1-e^{-j\omega\tau} - j\omega\tau e^{-j\omega\tau}}{-\omega^2\tau}$$

(c) 由图可知，$f_3(t) = \begin{cases} \cos(\frac{\pi}{2}t), & -1<t<1 \\ 0, & 其他 \end{cases}$

$$F_3(j\omega) = \int_{-\infty}^{\infty} f_3(t) e^{-j\omega t} dt = \int_{-1}^{1} \cos(\frac{\pi}{2}t) e^{-j\omega t} dt$$

$$= \int_{-1}^{1} \frac{1}{2}[e^{j(\frac{\pi}{2}-\omega)t} + e^{-j(\frac{\pi}{2}+\omega)t}] dt$$

$$= \frac{\pi\cos\omega}{(\frac{\pi}{2})^2 - \omega^2}$$

4.18 (1) $f(t)=e^{-jt}\delta(t-2)$

(5) $f(t)=\varepsilon(\frac{1}{2}t-1)$

解：(1) 基本信号$\delta(t)$的傅里叶变换为 $\delta(t)\leftrightarrow 1$

根据时移性质可得：$\delta(t-2)\leftrightarrow e^{-j2\omega}$

根据频移性质可得：$f(t)=e^{-jt}\delta(t-2)$

$\leftrightarrow e^{-j2(\omega+1)}$

(5) 常见信号$\varepsilon(t)$的傅里叶变换为：$\varepsilon(t)\leftrightarrow \pi\delta(\omega)+\frac{1}{j\omega}$

根据时移性质可得 $\varepsilon(t-1)\leftrightarrow [\pi\delta(\omega)+\frac{1}{j\omega}]e^{-j\omega}$

$=\pi\delta(\omega)+\frac{e^{-j\omega}}{j\omega}$

根据尺度变换性质可得：$f(t)=\varepsilon(\frac{1}{2}t-1)\leftrightarrow$

$2[\pi\delta(2\omega)+\frac{1}{j2\omega}e^{-j2\omega}]=\pi\delta(\omega)+\frac{e^{-j2\omega}}{j\omega}$

已知 $f(t) \leftrightarrow F(j\omega)$，试求频谱

4.20 (1) $tf(2t)$ (4) $f(1-t)$ (5) $(1-t)f(1-t)$

解：(1) 根据尺度变换性质可得：$f(2t) \leftrightarrow \frac{1}{2}F(j\frac{\omega}{2})$

根据频域微分特性可得：$(-jt)f(2t) \leftrightarrow \frac{1}{2}\frac{dF(j\frac{\omega}{2})}{d\omega}$

所以 $tf(2t) \leftrightarrow j\frac{1}{2}\frac{d}{d\omega}F(j\frac{\omega}{2})$

(4) 根据傅里叶变换时移性质可得

$f(1+t) \leftrightarrow F(j\omega)e^{j\omega}$

根据反转特性可得：$f(1-t) \leftrightarrow F(-j\omega)e^{-j\omega}$

(5) 根据频域微分性质可得：$tf(t) \leftrightarrow j\frac{d}{d\omega}F(j\omega)$

根据反转性质可得 $-tf(-t) \leftrightarrow -j\frac{d}{d\omega}F(-j\omega)$

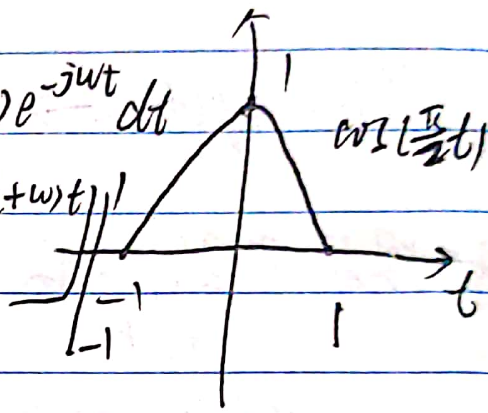
根据时移性质可得：$(1-t)f(1-t) \leftrightarrow -je^{-j\omega}\frac{d}{d\omega}(-j\omega)$

4.24、试用下列方法求所示余弦脉冲的频谱函数。

(1) 利用傅里叶变换定义

(2) 利用微分、积分特性。

(3) 将它看作门函数 $g_2(t)$ 与周期余弦函数 $\cos(\frac{\pi}{2}t)$ 的乘积。



(1) $F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt = \int_{-1}^{1}\cos(\frac{\pi}{2}t)e^{-j\omega t}dt$

$= \frac{1}{2}\left[\frac{1}{j(\frac{\pi}{2}-\omega)}e^{j(\frac{\pi}{2}-\omega)t} + \frac{1}{-j(\frac{\pi}{2}+\omega)}e^{-j(\frac{\pi}{2}+\omega)t}\right]\Big|_{-1}^{1}$

$= \frac{\sin(\frac{\pi}{2}-\omega)}{\frac{\pi}{2}-\omega} + \frac{\sin(\frac{\pi}{2}+\omega)}{\frac{\pi}{2}+\omega} = \frac{\pi\cos\omega}{\frac{\pi^2}{4}-\omega^2}$

(2) 对 $f(t)$ 求导：$f'(t) = -\frac{\pi}{2}\sin(\frac{\pi}{2}t)g_2(t)$

再求导，$f''(t) = -\frac{\pi}{2}\sin(\frac{\pi}{2}t)[\delta(t+1)+\delta(t-1)] - (\frac{\pi}{2})^2\cos(\frac{\pi}{2}t)g_2(t)$

$= \frac{\pi}{2}\delta(t+1) - \frac{\pi}{2}\delta(t-1) - (\frac{\pi}{2})^2 f(t)$

根据时域微分性可得 $(j\omega)^2F(j\omega) = \frac{\pi}{2}e^{j\omega} - \frac{\pi}{2}e^{-j\omega} - (\frac{\pi}{2})^2F(j\omega)$

得 $F(j\omega) = \frac{\pi\cos\omega}{(\frac{\pi}{2})^2-\omega^2}$

(3) $f(t) = \cos(\frac{\pi}{2}t) g_2(t)$

由 $g_2(t) \leftrightarrow 2Sa(\omega)$，$\cos(\frac{\pi}{2}t) \leftrightarrow \pi[\delta(\omega+\frac{\pi}{2}) + \delta(\omega-\frac{\pi}{2})]$

根据频域卷积定理得

$$F(j\omega) = Sa(\omega-\frac{\pi}{2}) + Sa(\omega+\frac{\pi}{2}) = \frac{\pi\cos\omega}{(\frac{\pi}{2})^2-\omega^2}$$