4.13. 解:

(b). 由图得:

$$f_2(t)=\begin{cases}\frac{1}{\tau}t, & 0<t<\tau\\ 0, & 其它\end{cases}$$

$f_2(t)$ 的傅里叶变换为: $F_2(j\omega)=\int_{-\infty}^{\infty}f_2(t)e^{-j\omega t}dt=\int_0^{\tau}\frac{1}{\tau}te^{-j\omega t}dt$

$$=-\frac{e^{-j\omega\tau}}{j\omega}-\frac{1}{j\omega\tau}\cdot\frac{e^{-j\omega\tau}-1}{j\omega}=\frac{1-e^{-j\omega\tau}-j\omega\tau e^{-j\omega\tau}}{-\omega^2\tau}$$

(c) $$f_3(t)=\begin{cases}\cos(\frac{\pi}{2}t), & -1<t<1\\ 0, & 其他.\end{cases}$$

$f_3(t)$ 的傅里叶变换为:

$F_3(j\omega)=\int_{-\infty}^{\infty}f_3(t)e^{-j\omega t}dt=\int_{-1}^{1}\cos(\frac{\pi t}{2})e^{-j\omega t}dt=\int_{-1}^{1}\frac{1}{2}[e^{j(\frac{\pi}{2}-\omega)t}+e^{-j(\frac{\pi}{2}+\omega)t}]dt$

$$=\frac{\pi\cos\omega}{(\frac{\pi}{2})^2-\omega^2}$$

4.18. 解:

(1) $\delta(t)$ 的傅里叶变换为 $\delta(t)\leftrightarrow 1$

则 $\delta(t-2)\leftrightarrow e^{-j2\omega}$ 故 $f(t)=e^{jt}\delta(t-2)\leftrightarrow e^{-j2(\omega+1)}$

(5). $\varepsilon(t)$ 的傅里叶变换为: $\varepsilon(t)\leftrightarrow\pi\delta(\omega)+\frac{1}{j\omega}$

~~F(jω)=~~ 则 $\varepsilon(t-1)\leftrightarrow[\pi\delta(\omega)+\frac{1}{j\omega}]e^{-j\omega}=\pi\delta(\omega)+\frac{e^{-j\omega}}{j\omega}$

故 $f(t)=\varepsilon(\frac{1}{2}t-1)\leftrightarrow 2[\pi\delta(2\omega)+\frac{1}{j2\omega}e^{-j2\omega}]=\pi\delta(\omega)+\frac{e^{-j2\omega}}{j\omega}$

4.20. 解: (1) $f(2t)\leftrightarrow\frac{1}{2}F(j\frac{\omega}{2})$

$(-2jt)f(2t)\leftrightarrow\frac{1}{2}\frac{d}{d\omega}F(j\frac{\omega}{2})$ 故 $tf(2t)\leftrightarrow j\frac{1}{2}\frac{d}{d\omega}F(j\frac{\omega}{2})$.

(4). $f(1+t)\leftrightarrow F(j\omega)e^{j\omega}$

故 $f(1-t)\leftrightarrow F(-j\omega)e^{-j\omega}$.

(5) ~~tf(t) - F(-jω)~~ $tf(t)\leftrightarrow j\frac{d}{d\omega}F(j\omega)$.

则 $-tf(-t)\leftrightarrow -j\frac{d}{d\omega}F(-j\omega)$

故 $(1-t)f(1-t)\leftrightarrow -je^{-j\omega}\frac{d}{d\omega}F(-j\omega)$.

4.24. 解: (1) $F(j\omega)=\int_{-\infty}^{\infty}f(t)e^{-j\omega t}dt=\int_{-1}^{1}\cos(\frac{\pi}{2}t)e^{-j\omega t}dt=\frac{1}{2}\int_{-1}^{1}(e^{j\frac{\pi}{2}t}+e^{-j\frac{\pi}{2}t})e^{-j\omega t}dt$

$$=\frac{1}{2}\left[\frac{1}{j(\frac{\pi}{2}-\omega)}e^{j(\frac{\pi}{2}-\omega)t}+\frac{1}{-j(\frac{\pi}{2}+\omega)}e^{-j(\frac{\pi}{2}+\omega)t}\right]\Bigg|_{-1}^{1}=\frac{\pi\cos\omega}{\frac{\pi^2}{4}-\omega^2}$$

(2) 对$f(t)$求导得：$f'(t) = -\frac{\pi}{2}\sin(\frac{\pi}{2}t)\, g_2(t)$.

再次求导：$f''(t) = -\frac{\pi}{2}\sin(\frac{\pi}{2}t)[\delta(t+1) + \delta(t-1)] - (\frac{\pi}{2})^2 \cos(\frac{\pi}{2}t) g_2(t)$.

$= \frac{\pi}{2}\delta(t+1) - \frac{\pi}{2}\delta(t-1) - (\frac{\pi}{2})^2 f(t)$

故有 $(j\omega)^2 F(j\omega) = \frac{\pi}{2} e^{j\omega} - \frac{\pi}{2} e^{-j\omega} - (\frac{\pi}{2})^2 F(j\omega) \Rightarrow F(j\omega) = \frac{\pi\cos\omega}{(\frac{\pi}{2})^2 - \omega^2}$

(3) 由波形可知：$f(t) = \cos(\frac{\pi}{2}t) \cdot g_2(t)$

由 $g_2(t) \longleftrightarrow 2Sa(\omega)$.

$\cos(\frac{\pi}{2}t) \longleftrightarrow \pi[\delta(\omega+\frac{\pi}{2}) + \delta(\omega-\frac{\pi}{2})]$.

则 $F(j\omega) = Sa(\omega-\frac{\pi}{2}) + Sa(\omega+\frac{\pi}{2}) = \frac{\pi\cos\omega}{(\frac{\pi}{2})^2 - \omega^2}$.