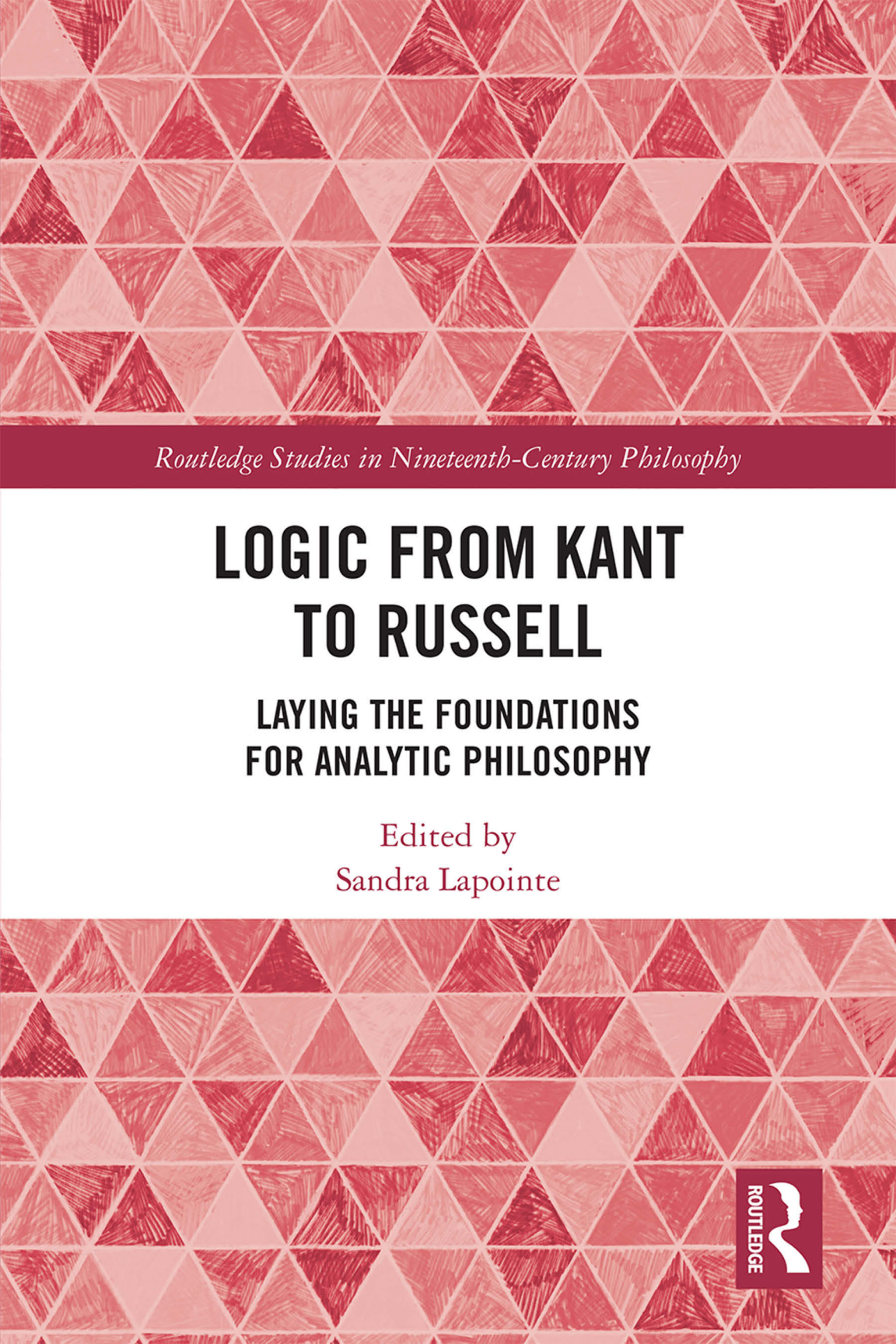
Routledge Studies in Nineteenth-Century Philosophy
LOGIC FROM KANT TO RUSSELL
LAYING THE FOUNDATIONS FOR ANALYTIC PHILOSOPHY
Edited by
Sandra Lapointe
ROUTLEDGE

Logic from Kant to Russell

> This book charts the rich and varied philosophical background against which mathematical logic developed.
>
> Peter Simons, Trinity College Dublin

The scope and method of logic as we know it today eminently reflect the groundbreaking developments of set theory and the logical foundations of mathematics at the turn of the 20th century. Unfortunately, little effort has been made to understand the philosophical context that led to these tremendous innovations in the 19th century beyond what is found in the works of mathematicians such as Frege, Hilbert and Russell. This constitutes a monumental gap in our understanding of the central influences that shaped 19th-century thought, from Kant to Russell, and that helped to create the conditions in which analytic philosophy could emerge.

The aim of *Logic from Kant to Russell* is to document the development of logic in the works of 19th-century philosophers. It contains 12 original essays written by authors from a broad range of backgrounds – intellectual historians, historians of idealism, philosophers of science and historians of logic and analytic philosophy. These essays question the standard narratives of analytic philosophy's past and address concerns that are relevant to the contemporary philosophical study of logic, language, mind and cognition. The book covers a broad range of influential thinkers in 19th-century philosophy and analytic philosophy, including Kant, Bolzano, Hegel, Lotze, the British Algebraists and Idealists, Moore, Russell, the Neo-Kantians and Frege.

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Logic from Kant to Russell

Laying the Foundations for Analytic Philosophy

Edited by Sandra Lapointe

Routledge
Taylor & Francis Group
NEW YORK AND LONDON

First published 2019
by Routledge
52 Vanderbilt Avenue, New York, NY 10017

and by Routledge
2 Park Square, Milton Park, Abingdon, Oxon OX14 4RN

Routledge is an imprint of the Taylor & Francis Group, an informa business

Library of Congress Cataloging-in-Publication Data
Names: Lapointe, Sandra, editor.
Title: Logic from Kant to Russell: laying the foundations for analytic philosophy / edited by Sandra Lapointe.
Description: 1 [edition]. | New York: Taylor & Francis, 2018. | Series: Routledge studies in nineteenth-century philosophy; 19 | Includes bibliographical references and index.
Identifiers: LCCN 2018039764 |
ISBN 9780815396321 (hardback)
Subjects: LCSH: Logic—History—19th century. | Analysis (Philosophy)
Classification: LCC BC38 .L6235 2018 | DDC 160.9—dc23
LC record available at https://lccn.loc.gov/2018039764

ISBN: 978-0-8153-9632-1 (hbk)
ISBN: 978-1-351-18224-9 (ebk)

Typeset in Sabon
by codeMantra

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Introduction*

Sandra Lapointe

Standard Narratives

This monograph aims to make sense of the way in which conceptions of logic changed over the course of the 19th century as well as to examine and ultimately do justice to the idea that Kant's views on logic substantially contributed to this development and to shape the field as we know it today. This might sound like a curious proposal, one that strikingly diverges from standard narratives of the history of logic and analytical philosophy. As far as logic goes, the 19th century is largely considered to have been at best the antechamber of the "Fregean Revolution"[1] and its aftermath in the work of Russell and Whitehead, Wittgenstein and the Logical Positivists, among many others over the course of the 20th century. One pervasive assumption underlying the standard narrative is that philosophers before 1879, that is, the year Frege's *Begriffsschrift* was published, misunderstood the essence of logic and its relation to the rest of philosophical knowledge (especially psychology and metaphysics), a situation that would supposedly be corrected, if not in Frege's own work, then at least by the first decades of the 20th century in that of his analytical successors. In the literature, narratives usually converge to offer the tale of a notoriously antagonistic relationship between early contributors to the new "logistic" or "symbolic logic" that became distinctive of analytical philosophy and their Idealist predecessors. Hylton's (1992) account of a Russellian turn away from Bradleyan Idealism or Coffa's (1991) proposed "semantic tradition" with its so-called common "enemy", i.e. Kant's doctrine of pure intuition, are prime examples of this narrative line.

Analytical philosophers today often concur in thinking of Kant's place in the development of logic as insignificant. As Volker Peckhaus, for instance, puts it:

* I would like to thank Clinton Tolley, Nick Stang, Erich Reck and Sean Morris for their helpful comments at various stages of drafting and editing.

1 The term is borrowed from Gillies (1992).

> The philosophical discussion in early 19th century Germany was determined by Kant and by the transformations of Kantian philosophy suggested by Hegel and other German idealists. In the preface to the second edition of his *Kritik der Reinen Vernunft* of 1787 Immanuel Kant (1724–1804) wrote that logic had followed the safe course of a science since earliest times. For Kant this was evident because of the fact that logic had not been allowed to take any step backwards since the time of Aristotle. But he regarded it as curious that logic had been unable to take any step forward either (Kant 1781, Bviii). Logic therefore seems to be closed and complete. Formal logic – in Kant's terminology the analytical part of general logic – played no prominent role in Kant's system of transcendental philosophy.
>
> (Peckhaus 2009, 3)

Undeniably by today's standards, most of the individual logical doctrines Kant endorsed were at best defective. But an interpretation of logic's past that is driven by today's standards may not aim to offer proper contextualisation, and in the absence of a proper contextuation the resulting narrative will likely fail to do justice to Kant's positions or to the role they played in logic's transformation and continuing development. For instance, one fact that has been vastly neglected and which Preti (*infra*) documents is that Kant' logical views were still considered to be relevant enough in late 19th century Britain to be the target of prominent critics. Assuredly, proper contextualisation would show that philosophers before Kant just like those who came in the century that followed understood the range of problems with which logic is concerned to be considerably broader than what we associate with formal logic today. As a result, even when it looks like concerns are similar across time, there is no straightforward way to project current standards back onto the past. Proper contextualisation would also show that Kant's impact had less to do with his possible improvement on formal logic's doctrinal content than with his reform of the metaepistemological framework within which such doctrines cohere.[2] Both points will be unpacked in what follows.

To be fair, Kant did make a few original proposals regarding individual doctrines that still resonate with contemporary formal logicians. For instance, Kant is often claimed to have been the first to suggest that existence is not a predicate, an idea a version of which is central to quantification theory. Other aspects of Kant's theories have also recently

2 By the "metaepistemological" framework of a discipline – as opposed to its theoretical or doctrinal core – I mean the set of assumptions that define its scope, method and place within philosophy and knowledge, more generally.

attracted the attention of non-classical logicians. In this volume, for instance, Graham Priest argues that Kant's treatment of the Antinomies foreshadows important aspects of dialetheism (see Priest, *infra*). In a very loose sense, then, Kant could be seen to have anticipated aspects of logic as we know it. Nonetheless, from today's perspective, the bulk of Kant's views – for instance, on conceptual analysis, the form of judgement or what makes for a valid inference – did not improve much over those of his immediate predecessors, e.g. Leibniz, Wolff and their contemporaries. Even though he sometimes sought to improve on various points – e.g. in the "False Subtlety of the Four Figures of Syllogism" (1762) or even in the discussion of the table of judgement in the first *Critique* – Kant's views on concepts, judgement and inference did not shift the discussion quite enough to constitute anything like the kind of transformations we find in, say, Bolzano or Lotze or more famously in Frege.

Kant's theories, then, were in certain respects conservative. Just like those of his German predecessors Leibniz and Wolff, his theory of judgement and inference is inextricably tied to his views on "representations" (*Vorstellungen*) and more specifically to the notion that concepts contain or include other concepts. This particular conception of concepts' structure has come to be known as the "decompositional" theory of conceptual analysis (Andersen 2004, 2005; Lapointe 2000, 2008, see also Lapointe *infra*). In context, theories of judgement and inference ultimately present themselves as the result of explications and/or upshots of such "inclusion" relations. Post-Cartesian philosophers' ambitions to simplify syllogistic theories by reducing inference to a few principles were also premised on the decompositional model of conceptual relations. Consequently, problems emerged for logicians – including Kant! – once it became clear that the syntax of conceptual inclusion relations was in principle too poor to provide the resources needed to model the logical complexity involved in mathematical and scientific reasoning.

Notoriously, the most exigent shortcomings of the decompositional conception of conceptual analysis were tied to its inability to capture polyadic predication and multiple quantification. Indeed, many have been drawn to the idea that Kant developed his views on the role of a putatively pure "intuition" in what he conceived as the "construction" of mathematical concepts at least in part in order to compensate for the inadequacy of the logical theories that underpinned previous epistemologies: Kant's doctrine of "schematism" is in many ways meant to pick up the slack where in, for instance, axiomatic disciplines deductive methods rooted in decompositional analysis ran aground (cf. Anderson 2004, 2005; Friedman 1990, 1992; Hintikka 1966).

When it comes to analytical philosophers' assessment of the development of logic over the post-Cartesian period, however, Kant's logical views, when he is deemed to have had views distinctive enough to be worth the mention, are only one of many targets. Historians of logic

often think of the post-Cartesian period both before and after Kant as one of general stagnation or decline.[3] As Bochenski puts it,

> For all his faults, Ramus [1515–1572 – SL] was a logician; Keckermann [1572–1608 – SL] too had some knowledge of the subject. The same can seldom be said of their successors until Bolzano, Peirce and Peano. Most historians of logic in the 17th, 18th and 19th centuries treat of ontological, epistemological and psychological problems rather than of logical ones. Furthermore, everything in this period, with few exceptions, is so conditioned by the then prevailing prejudices that we may count the whole period as part of the prehistory of our science.[4]
>
> (Bochenski 1961, 4)

> A further problem that belongs here is that of the so-called 'classical' logic [between 16th–19th century – SL]. One could understand it as a distinct variety, since while it consists of fragments of scholastic logic (taking over for example the mnemonic Barbara, Celarent etc.), yet these fragments are interpreted quite unscholastically, in an ancient rather than scholastic way. But the content of this logic is so poor, it is loaded with so many utter misunderstandings, and its creative power is so extremely weak, that one can hardly risk calling something so decadent a distinct variety of logic and so setting it on a level with ancient, scholastic, mathematical and Indian logic.
>
> (Bochenski 1961, 14)

Bochenski[5] was among a first wave of contemporary historians of logic with the Kneales (1962), Heinrich Scholz (1931; translated into English in 1961) and C.I. Lewis (1918) who argued that the introduction of first-order predicate calculus (i.e. quantification theory) was the single most important advance in the history of the discipline. It is quite unthinkable that one should disagree with them on this particular point. Of course, the credit for having sought to develop a new logic should not go to one single author, whether it be Frege (in the *Begriffsschrift* of 1879) or Russell and Whitehead (in the *Principia Mathematica* of 1910). Others like Bolzano,[6] Pierce, Peano, Dedekind, Hilbert and the

3 Risse's *Logik der Neuzeit* (1964) is a remarkably interesting outlier, a fabulous repository of information concerning the development of logic over the post-Cartesian period.

4 Bochenski was a supporter of the Nazis. This citation is not an endorsement of his ideas or actions.

5 Bochenski's History of Formal Logic (1961) is based on a German text published in 1956 under the title *Formale Logik* (München, Alber, 1956).

6 See Lapointe (*infra*) for a presentation of Bolzano's reform of logic.

young Husserl all saw the need for establishing the logical foundation of mathematics and scientific knowledge with the same urgency, and they all sought to contribute to shaping the discipline in ways that are consistent with what we know today. As a result of this activity in the last decade of the 19th century, very few if any of the individual doctrines put to work for the purpose of defining, for example, the structure of concepts and judgements, logical laws or valid inference leading up to the 17th and 18th centuries survived. The same holds for the conception of the connection between mathematics and logic (more on this below).

Radical disciplinary reforms of the sort logic experienced at the turn of the 20th century are exceptional, especially in philosophy. From the standpoint of the historian, they also raise exceptionally interesting questions: how, for one, should we deal with historiographical claims that predate the said philosophical revolution? By a historiographical claim about philosophy, I mean a claim whose purpose is to offer an adequate representation of a past author's contribution to the discipline or of some broader doctrinal or theoretical development. I use 'historiographical' as opposed to 'historical' to emphasise the fact that the type of claims I have in mind are claims which, implicitly or explicitly, result from our attempts to "interpret" and "represent" the information that is available through document sources, not claims that purport to describe past events and facts. The emphasis here should be on the notion that historiographical claims are types of "representations" that result from "interpretation". Assuredly, these terms need to be defined more precisely – the topic of future research – especially so as to make explicit important differences between historiographical claims and claims we otherwise understand as "factual", "empirical" or "descriptive".

Kant's Role in Logic

Throughout the 19th and well into the early 20th century, philosophers thought of Kant as a powerful influence on the development of logic. I argue below that this interpretation remains adequate in spite of the fact that 19th-century philosophers' conception of the scope of the discipline was much broader than the contemporary one. In particular, the kinds of transformation Kant was credited for are precisely those that carved the conceptual space for the idea that logic is a "formal" discipline independent of psychology and epistemology, a move that coincided with the increasing shift of logical investigations toward formalisms, calculi and/or semantics. If this is correct, although Kant himself did not conceive of logic in these terms, he played a crucial role in creating the conditions for it. What needs to be explained, then, is the type of "historiographical revisionism" that has led to today's virtual consensus among analytical

philosophers according to which Kant's contribution to the history of logic is at best minimal.

To be clear, the concern here is not that Kant might have been subject to historiographical injustice – given the standard conception of Kant's place within the canon, this is impossible. The concern is more broadly methodological: the fact that historians are prone to discard past historiographical claims raises questions that go straight to the methodological underpinnings of interpretation and, in particular, contextualisation in the history of philosophy. Examining these questions provides for better insight into historians of philosophy's various tasks. The fact that past historiographical claims about Kant's role in the development of logic have been quite drastically reassessed provides for a very opportune and richly documented case in point within our project: a better-informed comparative engagement with historiographical claims about Kant's role in logic, past and present, in our opinion is part of what is needed for a more adequate understanding of the discipline's development over the course of the 19th century.

Consider the following two groups of claims concerning post-Cartesian logic. (Each quotation is summarised for readers' convenience.)

Group 1:

Windelband: Kant's *Critique of Pure Reason* profoundly influenced the development of logic, which had been steadily 'Aristotelian' until then.

> A special inquiry into principles is, however, comparatively easy and free from danger in the case of a particular science whose main structure is relatively fixed and accepted. And we should perhaps have found ourselves in this position with regard to logic about a century and a half ago. It then stood as a well-built edifice firmly based on the Aristotelian foundation, to which subsequent exposition had in the course of time contributed changes in the arrangements of its parts, or made more or less prominent additions. **But, as is well known, this state of things was entirely changed by Kant.**
>
> (Windelband 1912, 1)

Blakey: Kant's *Critique of Pure Reason* is the point of origin of a significant new logical tradition in the 19th century.

> Kant's theory was the great starting point either in the way of supporting modifying or opposing to most of the logical treatises of the country during this present half century. **This theory obtained such a firm hold of the philosophic mind of the nation during the first twenty years of its history that the philosophers and logicians of Germany seemed spellbound and unable to set a single foot beyond the prescribed circle of the *Critique of Pure Reason*.** The entire mass

> of logical speculation of modern times rests upon an ingenious system of ringing the changes on the leading ideas or principles involved in Kant's views and those of his immediate followers and critics.[7]
>
> (Blakey 1851, 388)

Adamson: the development of logic in the 19th century is only intelligible in reference to Kant's *Critique of Pure Reason*.

> **The critical method**, which has so influenced general philosophy that all later speculation refers more or less directly to it, **has at the same time profoundly modified all later conceptions of the sphere and method of logic**. […] Indeed the grounds of distinction between the several doctrines thus brought into connection, and the significance of the terms by which they are expressed, are intelligible only when taken in reference to the Kantian system.
>
> (Adamson 1882, 110f)

Group 2:

The Kneales: the logical theories of modern logicians were neither original nor philosophically interesting.

> Although the subject survived in the elementary instructions of the universities, **it no longer attracted the attention of many of the best minds**. From the 400 years between the middle of the fifteenth and the middle of the nineteenth century we have in consequence scores of textbooks but **very few works that contain anything at once new and good**.
>
> (Kneale and Kneale 1962, 298)

Shapiro and King: only mathematicians – not philosophers – contributed to logic over the course of the modern period.

> By the middle of the fifteenth century little if any new work was being done. There were instead many simplified handbooks and manuals of logic. The descendants of these textbooks came to be used in the universities, and the great innovations of mediæval logicians were forgotten. Probably the best of these works is the *Port Royal Logic*, by Antoine Arnauld and Pierre Nicole, published in 1662. **When writers refer to 'traditional logic' they usually have this degenerate textbook tradition in mind. Since the beginning of the modern era most of the contributions to logic have been made by mathematicians.**
>
> (King and Shapiro, 3)

7 See also Ueberweg (1868, §28), who makes a similar claim about a "Kantian School".

Peckhaus: logic plays no role in Kant's system of philosophy, and Kant plays no role in the history of logic (Peckhaus 2009, as quoted above).

There is a considerable historiographical rift between Group 1, which epitomises positions that are typical for the 19th century, and Group 2, which is representative of the 20th and 21th century mainstream. Assuredly, there is a common-sense expectation that epistemological revolutions of the sort logic was subject to at the turn of the 20th century should invite a reassessment of past historiographical claims. But what lies behind "common-sense" is precisely what needs to be made clear.

Attempts to explain away past historiographical claims are rather common, at least in informal discussion. One could conceivably argue that Adamson, Blakey and Windelband had reasons "in context" to think of Kant as a reformer and as a trailblazer in logic. One might assume further that "the context changed" – but what does this mean? – in such a way as to render the same narrative inapt a few decades later. One could argue that, by the time Martha and William Kneale published their *Development of Logic* in 1962, a presumably "deeper" or otherwise "new" revolution – but in what respect? – had taken place and percolated into most aspects of the discipline. This radical "epistemological shift" – but what are those made of? – one might further surmise, had the effect of relegating what had previously been celebrated as a substantial logical innovation to an anecdote at best and calling for a reassessment of all previous narratives.

Such considerations are representative of an approach that coincides with the one the Kneales seem to have been happy to embrace:

> [T]his book is an account of the growth of logic, rather than an attempt to chronicle all that past scholars, good and bad, have said about the science[...] [O]ur primary purpose has been to record the first appearances of those ideas which seem to us most important in the logic of our own day. Such a programme is based on judgements of value, and we realize that our selection of material and still more our comments, especially in the later chapters, may seem eccentric to some readers. In defence of our undertaking we can only say that we have followed the plan which our interests suggested, and that we could not have written in any other way.
>
> (Kneale and Kneale 1962, v)

The Kneales' approach in the history of logic was not exactly new at the time. C.I. Lewis (1918), for instance, also explicitly espouses it in the preface to his survey of the history of mathematical, i.e. symbolic, logic:

> We must now turn back and trace in more detail the development of symbolic logic. A history of the subject will not be attempted if by history is meant the report of facts for their own sake. Rather,

> **we are interested in the cumulative process by which those results which most interest us today have come to be**. Many researches of intrinsic value, but lying outside the main line of that development, will of necessity be neglected.
>
> (Lewis 1918, 5, my emphasis)

That logic as we know it is the result of a process that can be represented as "cumulative" is a controversial position. In Lewis's case, it is all the more problematic that his survey effectively abstracts from all aspects of the context that are not already identified as anticipations of "mathematical" logic, ignoring most of the logical landscape. Lewis's historical survey deliberately deals only with precursors whose focus was on the development of symbolisms and calculi, including Leibniz, De Morgan, Schroeder, Boole, Grassmann, Jevons and Peirce, the narrative concluding with Frege and Russell. Lewis' approach, and the resulting interpretation and narrative are similar to that of many contemporary logicians in taking Russell and Whitehead's *Principia Mathematica* of 1910 as both the beginning of modern logic and the standard by which to evaluate all past theories:

> The publication of *Principia Mathematica* would seem to have determined the direction of further investigation to follow that general direction indicated by the work of Frege and the *Formulaire* [Frege's *Begriffsschrift* – SL]. The *Principia* is concerned with the same topics and from the same point of view. But we see here a recognition of difficulties not suggested in the *Formulaire*, a deeper and more lengthy analysis of concepts and a corresponding complexity of procedure. There is also more attention to the details of a rigorous method of proof.
>
> (Lewis 1918, 116)

Scholz (1931), an eminent member of the early German analytical school, also argued that the emergence of quantification theory (quantified first-order predicate calculus) at the turn of the 20th century forced a wide-ranging re-evaluation of past historiographical claims about logic. As Scholz put it:

> For what formal logic really is we know only because symbolic logic provided the conceptual equipment needed to answer this problem. In general, too, the extant gains registered by the modern symbolic treatment of logic have become such an essential factor in making pronouncements regarding the history of logic that we are constrained to say that an essential knowledge and mastery of the results of symbolic logic have become an indispensable condition for any and all fruitful study of the history of logic (Scholz 1931, v–vi [translation by K. Leidecker, New York, The Philosophical Library, 1961).

The methodology that underlies Lewis, Scholz and the Kneales' approach is not "wrong". Indeed, it coincides in many ways with what philosophers and historians today understand the purpose of "rational reconstruction" to consist in.[8] In this volume, Chapter 6 by Nicholas F. Stang and Chapter 2 by Graham Priest, for instance, both draw on this approach meaningfully. But the type of questions that drive rational reconstruction – e.g. "What can we learn from past logicians that is relevant to contemporary theories?" – do not have precedence and certainly do not constitute the only type of questions historians of logic and philosophy can be interested in. On the contrary, while rational reconstruction is abundant in contemporary scholarship – whether it is "historical" in any genuine sense is a question I will leave open – it corresponds to a specific kind of task philosophers investigating the past of their discipline may set for themselves.

The point therefore is not that the Kneales' narrative line or rational reconstruction more generally are in principle to be avoided in historical work. The point or part of the point I wish to make is that without a better understanding of the conceptual tools involved in interpreting past authors and, in particular, representing and assessing historiographical claims, it's not clear what would license us to dismiss Group 1's claims or endorse Group 2's. In the absence of a reflection on the methodological framework on the basis of which we formulate and evaluate historiographical claims, past historiographical claims that seem incompatible with more recent interpretations of the philosophical-historical data should not be simply dismissed. In my view, they should rather be properly leveraged as additional tools when painting the development of the discipline.

Rational Reconstruction vs Disciplinary History

Christopher Pincock and I argued in our "Introduction" to *Innovations in the History of Analytical Philosophy* (2017) that interpretation in the history of philosophy is never unprejudiced: at the very least, interpretation is always driven by the more or less clandestine "questions" and concerns that define the more or less well-defined set of "tasks" of historians, which may vary in each particular instance.[9] Questions we associate with rational reconstruction – such as "What can Kant teach us about logic today?" – represent only one amongst many types of questions that may guide historians of philosophy's endeavours and define the principles they adopt for structuring their interpretations. Another such question – the one which guides the book's project – can be put as follows:

8 Rorty (1984) is the classical locus for a description of rational reconstruction, especially the first section of the paper.

9 See Lapointe and Pincock (2017).

> How did Kant's views on logic inform the development of the discipline over the course of the 19th century and how did this development in turn come to shape the theories of early analytical philosophers?

The expression 'early analytical philosophers' in this context is meant to be understood broadly to include Bolzano's insightful theory of deductive systems, Frege's quantification theory, Russell and Whitehead's project of a foundation for set theory, Husserl's theory of "definite manifolds" and Hilbert's axiomatic programme, among many. On this account, Dedekind was also an early contributor to analytical philosophy and, as Erich Reck shows in Chapter 8 (*infra*), one whose logicist project serves to illustrate the richness and variety of the programmes in which philosophers would have been involved at the time. Early analytical philosophy as I understand it includes many others whose names may or may not be familiar: for instance, Sigwart, Herbart, Trendelenburg and Lotze. The task at hand involves tracking causal/textual connections with as much attention to detail as is reasonable so as to do justice to the fact that philosophical activity revolves around epistemic agents, i.e. people who produce doctrines and theories, some of whom may acquire canonical status while others do not.

Doing justice to the way in which the development of logic was impacted, initially by Kant himself, and later on by the various iterations of his positions in his successors over the course of the 19th century, to shape contemporary understandings of the discipline sets a task whose fulfilment can only ever be approximated. It involves producing interpretations and representations that are adequately contextualised; as such the latter should track a complex nexus of doctrinal influences, sometimes on the basis of merely partial and invariably underdetermining textual and documentary evidence. In Chapter 1 (*infra*), Jeremy Heis offers a splendid sample of this kind of work. Painting a more complete picture of the context in which some of the better-known logical achievements of the "Kantian School" were rooted is part of what is needed for a reassessment of standard narratives. The novel account of the history of logic envisaged here does not arise from new, hitherto inaccessible historical data – although it is clear that the studies included in this volume grow from consideration of philosophical work and corpora that have been generally neglected. Rather, it is rooted in the uprooting of one widely shared assumption, namely that the development of logic should be understood from the current, contemporary epistemic and normative standpoint.

In their historical work, the Kneales, Scholz and Lewis purport to answer some variant of the questions "What can I learn from x about y that is philosophically relevant today?" or "What did x have to say that is philosophically relevant to y as we understand it today?" What's

distinctive of this type of rational reconstruction is that it projects back onto x the philosophical norms and standards that concern y today, presumably in order to offer a philosophically relevant interpretation of x's treatment of y. While this approach is pertinent and adequate for certain specific philosophical purposes – for instance, in pedagogical contexts – historians of philosophy are not bound to ask questions of this form and may engage with widely different or vastly more differentiated questions and tasks. Indeed, if we are to judge by the literature, they typically do.

Importantly, the classical opposition between "rational" and "historical" reconstruction does not do justice to the manifold of possible question types that guide historians of philosophy's investigation and the answers to which require finely tuned dedicated methodological toolsets.[10] Compare, for instance:

A What were Kant's views on what we call 'logic' today? (rational reconstruction)
B What did Kant mean by 'logic' and 'form' when he claimed that logic has to do with form? (contextualisation)
C Were Kant's views on judgement influential in the 19th century? (doctrinal history)
D How did Kant's views on the scope, method and place of logic impact the development of the discipline? (disciplinary history)
E How do Kant's views on validity and inference compare to that of his immediate predecessors? (thematic investigations)
F Did Kant's views on logic shape those of early analytical philosophers and contribute to define who we are? (genealogy).[11]

The present volume's overall objective falls within the scope of (D), i.e. "disciplinary history". The approach is based on a question that is of a type eminently different from the questions Lewis, Scholz and the Kneales sought to answer, i.e. some variation of (A). The specific question that defines the present volume's task was put as follows:

> How did logic develop over the course of the 19th century and which part did Kant's theories and the theories that drew on Kant play in this development?

10 Lapointe and Pincock (2017) expands tentatively on what these toolsets would need to look like, but the conclusions are provisional and the bulk of the work is still forthcoming, in part in a planned volume on methodology in this history of knowledge.

11 Lapointe and Pincock (2017) discuss this conceptual framework in some detail, an endeavour for which there is no place here. Some of our ideas on methodological pluralism are indebted to Panaccio (2016).

While the individual essays differ in their degree of focus on this question, taken together, they contribute to the project it sets.

What Historians of Logic Get Wrong

The previous section aims to make conceptual space for the idea that standard historiographical methods do not exhaust all, nor even perhaps our best options when it comes to the history of logic. However, there is at least one other reason to reassess Kant's position in particular. Indeed, Kant's views on logic are in fact rarely properly represented, and this too contributes to their role in logic's developments being undervalued. This may seem like an odd claim to make in light of the fact that Kant's views on the foundations of mathematics have been singled out and discussed by virtually every thinker since the publication of the *Critique of Pure Reason*. But the mistake from the disciplinary historian's standpoint is precisely to think that we can project onto Kant the contemporary conception of the connection between the foundations of mathematics and logic without falling into anachronism.

Since the late 19th century, philosophers see a narrow, intricate and fundamental connection between logical and mathematical concerns, and rightly so. The contemporary understanding of the connection between logic and mathematics, however, cannot be projected onto the past without generating complications that make it impossible to understand how this conception of the connection between logic and mathematics arose – at the end of the 19th century – in the first place. Consider, for instance, that the mathematical community only started to deploy sustained, systematic efforts toward providing a logically sound axiomatic foundation for arithmetic at the beginning of the 19th century, Bolzano being one of the pioneers in the field. Assuredly, Kant's views on the epistemology of mathematics were important to his contemporaries. They were at any rate the target of generations of compelling criticisms, many noting that the main problem with Kantian epistemology is precisely that it short-circuits the effort toward a gapless logical foundation of axiomatic systems such as set theory. But these criticisms come out of, for example, the logicist projects of Frege and Russell and the related formalist projects of Hilbert. But those post-date the context of Kant's work by a century, and this is not a trivial factor in an attempt to understand the historical significance of Kant's views on mathematics and logic.

Early analytical philosophers generally failed to discuss and even to criticise in much detail what Kant had to say on individual formal logical doctrines and, much more relevantly for our purpose here, on the scope, method and place of formal logic as a whole. One possible reason for the latter might reside in the fact that Kant's views on the scope, method and role of logic within knowledge – by contrast to his views on synthetic a

priori knowledge and pure intuition in mathematics and pure physics – still in fact had currency among the same early analytical philosophers who so vehemently criticised his philosophy of mathematics. At the beginning of the 20th century, many of the same philosophers who were unsympathetic to Kant's epistemology of a priori knowledge nonetheless adopted his metaepistemological framework. One of the most interesting documents illustrating the situation comes from a representative of Logical Positivism in Germany, and incidentally one of the few to have written on the history of logic as such in the first decades of the 20th century: Heinrich Scholz.

Scholz, as we saw earlier, argued that, after the birth of "logistic", the history of logic would need a complete overhaul to reflect the new standards. But he nonetheless continued to think of Kant as having been a pioneer of the field and as such as defining those very standards. Scholz endorsed some of the main tenets of Kant's views on the scope, method and place of logic within knowledge. As Scholz sees it, we need to at least separate "formal" logic from its "transcendental" counterpart:

> We can note of course only in passing that the theory of categories became decisive in the development of an entirely new concept in logic with the absolutely original interpretation which Kant gave it. It is the famous concept of transcendental logic which Kant set up over against formal logic. We remember of course, that Kant also gave formal logic its name. This new transcendental logic has only a highly problematic connection with the 'forms' of Aristotelian logic which will not bear exact investigations. Not only can transcendental logic stand independently when this connection with formal logic is severed, but when disengaged it can be better appreciated for what it is.
>
> (Scholz 1931, 15)

Whether or not Scholz is right that Kant thought that transcendental logic could be "severed" from formal logic, this passage shows that Kant's metaepistemology was philosophically relevant to Scholz – a situation that was not idiosyncratic at the time, especially in Logical Positivist circles.[12] Of course, by the 1930s, Scholz's was not the consensus position. Although he does not reject it explicitly, nothing Lewis (1918) says, for instance, commits him *prima facie* to a Kantian metaepistemology.

12 One other way to put the point is to say, drawing on Friedman, that

> [N]ot only Reichenbach's but also Carnap's logical empiricism has been interpreted as a basically Kantian approach which maintains – contra more recent holistic naturalism(s) a la Quine and his many followers – the key Kantian division between the transcendental and the empirical levels of inquiry.
>
> (Pihlström and Siitonen 2005, 82)

What matters here is that whether or not Kant's metaepistemology was still predominant in 1930, the Kantian distinction to which Scholz appeals between "formal" logic and "transcendental" logic continued to be the catalyst of formal logic's development over the course of the 19th century and well beyond the point at which Kant's specific views on the nature of a priori knowledge had started to be exposed by his early analytical critics.[13]

What exactly is the Kantian metaepistemological framework? The distinction between what Kant himself calls 'pure general logic' and 'transcendental logic' was part and parcel of a larger proposal Kant had made as regards the division of labour within philosophy, one that crucially reshaped the theoretical landscape in the late 18th and early 19th centuries, giving rise to a range of new research programmes within philosophy. This included what would have been precursor disciplines not only to (i) today's "formal" logic but also to (ii) epistemology broadly construed, (iii) "empirical" psychology (and the study of mental phenomena broadly construed), and (iv) methodology and the philosophy of individual sciences. Kant's main contribution to formal logic – but it at once served epistemology, psychology and methodology equally well – was to propose the terms of an internal division and reorganisation of the concerns that had hitherto fallen, in bulk, within the domain of the *Vernunftlehre* and *philosophia rationalis*, i.e. "logic" as it was broadly construed back then.

This reorganisation is what provided for the first time a separate platform for a treatment of the questions we associate today with formal logic, namely questions that are distinct from those involved specifically in both epistemology (including concerns that range from justification to the theory of cognition) and psychology. It also made place for a rich and varied range of positions regarding the true nature of logic. In Chapter 7, for instance, Frederick Beiser offers a contextualisation of Cohen's *Logik der Erkenntnis* (1903) that shows him to embrace "methodology" (not "formal logic") as epitomising the most fundamental logical concerns. In Chapter 10, Sean Morris argues that Russell's early philosophy is also considerably informed by methodology as it was represented, in particular, in the work of Sigwart. What Scholz's testimony thus contributes to show is that the Kantian metaepistemological framework had still not been done away with in the 1930s. It documents the fact that, to the extent that the epistemology of a priori knowledge could still be understood as belonging to "transcendental logic" and opposed to "formal logic", the (very different) disciplinary boundaries we know today were still very much in the making.

13 Proust (1987), for instance, argues that both Wittgenstein and Carnap, although they shun the term 'transcendental', nonetheless incorporate aspects of the Kantian transcendental project in their logic.

Logic before Kant

The significance of the changes that Kant's new metaepistemological framework involved is best measured against the theories of his predecessors. In the 17th and 18th centuries, conceptions of logic in the Western world seem to have been relatively homogenous as regards scope, place and method. Philosophers including Arnauld and Nicole, Hume, Condillac, Leibniz, Locke, Wolff and even the young Immanuel Kant – let's call them 'post-Cartesian' philosophers – agreed broadly on what would have constituted the scope, method and place of logic: contemporary representations of the structure of human knowledge depict "logic" as one of the fundamental endeavours within science, and as such, the scope of logic was remarkably broad.[14]

What is also striking is the monolithic nature of the object of inquiry: reason or the understanding *as a whole*. Hume, for example, claims that the aim of logic is "to explain the principles and operations of our reasoning faculty, and the nature of our ideas" (Hume 1739, "Introduction"). As Wolff, one of the most influential German logicians of the 18th century, put it,

> [I]n order for us to know if we are skilled for philosophy or not, our first task must be to acquaint ourselves with the powers of the human understanding and its correct use in the recognition of truth. The part of philosophy in which this will be shown is called logic, or the art of reason or also the doctrine of reason.
>
> (Wolff 1712/1742, §10, 6)

Under 'operations of our faculties', 'nature of ideas' and 'the powers of the understanding', post-Cartesian philosophers included concerns belonging to a remarkably broad range of philosophical subdisciplines: epistemology, philosophy of science, philosophy of mind and language as well as psychology, among others. However, these concerns were not back then subject to this classification and were not in fact very often differentiated. For this reason, projecting contemporary disciplinary boundaries back onto post-Cartesian theories creates anachronism.

Why, then, should we assume that consideration of the concerns of post-Cartesian "logicians" bears on the history of logic? To the extent that the aim of the present volume is to understand the way in which logic developed, consideration of the way in which conceptions of the scope, method and place of logic within knowledge evolved should play a crucial role. In the present case, this would seem to require a good

14 Chrétien Frederic Guillaume Roth's *Genealogical Distribution of the Arts and Sciences* which was published in *Encyclopédie* (1759) offers an excellent illustration of this breadth (https://encyclopedie.uchicago.edu/content/arbre-genealogique).

understanding of the breadth of concerns that pertained to logic, understood as *Vernunftlehre* and *philosophia rationalis* in the 17th and 18th centuries, and the way this conception was progressively trimmed down, broken up and rearticulated over the course of the two centuries that followed. In particular, it requires appreciation for the fact that as a doctrine of "reason" or of the "cognitive powers", logic was tasked with not only defining the rules underlying truth preservation and deductive validity (the task defining the narrower scope of formal or symbolic logic today) but also providing an explanatory model for the nature and structure of representation, judgement, cognition and inference, as well as for both the formal *and the material* conditions of truth and human cognition, including, for example, how to avoid mistakes.

It needs to be stressed that Kant's conception of the scope and place of logic in the broader sense of a *Vernunftlehre* initially coincided with that of his predecessors. For the young Kant, logic's task was the investigation of cognition and of the perceptual, intellectual and rational powers. But Kant came to think of this broad, overall task as falling, so to say, into separate "research programs". In the *Critique of Pure Reason*, Kant set out the framework within which these programmes could cohabit – his metaepistemological framework – and focussed on one of them: an inquiry that targets the conditions of a priori cognition and which he calls 'transcendental logic'.[15] Kant's focus on transcendental logic needs, however, to be put in perspective: at least in principle, every other subdomain of Kant's theory of knowledge deserves proper treatment, and this is exactly what Kant's successors undertook and how epistemology, psychology and methodology or philosophy of science would emerge as individual research programmes.

As Kant sees it (cf. Kant 1781, B74–86), different types of processes are involved in cognition – Kant's way to frame this in some places is to talk of different types of rules that bear on different powers – which need to be studied separately. These rules vary in terms of their scope (viz. "general" or "special") and their nature (viz. "pure" and "applied"). In the "Introduction" to the section of the *Critique of Pure Reason* entitled "Transcendental Logic", these distinctions come together to define the Kantian metaepistemological framework. They can be represented by an orthogonal classification:

15 If we omit the substantial introduction, the book as a whole is divided into two very uneven parts: a "Transcendental Doctrine of Elements" (440 pages in the Cambridge English translation) and a "Transcendental Doctrine of Method" (80 pages). The Doctrine of Elements is itself divided into two subparts: the "Transcendental Aesthetics" (roughly 20 pages) and the "Transcendental Logic" (roughly 420 pages). The specificity of this putatively "transcendental" logic and its relation to the other parts of the critique of reason Kant makes clear in the introduction to the eponymous sections. Whether or not the division of labour between "Aesthetic" and "Logic" per se is a Kantian innovation is a question we'll leave open.

	General	*Special*
Pure	**"Formal" Logic** The (a priori) study of the mere form of the understanding, i.e. the "negative conditions" of truth, or what amounts to the same, for the thinking of objects in general	**Transcendental Logic** The a priori study of the conditions under which the thinking of objects of experience occurs
Applied	**Psychology** The study of the empirical conditions for the thinking of objects in general	"**Methodology**"[a] The study of the empirical conditions for the cognition of objects in specific domains

a 'Methodology' here does not mean the same as 'theory of method' as Kant uses the term in *Critique of Pure Reason*. Presumably, Kant's own theory of method would be included under the heading, but it's not clear what it would mean for the considerations we find in the "Theory of Method" in the *Critique of Pure Reason* to be "applied". As a theory of method for *transcendental* logic, it would seem that it cannot be applied "empirically", at least in the same sense as it would be in physics.

Leaving aside psychology and methodology for now (more on this below), what Kant's classification offers is the first clear attempt at an articulation of the distinction between "formal logic" and the "epistemology of a priori knowledge" in terms of their respective scope, method and place. Pure general logic, to the extent that it is pure, formulates a priori rules that hold for thinking or judging, and to the extent that it is general, these rules apply to all thinking. Transcendental logic, to the extent that it is transcendental, is also pure and thus yields rules that hold a priori. However, the rules in question apply to thinking about a subdomain of objects, namely the objects of experience or appearances. As such, it is reasonable (though perhaps not uncontroversial) to assume that Kant took the rules of transcendental logic to be "special".

Kant's influence on the history of logic comes from the fact that this metaepistemological framework, and with it the distinction between formal logic, transcendental logic, psychology and methodology, was almost immediately embraced by an astounding number of his successors. Discussions, constructive criticisms and rejection, but more often than not *mutatis mutandis* adoption of Kant's metaepistemological proposal, pervade the logical literature in the first decades of the 19th century. As we saw with Scholz (1931) above – the same holds for authors of Group 1 Adamson, Blackey and Windelband – these distinctions continued to inform conceptions of logic much beyond their initial reception. Kant had leveraged new conceptual resources – e.g. the distinction between a priori and a posteriori, pure and empirical, general and special rules, but also

between analytic and synthetic judgement and concept and intuitions – to articulate an account of knowledge as falling into different types of cognitive endeavours and, as such, as engaging distinct, yet connected explanatory resources. For the first time, philosophers found in Kant's metaepistemology the resources to articulate at once disciplinary boundaries and systematic connections between the following:

a formal logic as the study of the "form" of thinking (in Kant: pure general logic);
b the theory of a priori cognition as the study of the conditions of possibility of experience and the justification of a priori beliefs (in Kant: transcendental logic);
c psychology as the empirical study of thinking (in Kant: applied general logic);
d methodology as the empirical study of the rules that apply within specific sciences, i.e. domains of cognition (in Kant: special applied logic).

Kant's metaepistemological framework played an important role in defining the terms around which the relation between logic, epistemology, psychology and methodology/philosophy of science – as separate and independent disciplines – would henceforth be articulated. Whether these are different from the broad metaepistemological distinctions on which philosophers still rely today is not immediately obvious and is certainly an interesting question.

Logic in Kant and in Kant's Wake

In view of what precedes, the widespread claim that Kant's interest in logic was comparatively superficial needs to be rejected. Unfortunately, one made curious about Kant's views on the scope, method and place of logic is likely to find the task of contextualising them rather frustrating. Kant's views on logic were shaped on a continuum, as he taught the subject annually over the duration of his career. Assuredly, the evolution of Kant's thoughts on logic over 40 years is documented in the *Logic Lectures*. But the material we have – e.g. notes by students or note-takers – raises a number of questions.

When Kant started to teach logic in 1755, his views – they are discussed in Kant's *Habilitationsschrift*, the *Nova Dilucidatio* (1755) – seem to have reflected the contemporary mainstream. Meier's *Auszug aus der Vernunftlehre* (1752), the textbook Kant used, does not present itself as revolutionary relative to other logics of the same period, and in that sense at least it is representative of the context of Kant's early thought. Assuredly, Kant's views evolved, especially, as is well known, in the decade that predates immediately the publication of the *Critique*

of Pure Reason in 1781. Why Kant nonetheless continued to use Meier's textbook until the end of his career, when he retired in 1797, is open to speculation. Assuredly, Kant did amend his lecture notes profusely over the years, but the decision to not break completely with Meier is an odd one. Certainly, Kant would have had ample cause to do so. The point here is that one ought to be cautious when interpreting the views compiled in the *Jäsche-Logic* of 1800 or when ascribing these views to Kant. The book is one interpretation of the coherence of layers after layers of lecture notes which Jäsche organised with virtually no input from (an aging) Kant himself. Using these notes to understand what a Kantian logic would look like, as Jäsche attempted to do, is possibly an unsolvable philological puzzle.

While we lack sufficient evidence for determining what would have constituted Kant's own attempt at a textbook for "pure general logic", there is a remarkably rich and diverse number of such attempts by post-Kantians. Under 'post-Kantian logicians', I include here a broad range of individual philosophers who were influenced by Kant and wrote at length about logic, such as Maimon, Reinhold, Fichte and Hegel, as is customary, but also the plethora of more or less disremembered philosophers who occupied university positions and/or were part of the intellectual and philosophical environs in the German-speaking world and beyond in the decades that followed the publication of Kant's first *Critique*. The works of many of these authors, while invisible from the contemporary "canonical" perspective, were the fabric of the philosophical disciplines at the time.

All these authors addressed issues in their writings that make it clear that Kant's metaepistemological framework, i.e. his views on the scope, method and place of logic, were being put to work in almost all quarters of the German-speaking world and beyond. This convergence of views is partly what explains Friedrich Ueberweg's positing the existence of a 'Kantian School' of logic (1868, §29) in reference to Ludwig Heinrich von Jakob (1759–1827), Wilhelm Traugott Krug (1770–1842), Johann Gottfried Karl Christian Kiesewetter (1766–1819), Gottlieb Wilhelm Gerlach (1786–1864), Johann Gebhard Maass (1766–1823), Ernst Christian Gottlieb Reinhold (1793–1855), Jakob Friedrich Fries (1773–1843), Johann Friedrich Herbart (1776–1841) and Wilhelm Drobisch (1802–96).[16] With considerable overlap, post-Kantian logicians agreed

16 Heis (*infra*) discusses the views of some of them, explaining what makes them Kantian. Ueberweg thought little of the Kantian School:

> The logical works that proceed from the Kantian school, or which essentially share its tendency, refrain from entering upon the deeper problems, and do not make up for this want by perfect accuracy, sufficiency and clearness in the problems to which they have limited themselves.
>
> (Ueberweg 1868, §29)

on a range of issues. They were compelled, for instance, by the idea that cognition is a function of the interaction between concept (understanding) and intuition (sensibility), that general logic studies the form of the understanding, and they seemed to have agreed with Kant that the discipline that studies the form of thinking must have the highest degree of generality.[17]

One idea that has its roots in Kant's metaepistemological framework, and which continued to shape logic throughout the 19th century and beyond, is that pure general logic provides the foundational piece of the conceptual scaffolding required for the broader epistemological project of a theory of a priori cognition, i.e. the transcendental logic. As Kant sees it, pure general logic describes the rules without which "no thinking at all is possible" and, as such, it provides rules that are presumably at least normative – if not constitutive – of the thinking of all objects, i.e. anything. As such, these rules are involved in thinking a priori about objects of experience: that is, they inform transcendental logic as well.

This is an idea that continued to shape post-Kantian authors' positions until, as we have seen, at least Scholz in the early 1930s. Hegel is an important exception to the broad consensus on the importance of the distinction between thinking and cognising but, as Clinton Tolley spells out (Chapter 3), even Hegel takes logic itself to be about thinking, and, moreover, to be about thinking as something 'formal'. Otherwise the distinction between the study of thinking as a topic for formal logic and the study of cognising was put to work in a great number of other post-Kantian theories in Germany and beyond. For Wilhelm Traugott Krug, for instance, if the ultimate goal is

> theoretical or speculative philosophy [..., i.e.] the science of the original laws of that activity of the human mind which is called theoretical and consists in representing (*Vorstellen*) and **cognizing** (*Erkennen*)
>
> (Krug 1806, §1; my emphasis)

then we first need logic, which

> is meant to be a science of the original laws of the human mind as regards those activities which are called **thinking** (*Denken*) [...] and is therefore the science of the laws of the use of the understanding.
>
> (*ibid*, §7; my emphasis)

Krug's example is significant. As Patton shows in Chapter 5 (*infra*), Krug was an important vehicle for Kant's ideas in Britain in the first half of

17 See Lapointe (2012).

the 19th century, and we owe it in part to him that thinkers until Boole conceived of Logic as the "laws of thought". Generally, then, when Kant's successors discuss the interplay between an account of 'thinking' and an account of 'cognising' ('knowledge' was the term used to translate 'Erkennen' and 'Erkenntnis' in Britain), they invariably have in mind the Kantian conception of the distinction and relation between pure general logic as the study of the form of thinking/understanding and transcendental logic – or some novel, functionally equivalent doctrine, e.g. Krug's *Fundamentalphilosophie* – as the study of the conditions of experience and/or a priori cognition.

The notion that a study of the form of thinking (as defining the scope of pure general logic) is a precondition of the study of cognising comes across in a broad range of authors well beyond Germany and well into the 19th century:

> Before anything else, **an exact assessment of the human powers of cognition** through which will be determined what can be accomplished through the latter is thus necessary. §37. Here is presupposed that one already knows the dealings of the powers of representations and their laws. This is what, on the one hand, empirical psychology teaches us, [… and]; on the other hand, **logic, which develops** (**entwickelt**) **the merely general and necessary laws of thinking** from the concept of thinking.
>
> (Jakob 1800, §36/p. 11, my emphasis)

> We must first investigate the complete history of the formation of human cognition [anthropological logic, i.e. psychology – SL], in order to find therein the place of thinking and understanding. We can only then derive from this **the doctrine of the forms of thinking, the pure general logic itself, and afterward see how these forms of thinking are applied in human cognition** […].
>
> (Fries 1811, 7)

> Indeed the grounds of distinction between the several doctrines thus brought into connection, and the significance of the terms by which they are expressed, are intelligible only when taken in reference to the Kantian system. The peculiar sense attached to the term thought (from which follows naturally the formal view of logic), **the opposition between thought and knowledge (upon which rests the distinction between logic and theory of knowledge)**, the ultimate idea of the relation between thought, knowledge, and reality (upon which might be founded a distinction between logic, theory of knowledge, and metaphysics), are all Kantian in origin.
>
> (Adamson 1882, 110f, my emphasis)

The main difference between Kant and his successors who not only considered but adopted more or less integrally his metaepistemology was the focus of the investigative effort. The *Critique of Pure Reason* set out to establish "transcendental logic" as the new foundation of scientific knowledge. Kant, however, believed that "pure general logic" on which the latter is in turn established required comparatively less attention. Crucial issues such as, for instance, the theory of judgement – that is, a doctrine on which Kant's entire epistemological enterprise rests – are eminently neglected. In the *Critique of Pure Reason*, Kant's entire commentary on the "logical form of the understanding in judgement" is condensed in four points that cover a minute proportion of the book (Kant 1781, B95–102). It may be that Kant believed that these matters were already settled – as we know he suggests as much – and that logic was in fact "closed" and "complete". But Kant's immediate successors manifestly disagreed. They all dedicated substantial portions of their philosophical efforts to providing pure general, i.e. "formal", logic with an adequate treatment – and in some case, e.g. Fries, to providing a further anthropological, i.e. psychological, foundation in turn for the latter.

The point to take away is this: Kant's metaepistemological framework, and the Kantian distinction between pure general logic and transcendental logic in particular, was crucial in establishing the study of formal logic as an independent discipline. What Adamson, Blakey and Windelband – i.e. the authors of the earlier Group 1 – have in mind when they praise Kant's role in the development of the discipline is the metaepistemological framework just described. Kant's views on the scope, method and place of logic contributed to bringing about a general metaepistemological reckoning in philosophy in the German world and beyond. The fundamental logical reforms that took place over the course of the 19th century arose as a consequence of Kant's metaepistemology making place for the sort of research programmes that would lead to the separation of logical, psychological, epistemological and methodological concerns around the study of knowledge and cognition. As such, Kant's theories can be seen to have set up the conditions that progressively led to the invention of formal logic as a programme that is epitomised in the works of a vast plenitude of authors, albeit with different levels of sophistication and success, and which gradual, through a more or less traceable nexus of documents, converges in the logicist (e.g. Frege, Dedekind, Russell and Whitehead), formalist (Hilbert), algebraic (Schroeder and Boole), neo-Kantian (e.g. Cohen and Carnap) and even phenomenological (e.g. Husserl and Twardowski) approaches.

In light of this, what needs to be stressed is that historians of philosophy who seek to understand how the discipline evolved ought to resist the idea that whatever revolution produced logic as we know it arose *ex nihilo*. The disciplinary developments that led throughout the

19th century to the logical reforms we associate with early analytical philosophers were the result of a nexus of progressive doctrinal shifts whose richness and complexity have been widely simplified or even sometimes entirely overlooked in standard narratives. It is neither the case that modern logic was "born from Frege's brain unfertilized by external influences" (Dummett 1981, xxxv[18]) nor that the development of the discipline was cumulative and linear (cf. Lewis 1918, 5). Doctrinal shifts drive disciplinary development in ways that are neither cumulative nor linear. Philosophical doctrines evolve because they are, for example, criticised, deliberately modified, inadvertently reinterpreted, rejected or misinterpreted, giving rise to convoluted chains of texts. While some of the authors involved might have remained obscure or shunned or even anonymous, their work nonetheless shaped the context on the basis of which contemporary and subsequent theories need to be interpreted.

It might be tempting to leave aside the tedious work of doing justice to the actions of more or less "minor" figures that populate historical contexts, to avoid the sinuous and arborescent paths disciplinary developments usually take and to assume that there is a straight line between the "great" philosophers who tend to attract fascination. But each time we fail to do justice to the richness of the philosophical context, we draw inferences that do not truly do justice to our discipline's history and the way it evolved. By the same token, we mischaracterise an important aspect of philosophical rationality. Our philosophies are not shaped only by great influencers. Insistence on narratives that represent the history of philosophy as an epic tale of gods and titans generally has more to do with self-congratulations than with honest intellectual curiosity. Such narratives are also a serious impediment to an accurate representation of diversity in the discipline. Philosophies are shaped by problems, canons and debates that are anchored in the institutions in which philosophers participate. Past philosophers ought to be seen as members of overlapping epistemic communities and their work as being subject to pressures that emerge from these eminently complex contexts.

In the chapters that follow, various aspects of the discipline's development are being thrown into light for the first time, while others are being reassessed so as to do justice to the complexity of the context in which logic evolved, from Kant to Russell. Together they suggest the main lines of a new narrative on the origins and foundations of early analytical philosophy and possibly 20th-century philosophy more broadly construed, including Neo-Kantianism, Phenomenology, Logical Positivism and Pragmatism. While the range of topics covered is quite broad – the Kantian School, German Idealism, Bolzano, the New Analytic, Lotze, Dedekind, Cohen, the Cambridge "mental sciences"

18 Dummett (1981).

as well as Sigwart and Hilbert (in connection to Russell's early work) – the contributions of Fries, Herbart, Trendelenburg, Brentano, Mill and Husserl, to name the most obvious, should be flagged as obvious topics for future reflection. The decision to not include dedicated chapters on Frege and Russell was deliberate. Frege and Russell have acquired such immutable canonical status in analytical philosophers' narratives that the only way to make sure that they don't dominate the story is to provisionally refrain from giving them too much place. A reassessment of Frege and Russell's position in logic's development is required that does not presume of the direction logic was taking before Frege and Russell's came to inform it.

The 11 chapters that follow are an attempt at starting again from the beginning, in order to portray the kinds of transformation that created the conceptual space in which the radical logical reforms of the late 19th and early 20th centuries took place. They can be read chronologically, as covering various aspects of the development of logic between the publications of the *Critique of Pure Reason* (1781) and that of the *Principia Mathematica* (1910–13). They can also be considered thematically, to revolve around five tasks:

1 To illustrate the way in which Kant's views on logic impacted the work of his successors, for instance, in the "Kantian logical school" (Heis, Chapter 1) and, later on in Britain, in a range of authors from Hamilton to Boole (Patton, Chapter 5).
2 To provide insight into the work of philosophers who sought to develop their own original logical systems, either because, like Bolzano, they fundamentally rejected Kant's epistemology (Lapointe, Chapter 4) or, like Hegel, sought to provide an account of "thinking" that would putatively overcome the limitations imposed on it by Kant's own critical philosophy (Tolley, Chapter 3).
3 To show how philosophers of the 19th century anticipated some of the crucial developments of logic in the 20th century and, in doing so, to offer a new perspective on the origins of contemporary logical Platonism (Stang, Chapter 6) or non-classical approaches to the definition of logical consequence (Priest, Chapter 2).
4 To present the views of logicians whose theories make for an interesting counterpoint to those that established themselves as paradigmatic after Frege's seminal work, either because, like Dedekind, they encourage a closer examination of Frege's own logicist project (Reck, Chapter 8) or because, like Cohen, they make us realise that the idea that logic is distinctively formal still did not make consensus at the beginning of the 20th century (Beiser, Chapter 7).
5 To paint a richer picture of the context in which Russell came to his main positions and, in particular, of the various conceptions of logic that contributed to shape his views, whether it be the conception of

logic one would have been made to embrace as part of the "mental science" curriculum in Cambridge in the late 1990s (Preti, Chapter 9), the conception of logic as a "methodology" we find in Sigwart (Morris, Chapter 10) or the putatively "formalist" approach of Hilbert (Griffin, Chapter 11).

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1 The Logicians of Kant's School

(Or, If Logic Has Been Complete Since Aristotle, What's Left For a Logician To Do?)

Jeremy Heis

One of the most infamous claims in Kant's *Critique of Pure Reason* concerns the completeness of formal logic:

> [S]ince the time of Aristotle [formal logic] has not had to go a single step backwards, unless we count the abolition of a few dispensable subtleties or the more distinct determination of its presentation, which improvements belong more to the elegance than to the security of that science. What is further remarkable about logic is that until now it has also been unable to take a single step forward, and therefore seems to all appearance to be finished and complete.[1]
>
> (Bviii)

This infamous claim was subject to severe criticism in the early part of the 19th century from figures as diverse as Bolzano, De Morgan, Hegel, and Fries.[2] Of course, it is not surprising that figures such as these would be critical of Kant's claim, since their conception of the scope and content of logic differed fundamentally from his. What's more surprising is that there arose starting already in the 1790s and into the 19th century a group of logicians who self-consciously thought of themselves as orthodox Kantians and who wrote extensive and original works in formal logic. Not only did these logicians show an affinity with Kant's conception of logic, but their contemporaries and later logicians thought of them as forming a kind of school – what Friedrich Ueberweg, in his

1 Citations to Kant 1781/7 are according to the pagination in the first ("A") and second ("B") edition. Page references to Kant's other works are to the pagination in the Akademie [Ak] edition: Kant 1902–. References to the body of Kant 1800 (The "Jäsche Logik" or *JL*) are frequently to paragraph (§) number.

2 Cf. Heis 2012, 95–7.

history of logic in the 19th century, calls the "logic of Kant's school."[3] Ueberweg himself, who was very critical of the school, listed as members "Jakob, Kiesewetter, Hoffbauer, Maas, Krug, etc."

What made these logicians 'Kantian' is that they held to Kant's conception of logic as formal. This distinctive conception of logic had four parts. First, formal logic was distinguished from transcendental logic. Second, formal logic was considered the science of thinking, not of cognizing. Third, it was asserted that formal logic abstracts from all relation to an object. Fourth, formal logic was held to be independent of psychology and metaphysics.

Logicians in this school included Kant's students, such as Schultz and Kiesewetter, but also later logicians who wrote after Kant's death, such as Krug and Esser. Here is an undoubtedly incomplete list of some of the logicians in this Kantian school, along with some of their major works.[4]

Schultz, Johann. 1789. *Prüfung der Kantischen Critik der reinen Vernunft*. Vol. 1.
Jakob, Ludwig Heinrich. 1791. *Grundriss der allgemeinen Logik*. 2nd ed.
Kiesewetter, J.G.C. 1791. *Grundriss einer allgemeinen Logik nach Kantischen Grundsätzen*.
Hoffbauer, J.C. 1792. *Analytik der Urtheile und Schlüße*.
Maass, J.G.E. 1793. *Grundriss der Logik*.
Kant, I. 1800. *Logik*. Ed. Jäsche.
Krug, Wilhelm Traugott. 1806. *Denklehre oder Logik*.
Herbart, Johann Friedrich. 1813. *Lehrbuch zur Einleitung in die Philosophie. Zweiter Abschnitt. Die Logik*.
Esser, Wilhelm. 1823. *System der Logik*.
Drobisch, Moritz. 1836. *Neue Darstellung der Logik*. ^{2}1851, ^{3}1863.
Hamilton, William. 1860. *Lectures on Logic*. (Delivered 1837–8.)
Mansel, Henry Longueville. 1851. *Prolegomena Logica: An Inquiry into the Psychological Character of Logical Processes*.

3 Cf. Ueberweg [1857] 1871, §29.

4 This list of logicians of Kant's school is no doubt incomplete and open to debate. In putting together this list, I have more or less followed Ueberweg. In particular, among the German logicians, I list all of the logicians whom Ueberweg identifies as among "The Logic of Kant's School." This list admittedly omits logicians who plausibly belong, such as Mehmel and Krause. Ueberweg also lists other logicians whose works are "more or less related to this formal view-point": Christian Twesten, Ernst Reinhold, Carl Friedrich Bachmann and Friedrich Fischer. I don't discuss these "more or less related" works in this chapter, partly because my expertise is limited, and partly because I am trusting Ueberweg's opinion that they depart in one way or the other from the conception of logic definitive of Kant's school.

To Ueberweg's list, I have added the British logicians Hamilton and Mansel, for reasons I will shortly explain.

One omission of note: I exclude Fries, since he does not hold the distinctly Kantian view that formal logic is independent of psychology (cf. Fries [1811] 1837, 4–5).

Not surprisingly, most of these logicians were German and wrote in the few decades after the publication of the first *Critique*. However, the last two logicians on the list were British, and that requires some explanation. But first some background about the history of logic in Britain in the early 19th century. In 1826, Archbishop Whatley ([1826] 1866) restored the fortunes of logic in Britain by arguing that logic is a science, not an art. As a science, logic contains a law, namely the *dictum de omni et nullo*, that explains the validity of the figures of the syllogism. And since logic is not an art, it's not meant to be a tool of discovery, nor a "medicine of the mind." Whately used this conception of logic to defend logic against its detractors, who pointed to the alleged sterility and lack of utility of the study of logic. A few years later, William Hamilton defended Whately's view of logic, but argued that in fact Whately's point of view had already been expressed 50 years earlier by Kant in the *Critique of Pure Reason*. Hamilton ([1833] 1861) argued that Kant had already maintained that formal logic is a science; that the logical laws explain the validity of forms of inference; that, as a canon and not an organon, it is not a tool of discovery; and lastly, that it was not designed to cure errors in reasoning. These points about Kant's conception of formal logic were captured in Kant's distinguishing formal logic from special logic and applied logic, respectively. Hamilton, having defended Kant's conception of logic, in the following years lectured extensively on formal logic using the texts of the German Kantian logicians Krug and Esser. These lectures were formally published many decades later, after a wide circulation, in 1860 (cf. Hamilton [1860] 1874). Among Hamilton's students was Henry Mansel, who wrote *Prolegomena Logica* in 1851, defending a kind of Kantian conception of formal logic.

The very existence of a Kantian school seems surprising, since it's not clear, from our point of view, what exactly the Kantian school took itself to be doing. After all, if logic has been complete since Aristotle, what's left for a logician to do? In this chapter I want to answer this question. I'm going to identify four representative questions or issues that the logicians in this school considered and debated. In each case I'll argue that this was a question that a reasonable reader of Kant might think Kant had left unsettled, and then I'll give a representative sample of the kinds of answers that logicians in this school gave to these questions. My goal in this chapter is not to be exhaustive, but to introduce the reader to the kinds of questions, debates, and philosophy that were done by logicians in the Kantian school – a school that at least in the last century or so has been more or less absent from the historiography of logic. Since my goal in this chapter is to give the reader a sense of the questions asked by the Kantian logicians and to give the reader a sense of the kinds of answers that they gave, my goal in this chapter will not be to evaluate the answers given, either as interpretations of Kant or as philosophically

defensible positions in their own right. Though I will occasionally editorialize, my goal here is simply to present the various views and not to evaluate them. I'll identify four questions that the Kantian logicians attempted to answer.

1 What is the relationship between formal logic and analyticity?
2 What are the logical laws and how are they related?
3 What does it mean to say that logic is formal?
4 Are all concepts formed through comparison, reflection, and abstraction?

I'll address each of these questions in turn in the following four sections of this chapter.

What Is the Relationship between Formal Logic and Analyticity?

It's clear from Kant's characterizations of analytic judgments that every analytic judgment is meant to be knowable through logical laws alone. At various points he asserts that analytic judgments are "thought through identity" (A7/B10), that "their certainty rests on identity of concepts" (*JL*, §36), and that they are knowable through the principle of contradiction alone.

> For, if the judgment is analytic, whether negative or affirmative, its truth can always be adequately known in accordance with the principle of contradiction. The reverse of that which as concept is contained and is thought in the knowledge of the object, is always rightly denied. But since the opposite of the concept would contradict the object, the concept itself must necessarily be affirmed of it. The principle of contradiction must therefore be recognized as being the universal and completely sufficient principle of all analytic knowledge.
>
> (A151/B190–1)

Since the principles of identity and non-contradiction are logical laws, and are sufficient for knowing all analytic judgments, it follows of course that, on Kant's conception, all analytic judgments are derivable from logical laws alone. But is the converse true? Is it true that every judgment knowable through logical laws alone is an analytic judgment?

Kant does not say so, at least not directly. This may seem surprising. At least since Frege, analytic judgments have often been characterized as those that are provable from logical laws plus definitions. From this conception of analyticity, it of course follows that all logical propositions are themselves analytic.

> The problem becomes, in fact, that of finding the proof of the proposition, and of following it up right back to the primitive truths. If, in carrying out this process, we come only on general logical laws and on definitions, then the truth is an analytic one.
>
> (Frege [1884] 1950, §3)

However, this conception of analyticity not only differs from Kant's, but was presented a full century after the *Critique of Pure Reason*. What's more, this conception of analyticity seems to have been novel with Frege. I have been unable to find any source before Frege who asserts that analytic truths are those provable from logical laws plus definitions.

Of course, absence of evidence is not evidence of absence. But there is a further piece of convincing evidence here. Couturat in his *Les Principes des Mathématiques*, in 1905, includes an appendix on Kant's philosophy of mathematics and his conception of analyticity. Couturat surveys various conceptions of analyticity and asserts that Frege's definition was original with him. He writes:

> One should therefore say, in order to remain true to the spirit, if not the letter of Kantian doctrine: a judgment is analytic, if it can be proved from definitions and laws of logic alone.
>
> (Couturat 1905, 246)

In a footnote Couturat cites Frege's *Grundlagen* and a paper by Gerardus Heymans from later in the 1880s (cf. Heymans 1889; cf. also Heymans 1886). Concerning Frege's *Grundlagen*, he writes:

> This latter work is one of the few in which the Kantian theory of arithmetic is handled with the greatest energy and depth…But it is also the only one which gets no mention in the bibliographies related to Kant. How useless are bibliographies!
>
> (Couturat 1905, 246)

Frege's characterization of analyticity, he admits, is not true to the letter of the Kantian definition. He gives two arguments for why post-Kantian developments were necessary in order to make this friendly amendment possible. First, it had to be clearly recognized that there are many other logical laws besides the principle of non-contradiction (PNC), and that there are other logical relations among concepts besides conceptual containment. Second, "what is thought in a concept" needed to be de-psychologized. Understood psychologically, this is a notion that is relative to a subject and a time; but, he argues, the definition of a concept is an objective matter, not relative to a subject. It is only these later developments in logic and philosophy, Couturat argues, that allow Frege's definition to seem like the friendly amendment to Kant's definition that it is.

One can dispute whether or not Frege's definition really is an improvement over Kant's, but I think the historical point is convincing: the conception of analyticity that makes it immediate that analytic truths are logical truths and *vice versa* did not arise until almost a century after Kant's own writing. Given the fact, then, that Kant never asserts that all logical truths are analytic, the question then becomes whether or not his followers took him that way. Was it obvious to those in the Kantian school that all logical truths are analytic? Now, it's true that some logical laws are clearly analytic in Kant's view. In particular, he argues that the PNC itself is analytic (cf. A152). But is it the case that all of the logical laws are analytic? Or are all the propositions of pure general logic analytic?

Among the first two generations of Kantian logicians, which included Kant's students and those writing in the first few decades of the 19th century, I found only one philosopher who asserts that all of formal logic is analytic. This is not in a logic text but in a passing remark in Schultz's *Prüfung* from the 1780s. Schultz writes:

> In general logic there is a pure part, which consists in propositions that are obviously a priori, but which are not synthetic, but rather are all analytic.
>
> (Schultz 1789, 45)

This is as clear a statement as you're going to find. However, Schultz's argument for this claim unfortunately shows a serious misinterpretation of Kant's view. The passage continues

> Insofar as it determines the necessary rules of all thinking theoretically, without regard to its actual execution or application in particular cases, the understanding is concerned neither with any objects, nor with our sensibility, but merely only with itself, and the analysis of its form; consequently, the theoretical part of general logic as mere analysis of our form of understanding, is an entirely pure science, to which nothing empirical must be mixed, and whose rules are obviously a priori, but not synthetic, but rather analytic.
>
> (Schultz 1789, 45–6, emph. added)

Schultz is certainly picking up on a real theme in Kant's writings on logic. Kant believes that formal logic results from an analysis, namely, of the actions [*Handlungen*] of our faculty of the understanding.

> Since merely formal logic, so conceived, abstracts from all content of cognition (whether it be pure or empirical), and concerns itself merely with the form of thinking (of discursive cognition) in general, it can also include in its analytic part, a canon for reason, the form

> of which has its secure precept, into which there can be *a priori* insight through mere analysis of the actions of reason into their moments, without taking into consideration the particular nature of the cognition about which it is employed.
>
> (A131/B170, emph. added)

So Schultz is clearly correct that formal logic is 'analytic' in the sense that it arises through an analysis of our faculty. However, this emphatically does not show that the propositions of formal logic are themselves analytic truths, any more than it shows that the propositions of the "Transcendental Analytic" in the *Critique of Pure Reason* are all analytic.

Indeed, in the *Jäsche Logic* (Ak 9: 16), Kant makes clear that this notion of 'analytic' contrasts not with 'synthetic' but with 'dialectic.' And he claims that logic is an analytic in the sense that it's a "canon," as opposed to a dialectic, which would be an "organon" for this or that science. Pure general logic or formal logic is an analytic or a canon; a special logic or dialectic is an organon. This same kind of contrast was picked up by later Kantian logicians. For instance, Jakob Esser (Esser 1823) divides pure logic into two parts: the analytic, which he calls the doctrine of elements, and the synthetic, which he calls the doctrine of method. In this way, he follows the distinction in the *Jäsche Logic* between an analytic, which is a canon, and a dialectic, which is an organon. However, it does not seem that Esser makes the additional (mistaken) step to claim that the doctrine of elements is composed of analytic truths and the doctrine of method is composed of synthetic truths.[5]

Besides this one-off and frankly confused statement from Schultz in the 1780s, a sustained statement in defense of the claim that the propositions of logic are all analytic doesn't appear, as far as I know, until 70 years after the publication of the *Critique* in the work of Henry Mansel. He argues that every law of logic is analytic or identical, since they are laws of the mind's conformity with itself.

> [The fact that] the fundamental principles of pure thinking are, as they seem to be, analytical or identical… points to the important fact that these principles are laws of mind…These [laws] are the highest and simplest forms of identical judgments, to one of

5 Indeed, the contrast between logic as an analytic and logic as an organon is reflected in the very titles of other early 19th-century German texts of 'formal' logic: Hoffbauer's *Analytik der Urtheile und Schlüsse*; Mehmel's *Versuch einer vollständigen analytischen Denklehre*; Twesten's *Die Logik, insbesondere die Analytik* and *Grundriß der analytischen Logik*; and Krause's *Vorlesungen über die analytische logik und Enzyklopädie der Philosophie*. But, again, this contrast is emphatically not to be confused with the contrast between analytic and synthetic judgments. (Thanks to Sandra Lapointe for pointing out these titles to me.)

> which all analytical thinking may ultimately be referred: and all pure thinking may be shown, on psychological grounds, to be of strictly analytical character. The necessity arising from these laws is that of harmony of thought with itself, – of its conformity to its own nature.
>
> (Mansel [1851] 1860, 159–60, 202)

Mansel's argument here is interesting. Since logical laws are grounded in the nature of the understanding itself, this means that the propositions of formal logic express the mind's conformity with itself; thus, he claims, these expressions of self-identity should themselves be identical or analytic judgments.

Though there were only a few Kantian logicians who argued explicitly that all propositions of formal logic are analytic, this does not mean that 19th-century 'Kantian' logicians did not reflect in other ways on the relationship between logic and analyticity. Instead, these logicians asked a different kind of question: is formal logic concerned with the forms of all judgments or only the forms of analytic judgments? This question may be surprising to Kant's contemporary readers, since (I believe) few if any readers now question that Kant intended formal logic to be a canon for all judging regardless of subject matter. Fries in 1824 seems to be the first person to express the contrary view:

> Philosophy is partially formal (namely Logic, the doctrine of the understanding), partially material (which we should call metaphysics, or following Kant, doctrine of the understanding). Philosophical logic is the system of analytic judgments, that is, to it belongs all those philosophical cognitions that contain only the laws of the thinkability of things for themselves, without regard for any special kind of objects that we think about. Metaphysics on the other hand is the system of synthetic judgments, of which we become conscious only through thinking (Fries, *System of Metaphysics*, 1824, §10; cf. also Fries, *System der Logik*. 1811/37, §40).[6]

Now Fries was not quite a logician of Kant's school in my sense, since he criticized Kant for too strongly separating formal logic from psychology. However, a few decades later, Mansel gave an explicit argument that formal logic concerns the forms of only analytic judgments. He began

6 Fries's statement here, as well as in 1811/37, §40, does not seem unambiguous to me. Perhaps he means that all the propositions of logic are analytic and all of the propositions of metaphysics are synthetic – a plausible plain reading of his words, but an implausible conception of metaphysics. The better reading of Fries, I contend, is that he means that the propositions of logic are *about* analytic judging, and the propositions of metaphysics are *about* synthetic judging.

this argument by first claiming that logic is the "science of the laws and products of pure or formal thinking." He then defines pure or formal thinking in this way:

> Pure judgments are those in which the given concepts are of such a character that their mutual relation of agreement or disagreement can be determined by an act of thought alone.
>
> ([1851] 1860, 220)

These pure judgments are, then, precisely the analytic ones, and so logic becomes the science of the laws and products of analytic judgment. This view, though it has few defenders now, had a surprising number of defenders in the mid- to late 19th century. Ueberweg defends it (cf. Ueberweg, [1857] 1871, §2), as does Hermann Cohen in his *Kants Theorie der Erfahrung* (cf. Cohen 1885, 242). Cohen argues that formal logic, as Kant understands it, abstracts from all relation to an object. And analytic judgments, he claims, are not actually about objects but about concepts. So, formal logic can only tell us about analytic judgments, not about synthetic ones.

What Are the Logical Laws? How Are They Related?

A distinctive feature of Kant's conception of logic is his rejection of Wolff's reductionism. Wolff had a reductive program that reduced all forms of judgment to categorical ones, all kinds of truth to containment, all inferences to syllogistic, and all laws of logic to the PNC. Kant rejected this reductionism. In this way, he introduced a pluralist program into logic. Indeed, the Kantian school was a middle way between reductionist programs, such as Wolff's, and 'derivationist' programs, such as Hegel's and Fichte's. In this respect, the Kantian school stood between the better known philosophers writing logics in the German idealist tradition, on the one hand, and the British tradition in the 19th century, on the other hand, which tended to be reductionist. This conception introduced a kind of program into formal logic to identify the fundamental laws of logic, without reducing them or deriving them all from one another, and to explain how they interrelate with one another.[7]

Concerning the most basic point, enumerating the fundamental principles of logic, the *Jäsche Logic* is ambiguous. In one place, *JL* lists three fundamental principles of logic:

7 An interesting exception here is Herbart, who seems to deny that the principles of identity, contradiction, excluded middle, and sufficient reason have any substantive role in formal logic. Indeed, he criticizes Kant for moving the PNC and PI into logic, "where they are useless" (Herbart [1813] 1850, 82).

> Thus we will be able to advance three principles here as universal, merely formal or logical criteria of truth; these are
>
> 1 the principle of contradiction and of identity (*principium contradictionis and identitatis*), through which the internal possibility of a cognition is determined for problematic judgments;
> 2 the principle of sufficient reason (*principium rationis sufficientis*), on which rests the (logical) actuality of a cognition, the fact that it is grounded, as material for assertoric judgments;
> 3 the principle of the excluded middle (*principium exclusi medii inter duo contradictoria*), on which the (logical) necessity of a cognition is grounded – that we must necessarily judge thus and not otherwise, i.e. that the opposite is false for apodeictic judgments.
>
> (Ak 9: 52–3)

To begin with, this quotation – along with passages in the first *Critique* where analytic judgments are said sometimes to be grounded on identity (A7/B10) and sometimes on contradiction (A151) – raises the question of the relationship between the principles of contradiction and of identity: Are they the same principle or different principles? What's more, this list of the laws of logic doesn't include Kant's replacement for the *dictum de omni et nullo*. That principle is articulated in this way in the *Jäsche Logic*:

> What belongs to the mark of a thing belongs also to the thing itself; and what contradicts the mark of a thing contradicts also the thing itself.
>
> (*JL*, §63)

Kant believes that this dictum explains the validity of the figures of the syllogism (*Heschel Logic*: Kant 1992, 97), but he's unclear here and elsewhere whether it's an independent principle or whether it is derived from these other laws.

These are the sorts of questions that logicians in the Kantian tradition had to tackle. To give a sense of this, I begin by listing the various characterizations of the laws of logic given by Krug and Esser, who agree that there should be four fundamental laws of logic but disagree in significant ways on how these laws of logic should be formulated. Here are Krug's four laws of logic (Krug 1806, §§13–22):

1 The principle of absolute identity: "Everything is identical to itself," or "The concept of a thing is identical to the sum of its marks."
2 The principle of thesis: "Posit nothing contradictory but rather only what agrees!"

3 The principle of antithesis: "Only posit one among opposing determinations of a thing, and if one is posited, you must withdraw the others."
4 The principle of synthesis: "Posit nothing without ground."

Note the interesting feature that laws 2, 3, and 4 have an imperatival form.

Esser's formulation of the laws of logic is different (Esser [1823] 1830, §§14–30):

1 "All that is identical to an object must be attributed to it."
2 "To every object must be denied all that is opposed [Gegentheil] to it."
3 "To every fully determinate object every possible mark either belongs or does not."
4 "If one of two opposing marks should be affirmed or denied of an object, then there must be a sufficient ground on account of which this is attributed or denied of it."

Note that Esser's laws do not have an imperatival form, but rather all concern the marks that belong to or don't belong to an object.

A look at various formulations of the PNC will again bring out the variety of ways in which the laws of logic were formulated in this tradition.

- Jakob: "Contradictory representations cannot be united in one consciousness" (Jakob 1791, §121).
- Kiesewetter: "Manifolds that are contradictory cannot be united in a unity of consciousness" (Kiesewetter 1791, ad §97).
- Krug: "Posit nothing contradictory but rather only what agrees!" (Krug 1806, §18).
- Herbart: "What is opposed [Entgegengesetztes] is not identical" (Herbart [1813] 1850, §39).
- Esser: "To every object must be denied all that is opposed [Gegentheil] to it" (Esser [1823] 1830, §§18).

I mentioned earlier that Kant sometimes formulates the principles of identity and non-contradiction as one principle, sometimes as two. There was no consensus among the Kantian logicians on this question either. Jakob, for instance, holds that the principle of identity (PI) is in fact derivable from the PNC (cf. Jakob 1791, §121), as does Herbart (cf. [1813] 1850, §39). Esser (cf. [1823] 1830, §21) and Krug, on the other hand, held that the PNC follows from the PI. Here is Krug's argument:

> Because the concept and its marks are identical, if someone were to think an object through its mark, this thinking would be impossible,…if one wanted to add to its concept an opposing mark.
>
> (Krug 1806, §17)

Herbart, for his part, claims that the PNC and PI are equivalent [gleichgeltend] and imply the principle of excluded middle (cf. Herbart [1813] 1850, §39). Still other Kantian logicians, such as Hoffbauer (cf. 1792, §23) and Mansel (cf. [1851] 1860, 168), argued that the PNC and the PI are not derivable from one another.

We saw earlier that Kant's official list of logical laws sometimes does not include the *dictum de omni et nullo*, leaving open the question of its relationship to the other logical laws. For instance, the *Jäsche Logic* (§§63, 76, 78) argues that the rules of syllogistic follow from the dictum (while the rules of hypothetical inference – modus ponens and modus tollens – follow from the principle of sufficient ground, and the rules of disjunctive inferences follow from the law of excluded middle). This then gives the dictum an independent status within logic. But in logic lectures delivered in the 1780s (cf. The Dohna-Wundlacken Logic, Ak 24: 773), it is claimed explicitly that the dictum is a consequence of the PNC. In other logic lectures, this seems to be denied. In the Heschel Logic, we find:

> A proposition that is to become the possibility of inferences of reason cannot in turn be proved, …for one would have to presuppose it in order to prove it. The highest *principium* of categorical inferences is this: *nota notae est nota ipsius*.
>
> (Kant 1992, 98)

Indeed in the Heschel Logic (Kant 1992, 97), he argues that the PNC holds for propositions only and not for inferences of reason; thus, it could not imply the *dictum* since the *dictum* is a principle for inferences of reason.

How do the Kantian logicians address this question? Esser, for instance ([1823] 1830, §88), has an interesting argument that the *dictum* is derived from the PI, together with the PNC and a further principle that identity is transitive.

The question of the relationship of the PNC and the *dictum* leads to a more general question: is the PNC the highest principle of formal logic, as it were, a first among equals? There are some places where Kant can be read in this way. For instance, at A55/B79–80, Kant argues

> But concerning the mere form of cognition (setting aside all content), it is equally clear that a logic, so far as it expounds the general and necessary rules of understanding, must present criteria of truth in these very rules. For that which contradicts these is false, since the understanding thereby contradicts its general rules of thinking and thus contradicts itself. But these criteria concern only the form of truth, i.e. of thinking in general, and are to that extent entirely correct but not sufficient. For although a cognition

> may be in complete accord with logical form, i.e., not contradict itself, yet it can still always contradict the object. The merely logical criterion of truth, namely the agreement of a cognition with the general and formal laws of understanding and reason, is therefore certainly the *conditio sine qua non* and thus the negative condition of all truth; further, however, logic cannot go, and the error that concerns not form but content cannot be discovered by any touchstone of logic.

Here Kant argues that logic presents a criterion of truth. This criterion of truth amounts only to the negative requirement that a cognition be in complete accord with its logical form, i.e. that it not contradict itself. This suggests that the PNC is in a sense the master principle of formal logic.

Krug considers a similar kind of question, but comes to a different conclusion, namely that the PI is the first among equals.

> Identity is properly indicated by the formula: A = A. In this formula, the thing *posited* through the concept is *opposed* to itself and *posited as equal* in this opposition. One can therefore resolve this formula into the proposition: *everything is identical to itself*, and this is called the principle of absolute identity. It is the principle of *thesis*, *antithesis*, and *synthesis* in thinking in general.
>
> (1806, §17)

Recall from earlier that Krug calls the second, third, and fourth laws of logic the principles of "thesis," "antithesis," and "synthesis." Krug's argument for this claim has a strongly Fichtean flavor. The "fact of consciousness" is the certain material starting point of philosophy: "the philosophizing subject itself, that is, the I looking away (abstracting) from the outer (the given) and looking into the inner (reflecting)" (Krug 1806, p. viii). Because Krug believes that the fact of consciousness is the material starting point of philosophy, Krug also thinks that the starting point for logic should be the 'I think.' So we begin with the 'I think.' Now, that which I think must be thinkable. Thinking aims for truth and truth is fundamentally, he argues, self-agreement. So the highest principle of logic is A = A (*ibid*, §§13–14).

A surprising feature in Kant's list of logical laws is the appearance of the logical principle of sufficient reason. He characterizes the logical principle of sufficient reason in various ways. For instance, in the *Jäsche Logic*:

> But one cannot infer conversely that if no false consequence flows from a cognition, then it is true; for one can infer true consequences from a false ground.

> If all the consequences of a cognition are true, then the cognition is true too. For if there were something false in the cognition, then there would have to be a false consequence too.
>
> (*JL*, §52)

Given the general lack of clarity as to what exactly the logical principle of sufficient reason amounts to, what exactly its position is within formal logic remained unclear as well. Mansel, for instance, argued that the principle of sufficient reason is not formal but material and so not a part of formal logic after all (cf. Mansel [1851] 1860, 182). Unfortunately, Kant argued that the principle of sufficient reason is the ground for the validity of *modus ponens* and *modus tollens*. Mansel bites the bullet here and argues that *modus ponens* and *modus tollens* are not inferences of formal logic (cf. *ibid*, 194). A less extreme position is taken by Esser, who argues that the logical principle of sufficient reason is a part of formal logic, but has a special status inasmuch as it belongs in the doctrine of method (or the "synthetic," cf. Esser [1823] 1830, §26), not in the doctrine of elements (to which belong all the other logical laws). Herbart – ever skeptical of substantive uses of logical laws – is unconvinced that there is a "logical" principle of sufficient reason (cf. Herbart [1813] 1850, §39, Anmerkung) and makes no use of it in explaining the validity of *modus tollens* and *modus ponens* (cf. *ibid*, §64ff.).

What Does It Mean to Say That Logic Is Formal?

The central and distinctive claim of Kant's philosophy of logic is that logic is formal.

> [Pure] general logic abstracts from all content of cognition, i.e., from any relation of it to the object, and considers only the logical form in the relation of cognitions to one another.
>
> (A55/B79)

This claim gives rise to many philosophical and interpretive questions. For example, is logic formal because it concerns only a certain kind of thinking, formal thinking? Or is it concerned with all thinking but only with respect to its form? And what exactly are forms of thinking? In what precise sense does logic abstract from any relation of a thought to an object?

In this section I want to look at how Krug, Esser, Mansel, and Hamilton approach these topics. (I'll also discuss Drobisch's and Herbart's understanding of formality in the beginning of the next section.) Let's start with the first question: does logic concern thinking in abstraction from its relation to objects, or does it concern a certain kind of thinking, namely formal thinking? The first view, which I believe is now the common

reading, was defended by Esser ([1823] 1830, §§9–11), who argued in this way. Logic concerns all thinking, but taken in abstraction. There are no pure thought-acts open to observation that have only form and do not in any way concern objects. Instead, all thinking is about objects, even if those objects are merely possible objects. So the proper procedure in logic is to consider some act of thinking – and any thinking about any object will do – but abstract away from the objects of the thought.

The contrary view was given by Krug, who argues that logic concerns a certain kind of thinking, namely formal thinking.

> This function [i.e. thinking] can be considered … as a mere thinking, by means of which given representations are related only to one another, to become conscious of their particularities and relations, without further reflecting on the object, to which they may be related.
>
> (1806, §8)

Krug thus isolates "formal," "analytical," or "logical" thinking, which he distinguishes from "material," "synthetic," or "metaphysical" thinking, which "determines an object." The idea is that there are certain thought-acts through which given representations are related only to one another, and there is another kind of thought-act that relates representations to the objects themselves. Logic concerns only the former kind; it is abstract in the sense that it considers only a proper subset of the acts of thinking. A few decades later, as we saw in the previous section, Mansel takes up this idea, equating formal thinking with analytic judgment and thus arriving at the view that logic concerns only analytic judgement.

Kant claims that logic concerns only the logical form of thinking. What precisely are these forms of thinking? Krug glosses forms of thinking as original modes of activity (*Handlungsweise*) of the I "through the nature of the capacity to think in general" (Krug 1806, §11). This idea – that forms of thinking are modes or manners of the act of thinking – is taken up and expanded by Esser.

> Logic is customarily called a formal science; obviously this does not mean that logic has only a form and not an object; obviously the form of human thinking is the object of logic.
>
> ([1823] 1830, §3)

The form of thinking is the proper subject matter of logic. Now what, specifically, are these forms? Since logic is a priori, Esser argues, its object must be the general and necessary form of human thinking. Now, he argues, necessity is grounded in laws. So the necessity of the form of thinking must be grounded in laws of the faculty of thought itself, and not laws of anything outside of thought. Moreover, laws, he claims, are

general rules by which powers are brought into activity. Putting this all together, thinking must be a power or capacity with laws that flow from its own nature. Forms of thinking, then, are these internal laws of the activity of thinking (cf. Esser [1823] 1830, §§5–6). (Sometimes he makes the point slightly differently, treating forms of thinking not as laws for thinking, but as lawlike activities of thinking.)

Esser argues that the existence of laws explains both the generality and the formality of logic.

> Every law relates in no way to single or particular objects of thinking, but rather to all objects without distinction. They can only determine the way and manner (the form), in which the thinking mind [*Geist*] takes up according to its original direction the particular objects without distinction.
>
> (Esser [1823] 1830, §6)

Esser here is elaborating on the idea we canvassed just a few paragraphs back: that logic concerns all thinking, not just "formal thinking." Though all acts of thinking concern objects, he claims, since thinking has its own internal laws, these laws can explain the generality and formality of logic. Laws are general, and the internal, necessary laws of thinking are its form.

Since Esser emphasizes that logic concerns all thinking and that all thinking concerns an object, this makes it important for him to elaborate on what exactly abstraction amounts to. In what sense do we abstract from relation to an object? Esser explains the possibility of abstracting by distinguishing between two aspects of thinking.

> By thinking we understand any designation of an object [Gegenstand] through a mark, that is, through a concept. And in this designation of an object through a mark, that is, in this thinking we distinguish by means of abstraction two things from one another: 1. The object [Objekt], which is thought (the matter), and 2. The way and manner [Art und Weise] in which it is thought (the form) of our thinking.
>
> (Esser [1823] 1830, §3)

Logic is abstract because it concerns only the manner in which the object is thought and not in any way the particular object that is thought.

This distinction is expanded and ramified in Hamilton's lectures, which in fact explicitly draw on Esser's account. Hamilton claims that within thinking we can distinguish three aspects: first, the thinking subject itself; second, the object about which we think; and third, "a relation between subject and object of which we are conscious – a relation always manifested in some determinate mode or manner" (Hamilton [1860] 1874, 73). Hamilton calls this third aspect the "form of thought." Hamilton

thus differs from Esser in thinking of the form of thought not as a law of thinking (which Esser sometimes glosses as a mode or way of thinking), but as a certain kind of relation between the subject and the object. Within this relation, he argues, one can distinguish two sides: "an act, operation, or energy" and the "product of such an act" (*ibid*, 74). He claims that only the latter belongs to logic. Logic therefore achieves its abstract character from abstracting away consideration of the act of thought itself, leaving the logician to consider only the product of that act.

Are All Concepts Formed through Comparison, Reflection, and Abstraction?

It is an historical fact that, especially after Hegel's attack on the theory,[8] the account of concept formation by comparison, reflection, and abstraction was very strongly associated with self-styled "formal logicians" – especially formal logicians of the Kantian school. This association was central in 1840, when Adolf Trendelenburg – as part of his forceful attack on the very idea of formal logic in the Kantian sense[9] – identified the theory of concept formation by comparison, reflection, and abstraction as a paradigmatic error of formal logic in the Kantian sense. Trendelenburg directed his attack against Moritz Drobisch, who had articulated in 1836 a Kantian theory of logic and defended the theory of concept formation by comparison, reflection, and abstraction. Trendelenburg identified Drobisch's *Neue Darstellung der Logik* as perhaps the clearest and most sophisticated presentation of formal logic in the Kantian sense (Trendelenburg 1840, 7). For Trendelenburg, then, a successful attack on Drobisch's theory of concept formation amounted to a refutation of the very idea of formal logic in Kant's sense. In this last section I'll give a brief overview of the Trendelenburg-Drobisch debate, which was one of the most important episodes in logic in mid-century Germany.

Let's begin by looking at the official theory of concept formation, articulated in Kant's *Jäsche Logic* and elsewhere as well.[10]

> To make concepts out of representations one must thus be able to compare, to reflect, and to abstract, for these three logical operations of the understanding are the essential and universal conditions

8 For details, see Heis 2014, 282–5.

9 Trendelenburg believed that the very idea of formal logic depended on objectionable features of Kant's philosophy: "For the first time in *Kant's* critical philosophy, in which the distinction of matter and form is robustly conceived, *formal logic* clearly emerges and actually stands and falls with Kant" (Trendelenburg 1870, 15; cf. Trendelenburg 1840, 4). By 'formal logic,' Trendelenburg has in mind all four characteristic features of the Kantian school, which I listed earlier in the second paragraph of this chapter.

10 See, for example, the first Introduction to *Critique of Judgment*, Ak 20: 212n.

> for generation of every concept whatsoever. I see, e.g., a spruce, a willow, and a linden. By first comparing these objects with one another I note that they differ from one another in regard to the trunk, the branches, the leaves, etc.; but next I reflect on what they have in common among themselves, trunk, branches, and leaves themselves, and I abstract from the quantity, the figure, etc., of these; thus I acquire a concept of a tree.
>
> (*JL*, §6)

Now there are three important but puzzling features of this account – puzzling features that will be relevant for the dispute between Trendelenburg and Drobisch. First, concepts can be formed by comparison, reflection, and abstraction not only from other concepts but also from intuitions.[11] When a concept is formed from an intuition, the concept formed thus differs in its logical form from the representation it is formed from: the concept is general but the intuition is singular. But this poses a puzzle: *how could taking a singular representation and leaving out part of it make it general?* Call this 'the generality puzzle.'

Second, in Kant's view, abstraction is merely negative and it does not create content.

> General logic abstracts from all content of cognition, and expects that representations will be given to it from elsewhere, wherever this may be, in order for it to transform them into concepts analytically. …Prior to all analysis of our representations [space and time] must first be given, and no concepts can arise analytically as far as the content is concerned.
>
> (A76–8/B102–3; cf. *JL*, §6, note 2)

Kant's idea is that concept formation can change the logical form in which content is represented, but it cannot introduce new content itself. Now this raises a second puzzle: *where then does this content come from?* Doesn't this admission leave the official theory of concept formation radically incomplete? Call this 'the content puzzle.'

Third, in the first introduction to the *Critique of Judgment*, Kant identifies a priori but subjective principles that make concept formation possible. These are principles for what Kant calls "reflective judgment," which he defines as "an ability to reflect in terms of a certain principle on a given representation so as to make a concept possible" (Kant 2000, Ak 20: 211). One such subjective principle is the principle of the purposiveness of nature, which asserts, roughly, that a system of concepts can be formed that is parsimonious, comprehensive, and gap-free. But this

11 See Kant, *What Progress?*, Ak 20: 273–4.

subjective maxim that guides our concept formation, since it is a principle for the systematicity of science, should belong to the doctrine of method of formal logic. However, the account of concept formation by comparison, reflection, and abstraction is located in a different part of logic, the doctrine of elements. This again poses a puzzle: *how do these methodological principles for concept formation interact with the 'official account'?* Are they equivalent to it? Do they replace it, or perhaps supplement it? Call this 'the methodology puzzle.'

The Trendelenburg-Drobisch debate was carried out over three editions of Drobisch's *Neue Darstellung der Logik* (1836, 1851, and 1863) and the three editions of Trendelenburg's *Logische Untersuchungen* (1840, 1862, 1870).[12]

In his *Neue Darstellung*, Drobisch defends a distinct version of the Kantian view of logic – a version of Kantianism that was first sketched in Herbart's *Lehrbuch zur Einleitung in die Philosophie* (cf. Herbart [1813] 1850). In particular, Drobisch follows Herbart in articulating two aspects of the Kantian conception of logic – two aspects that will be relevant to the debate over concept formation.

First, he claims that logic is formal, which he defends in a distinctive way. In his view, philosophy in general is concerned with concepts and not with objects. In Herbart's slogan, philosophy is the "working out" [Bearbeitung] of concepts (cf. Herbart [1813] 1850, 57). Philosophy "presupposes the object as already known" and so considers only our concepts of objects (Drobisch 1836, §3); in Herbart's words, "logic presupposes the concepts as known" ([1813] 1850, 42; cf. [1808] 1850, 467). Logic as formal philosophy is then concerned with "the most *general relations*" among concepts (Herbart [1813] 1850, 33) and not with their matter or "particular content" (cf. Herbart [1813] 1850, 42). As Drobisch puts it,

> The relations [of concepts] are either those which belong to all concepts, independently of the special [conditions] by which they are thought, thus relations that belong to all concepts, or they are those relations which are dependent on those special [conditions] and so are limited to certain classes of concepts. The former general conceptual relations are the subject *of logic*, as the first part of philosophy. It is also called *formal philosophy*, because it is directed to the consideration [not] of the matter that the concept contains and therefore only to the formal differences between them.
>
> (Drobisch 1836, §5)

12 In order to make it easier to locate references across the various editions of Trendelenburg's and Drobisch's books, I cite their works by section number, which (unlike page numbers) tend to be constant across the three editions of their works.

Second, Drobisch, like Kant (1800, Ak 9: 14, 16) and Herbart (1825, 173), believes that logic is normative and not descriptive.

> [Logic gives] the conditions of law-like connections among concepts. … But it is no description of thinking as it actually is, but rather a prescription [Vorschrift], how it *should* be; no natural laws of thinking, but rather a law book for thinking. Logic must *postulate* the formation and comparing of concepts. It avails itself of *actual* thinking as a fact, in order to cognize the conditions of its law-likeness; in no way does it come to be from a mere observation of one's own thinking.
>
> (Drobisch 1836, §9)

Logic, as normative but not descriptive, accepts the fact that concepts can be formed and compared; it doesn't investigate into the causal psychological laws that make this possible.[13] The normativity of logic is thus part of Herbart's and Drobisch's anti-psychologism about logic. As Herbart insists, "in logic it is necessary to ignore everything psychological" ([1813] 1850, §34).

Indeed, Drobisch's and Herbart's anti-psychologism goes one step further. They claim that logic, unlike psychology, is concerned only with the unchanging contents of concepts and their unchanging relations – and not in any way with the mental conditions of the temporal acts of forming these concepts.

> All our thinking can be considered from two sides: partly as activities [Thätigkeiten] of our mind [Geist], partly in view of *what* is thought through them. In the latter relation, they are called *concepts*, which word, inasmuch as it indicates that which is *conceived*, requires that we abstract from the manner and way that we may discover, produce, or reproduce the thoughts.
>
> (Herbart [1813] 1850, §34;
> cf. Herbart [1808] 1850, 467)

> Our thoughts are called *concepts*, insofar as we, abstracting from the way they came to be, attend only to *what* is thought in them. This what is called its content (complexus) [Inhalt]. It lies already in the explanation of the concept, that its content is independent of

13 In similar terms, Herbart denies the logic is concerned with mental [geistlich] laws ([1813] 1850, §34) or natural laws (§35, Anmerkung) of the mind; it is not concerned with thinking as an *event* (§34) and does not give a "natural history" (§35) of thinking.

> the changing mental conditions of the subject by whom it is thought, and that it therefore bears within itself the character of something abiding and unchanging.
>
> (Drobisch 1836, §11)

As Drobisch puts it, quoting Herbart's memorable phrase (cf. [1808] 1860, 467), "concepts as such are present only once."[14]

In addition to defending the roughly Kantian view that logic is formal and normative, Drobisch also defends the orthodox Kantian view that concepts are formed by abstraction.

> In every compound concept one can think away, *abstract* [abstrahiren] individual marks. The concept, which is then still left over, is called, in relation to the concept from which it came to be by abstraction, the *next highest* [i.e. the genus]. Every concept has as many next highest concepts as it has marks.
>
> (Drobisch 1836, §14)

Putting this passage next to *Jäsche Logic* §6 makes clear just how similar this view is to the Kantian one.

Trendelenburg initiated his polemic against the theory of concept formation from Drobisch's book in the first edition of his *Logische Untersuchungen*. An overarching goal of Trendelenburg's was to refute the very idea of formal logic in the Kantian sense and return to a pre-Kantian (and indeed Aristotelian) conception of logic: not as formal, but as an investigation into the metaphysical structure of the world that makes it possible for us to make judgments, form inferences, and develop sciences about it. This metaphysical structure, roughly, Trendelenburg found in Aristotle's metaphysics. Trendelenburg believed that Drobisch and the other formal logicians in fact had no theory of concept formation, despite what they claimed. Indeed, he found it revealing that Drobisch seemed to admit that he had to presuppose the very thing that, in Trendelenburg's view, logic is supposed to explain.

> Formal logic from the beginning presupposes the concept as given [in footnote, Trendelenbrug cites Drobisch 1836, §3, §11, and §14]. If we would understand a concept in its full meaning as that representation of a thing that discloses the ground of the thing, then in fact everything actually would be presupposed: we would take as given in the beginning what science has to first achieve as its final goal.
>
> (Trendelenburg 1840, Ch. I, §4)

14 Herbart further clarifies that concepts, though they have a fixed character and fixed relations to other concepts, are neither real objects [reale Gegenstände] nor actual acts [wirkliche Acte] of thinking (cf. Herbart [1813] 1850, §35 Anmerkung).

Trendelenburg's idea is that philosophy is supposed to explain how we form concepts and what it is about our thought and the world that makes it possible for us to form concepts that capture the structure of the world. And though Drobisch claims that we form concepts by comparison, reflection, and abstraction, Trendelenburg finds this theory empty and at best misleading.

> According to this way of thinking, families, genus, species come to be only through omitting marks. They are arbitrary constructs [Gebilde] of the abstracting, that is, evaporating, thinking; but never does there appear a law of this process, from within or from without.
>
> (Ch. I, §4)

In Trendelenburg's view, it's the job of the logician to articulate laws for how we should compare, reflect, and abstract. After all, contra Drobisch, it is not indifferent which marks one abstracts from a given concept. We cannot just abstract any marks willy-nilly and expect to arrive at concepts fit for science.

> Are the marks of the concepts so indifferent [of equal value] to one another, do they stand in this way in one line, that it is all the same which mark one abstracts first? What then is the significance of the expression of subordination of concepts?
>
> (Trendelenburg 1840 Ch. XIII, §7; 1862 and 1870, Ch. XV, §7)

Of course, in a roughly Aristotelian view, concepts are not just an unordered list of marks: some of those marks specify the genus, some of those marks specify differentia, and determining which is which is no more and no less than determining what laws govern the things we're thinking about. Indeed, Trendelenburg thinks it incredible that Drobisch does not face the puzzles concerning content, generality, and methodology that I mentioned earlier. For Trendelenburg, our capacity to form general representations that have content depends on the deep correspondence between thought and being, between our capacity to form concepts and to make judgments and the world's metaphysical nature as composed of substances that carry out certain activities.

Towards the end of his book, Trendelenburg sketches an alternative methodology of concept formation that highlights the role of judgments in forming concepts.

> The concept arises in a similar way from the first judgment of mere activity, as the substance arises from formative activity [gestaltenden Tätigkeit]; and as substance is expressed in activity, so too

> is the subject in the predicate, the concept alive in the judgment. (Trendelenburg 1840, Ch. XII, §3, 145; 1862, Ch. XIV, § 3, 236; 1870, Ch. XIV, § 3, 234)

His claim is that we first form judgments and then derive concepts from them. With these we can form new judgments, from which we can then derive new concepts, etc.

> In this way, the subjectless judgment is the first (e.g. It is lightning). By fixing the concept (e.g. lightning), it grounds the complete judgment (e.g. the lightning is conducted by iron), and the complete judgment comprehends its result anew in a concept (e.g. lightning conductor). (Trendelenburg 1840 Ch. XII, §3, 148–9; 1862, Ch. XIV, § 3, 238; 1870, Ch. XIV, § 3, 238)

Trendelenburg's idea is that our first knowledge concerns activities [Tätigkeiten] – that is, things that substances do. Through forming judgments about these activities (e.g. it is lightning), we come to fix on a concept (e.g. the concept of lightning), which is itself a kind of substance. Once we do that, we can then attribute to this substance new accidents, such as being conducted by iron, from which we can form yet further compound concepts (i.e. of substances together with their accidents). In this brief way, Trendelenburg highlights the interplay between judging and concept formation, and the relationships between concept formation and the formation of natural laws. These connections, he believes, are missing from Drobisch's theory of concept formation. Indeed, they have to be, as long as Drobisch adheres to the mistaken Kantian view that logic is formal.

The 1851 second edition of Drobisch's *Neue Darstellung* contains significant additions and clarifications, many of which were explicitly in reply to Trendelenburg's criticisms. These new additions are also Drobisch's attempts to confront the three puzzles with Kant's account of concept formation that I highlighted earlier in this section. To begin, Drobisch gives a clarified and expanded account of concept formation. First, he grants Trendelenburg's basic point that concept formation is not independent of forming judgments, but in fact concepts are formed through forming judgments.

> Thinking forms concepts from the representations, by bringing to consciousness that which belongs to the What of the represented and abstracting [absondern] what does not belong to it. This happens in judgments, which are therefore partly affirmative, partly negative. Insofar as the concept contains a manifold of what is represented, the concept is the unity of this manifold, and the form of combination of the manifold is the form of the concept itself. But

> the judgments substantiate this combination as one coming to be gradually for thinking, and are therefore the forms of genesis of the concept in thought. So it is now certain that the formation of concepts and the grasping of their form would be impossible without the thought activities of judging; and so the judgments doing the forming must precede the formed concepts.
>
> (Drobisch 1851, §10)

In addition to granting Trendelenburg's point that the formation of a concept depends on a prior judgment, he also acknowledges that the theory of definition must constitute part of the story of concept formation as well.

> When logic finds in the judgment the form of the genesis of the concept, and teaches to compose it from its elements through the definition, it concerns itself with the correct formation of concepts.
>
> (Drobisch 1851, p. x)

Of course, the theory of definition in Kant's view is part of the doctrine of method of logic, and so we see Drobisch recognizing that the process of forming concepts depends on conforming to the norms for the formation of scientific theories. These two additions then constitute Drobisch's answer to what we earlier called the methodology puzzle.

Trendelenburg had argued that Drobisch's account of concept formation treats concepts as effectively already given. We also saw earlier a certain puzzle about content: how is it that concepts can come to have their content, if not by abstraction? Drobisch faces this puzzle straight on in the second edition, biting the bullet and admitting that some concepts are simply given and not in any way generated by thought. "Of course, the genus is generated by thought. But the immediately given concepts of objects are not generated by thought" (Drobisch 1851, note to §17). This means that the theory of concept formation cannot explain how concepts come to have content to begin with, and thus the account of concept formation in logic is not meant to explain the possibility of content. Indeed, it presupposes it, by presupposing certain immediately given concepts from which higher concepts can be generated.[15]

However, this raises a further puzzle: how to articulate the notion of 'generality' or 'abstraction' that is at work in the theory of concept formation by comparison, reflection, and abstraction. Is this notion of

15 Herbart says little to nothing about concept formation by comparison, abstraction, and formation. However, he does seem to hold the view that Drobisch settles on in the second edition: that logic presupposes some given, determinately contentful concepts from which other concepts can be formed. "Concepts are always presupposed as ready to hand and finished [vorhanden und fertig], from whose combination new concepts are to arise" (Herbart [1808] 1850, 468).

generality or abstraction the same notion at work in his claim, made already in the first edition, that all concepts are 'abstract' in the sense that their content is objective and can be grasped through many acts of thinking by many distinct thinkers? In the second edition he clearly says that the answer is "No." The abstract generality brought about by concept formation through abstraction is distinct from the kind of objectivity of content that belongs to all concepts, whether they be given or generated by thought. He more clearly characterizes the generality characteristic of a concept in terms of its repeatability, graspability, and constant character, even as it is brought into relation to other concepts in repeated representations. He contrasts this notion of generality with a notion of generality that's "generated through abstraction" (Drobisch 1851, §9). For example, the representation of sky blue can be found in flowers, paintings, or clothing. If the representation of sky blue is isolated from all these relations, we obtain the general representation or concept <sky blue>. But this representation is in no way abstract, but it is instead fully individual and particular. The combinations that it occurs in (for example, my representation of this flower, of this painting, or of this clothing) are then not species of <sky blue>. <Sky blue> is not the genus of which my representation of this flower is a species.

Drobisch in this way makes clear a distinction that was not clear in the first edition of his work. In fact, Hamilton, whose lectures on logic cite the first edition of Drobisch's *Neue Darstellung* on this very point (Drobisch 1836, §14), does not clearly distinguish the act/content distinction from the species/genus distinction.

> We are conscious to ourselves that we can repeat our acts of consciousness; that we can think the same thought over and over… This relation of absolute similarity which subsists between the repetitions of the same thought, is found to hold between our representations of the resembling qualities of objects. … Now, in so far as we exclusively attend to the resembling qualities, we, in the first place, obscure or remove out of view their non-resembling characters i, o, u, while we remain exclusively conscious of their resembling qualities y, y, y. … In other words, we classify B, C, and D under y; y is the genus, B, C, and D are its individuals or species.
>
> (Hamilton [1860] 1874, Lecture VII, 124)

This clearly demonstrates the kind of confusion that Drobisch, incited by Trendelenburg, tries to ward off in the second edition of his work. Hamilton begins by noting the objectivity of content – the fact that many distinct acts of thinking are of the same thought. But he immediately confuses this objectivity with the way in which a more abstract concept is formed from a lower concept.

In this chapter, I've posed the question, "If logic has been complete since Aristotle, what's left for a logician to do?" One would think that the answer to this question would be "Nothing," in which case the mere existence of the logicians of the Kantian school, as Ueberweg calls them, would seem utterly mysterious. But in fact we've seen that Kant left behind a great number of open questions, the answering of which provided important work for later logicians – even logicians who attempted to be orthodox Kantians. What's more, in some of these cases, for instance, in the articulation of the logical laws, these new projects were stimulated by Kant's innovations within formal logic. In other cases, such as concerning the formality of logic and the nature of concept formation, later logicians tried to clarify and fill in the details left open by Kant's work. In this chapter, I have refrained from evaluating the contributions of the Kantian logicians. Nevertheless, this chapter has clearly demonstrated the historical fact that there existed a vibrant tradition of logic in the Kantian tradition – a tradition that has been more or less forgotten and absent from historiography of logic. I have also shown, I believe, that the logicians of this school did far more than simply repeat the sayings of the Master, but were alert to the formal and philosophical issues left behind in Kant's wake.

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2 Kant's Excessive Tenderness for Things in the World, and Hegel's Dialetheism

Graham Priest

Introduction

Dialetheism is the view that some contradictions are true. The very opposite has been high orthodoxy in Western philosophy since Aristotle's muddled but influential defence of the Principle of Non-Contradiction in the *Metaphysics*.[1] Drawing on modern developments in paraconsistent logic, the view has now become more palatable than before. The history of Western philosophy has, however, produced a few thinkers who stood up against the orthodoxy, the most notable of these being Hegel. But he did not get there alone. The ground was laid by Kant in his account of the Antinomies in the first *Critique*. In the end, Kant was not prepared to follow the logic of his arguments into dialetheism and suggested somewhat unsatisfactory ways out of their contradictory conclusions. Hegel was made of sterner stuff. He rejected what he saw as Kant's 'excessive tenderness for things of the world' and promoted dialetheism to the centre stage of his philosophical thought: his logical dialectics.[2] In this essay, we will look more closely at the details of how this happened.[3]

Hegel the Dialetheist

First of all, let us be clear that Hegel was a dialetheist. Hegel explicitly claims that reality may be contradictory. For example, in the *Logic*, he says,[4]

> … ordinary experience itself declares that at least there are a number of contradictory things about, contradictory arrangements, and so forth, the contradiction being present in them, and not merely in an external reflection.

1 See Priest (2006a), Chapter 1.
2 Contemporary developments in dialetheism tend to be motivated by quite different considerations, a major one of which is the paradoxes of self-reference. For a review of dialetheism, see Priest, Berto, and Weber (2018).
3 The following essay draws heavily on Priest (1990, 2002), Chapters 5–7.
4 Quotations from the *Logic* are taken from Johnston and Struthers (1929). The following comes from Vol. II, p. 67.

And in case one might suspect that he does not mean by 'contradiction' something of the form A ^ ¬A, he says a few lines later:

> External, sensible motion is itself its [Contradiction's] immediate existence. Something moves not because it is here at one point of time and there at another, but because at one and the same point of time it is here and not here, and in this here both is and is not. We must grant the old dialecticians the contradictions which they prove in motion; but what follows is not that there is no motion, but rather that motion is existent Contradiction itself.

For what it is worth, Hegel also espouses a dialetheic solution to the Liar paradox—just as many of the contemporary dialetheists do. In his comments on Eubulides in his *Lectures on the History of Philosophy* (Part 1, Chapter 2, C.1.b), he says that the liar sentence:[5]

> both lies and does not lie… For here we have a union of opposites, lying and truth, and their immediate contradiction…

He also berates the error of those who have tried, futilely, to give a 'one sided' answer to the question of the status of the liar.

Of course, a number of Hegel exegetes, who themselves could not countenance the possible truth of a contradiction, misapplied the Principle of Charity and insisted that Hegel cannot be interpreted literally here. In his mouth, 'contradiction' must mean something else.[6] And in all fairness, it must be agreed that Hegel does use the word in a variety of ways. However, to insist that he never means 'contradiction' in the logician's sense does such violence to the text that this can only result in misinterpretation.

This will become clear if we chart Hegel's path into dialetheism. The path is via Kant; and, in particular, what he says in the section of the first *Critique* termed the *Antinomy of Pure Reason*. However, before we can get to this, we need to start further back and look at Kant's views concerning phenomena and noumena.

Kant: Phenomena and Noumena

For Kant, phenomena are, essentially, those things that are perceivable via the senses. I use 'thing' in a fairly loose way here, to include objects such as buildings, countries, and stars; and events such as the extinction of the dinosaurs, plane journeys, and the death of Hegel. Noumena are things which are not phenomena. To the extent that they can be "brought before the mind" at all, they can be conceived but not perceived.

To understand how this distinction functions for Kant, it is necessary to be very clear about his views concerning perception. Kant thinks

5 Haldane and Simpson (1955), p. 460.

6 See, for example, Acton (1967), esp. p. 444.

that objects in themselves cannot be perceived, or intuited, in his jargon; what are perceived are our mental representations of such objects. As he explains (A109):[7]

> Appearances are the sole objects which can be given to us immediately, and that in them which relates immediately to the object is called intuition. But these appearances are not things in themselves; they are only representations, which in turn have their object—an object which cannot itself be intuited by us, and which may, therefore, be named the non-empirical, that is, transcendental object = x.

The phenomena, or representations, perceived are a result of something contributed by the things in themselves but also of the *a priori* structure our mind employs to constitute the representations (intuitions). In particular, space and time are not features of things themselves but are a most important such structure. For Kant, a horse is a spatio-temporal representation of an object, but what the representation is a representation of (which the rest of us might call a horse) is neither perceived nor in space and time. As he puts it (A30=B45):

> The transcendental concept of appearances in space… is a critical reminder that nothing intuited in space is a thing in itself, that space is not a form inhering in things themselves as their intrinsic property, that objects in themselves are quite unknown to us, and that what we call outer objects are nothing more than mere representations of sensibility, the form of which is space. The true correlate of sensibility, the thing in itself, is not known, and cannot be known, through these representations; and in experience no question is ever asked in regard to it.

The Categories and Their Applicability

Next, we must turn to Kant's views on the Categories. Categories are concepts of a certain kind. Kant calls them 'pure', meaning that they have no empirical content (unlike, for example, the concept *horse*). Kant abstracts them from what he took to be the logical forms of judgements, or statements, as we might now put it. In the neo-Aristotelian logic he endorsed, every judgement has a quality, quantity, relation, and modality. And it may have each of these in one of three ways. Corresponding to each of these ways is a Category. In the *Prolegomena to any Future Metaphysics*, these are listed as follows:[8]

7 Quotations from the *Critique* are taken from Kemp Smith (1933).

8 Quotations from the *Prolegomena* are from Beck (1950). The list is from Section 21, except that I have reversed the order of the three quantities, following Bennett (1966), p. 77. It is perhaps stretching the point a little to say that the Category of modality is a matter of logical form in the modern sense, for Kant takes this to be semantic rather than syntactic. See A74=B100 ff. However, we may ignore this subtlety here.

	Logical Form	*Category*
Quantity	Singular	Unity
	Particular	Plurality
	Universal	Totality
Quality Relation	Affirmative	Reality
	Negative	Negation
	Infinitive	Limitation
	Categorical	Substance
	Hypothetical	Cause
	Disjunctive	Community
Modality	Problematic	Possibility
	Assertocic	Existence
	Apodictic	Necessity

To illustrate, consider, for example, the judgement 'Some capitalists may not be compassionate'. This has particular quantity, negative quality, categorical relation, and problematic modality. It thus deploys the Categories of plurality, negation, substance, and possibility. Or again, the statement 'If a piece of metal is heated then, necessarily, it expands' has universal quantity, affirmative quality, hypothetical relation, and apodictic modality. It thus deploys the Categories of totality, reality, cause, and necessity.

The precise details of this matter are not very important for our purposes. The main point to note here is that the Categories are abstracted from the logical forms of judgements and, crucially, that each judgement deploys one or more such Category, as Kant himself remarks in the following corollary (A245=B302):

> [The Categories] cannot themselves be defined. The logical functions of judgments in general, unity and plurality, assertion and denial, subject and predicate, cannot be defined without perpetrating a circle, since the definition itself must be a judgment, and so must already contain these functions.

Having sorted out the Categories, the next point to note is Kant's view that they can be (meaningfully) applied only to phenomena. As Kant puts it in the *Prolegomena*,[9]

> even if the pure concepts of the understanding are thought to go beyond objects of experience to things in themselves (noumena), they have no meaning whatever.

9 Beck (1950), Section 30.

He comes back to this point again and again in the *Critique* (for example, A95, Bl47, Al39=Bl78, A239=B298).

Perhaps his major argument for this concerns the criteria for the application of the Categories. Kant notes that to apply a Category, it is necessary for us to have some criterion, or *schema*, in his jargon, for its applicability. In the 'Schematism of the Pure Understanding', Kant gives what he takes to be these criteria. He does not deny that, logically, there could be other criteria; but, as a matter of fact, these are the only criteria that we have, or that beings constituted like us could have.

Now, it turns out that the criteria for all the Categories involve time. To give a couple of the simpler examples (A143=Bl83 ff.), 'the schema of substance is permanence in real time', and 'the schema of necessity is existence of an object at all times'. It follows that it makes sense to apply the criteria only to those things that are in time: phenomena. As Kant puts it (Al45=Bl84 ff.),

> We thus find that the schema of each Category contains and makes capable of representation only a determination of time… The schemata of the pure concepts of the understanding are thus the true and sole conditions under which these concepts obtain relation to objects and so possess significance. In the end, therefore, the Categories have no other possible employment than the empirical.

The correctness of Kant's criteria is not beyond argument; but what he takes them to be is not.

The Transcendental Illusion and the Antinomies

With this background, we can now move to Kant's discussion of the Antinomies of Pure Reason.

The section of the *Critique* called the *Transcendental Dialectic* concerns certain objects, which Kant calls *Transcendental Ideas*. Given some phenomenon, we can consider its conditions of a certain kind. According to Kant, Reason then forces us to construct the totality of all conditions of that kind. As he puts it (A409=B436):

> Reason makes this demand in accordance with the principle that *if the conditioned is given, the entire sum of the conditions, and consequently the absolutely unconditioned… is also given.*

The resultant totality does not itself possess conditions of the appropriate kind—or it would not be the totality of all such conditions. This is why Kant calls it the unconditioned. It is therefore a noumenon (if it is anything at all): any phenomenon must have conditions of space,

time, etc. The unconditioneds are exactly the Transcendental Ideas. According to Kant, there are three ways of totalising, corresponding to the three kinds of syllogism: categorical, hypothetical, and disjunctive (though the correspondence is tenuous to say the least). We thus come to three Transcendental Ideas: the Soul, the Cosmos, and God. (One should note, though, that this is somewhat misleading, since it is going to turn out that there are four distinct Cosmological unconditioneds.)

Each Transcendental Idea brings in its wake a family of arguments, which Kant calls, respectively: the Paralogisms, the Antinomies, and the Ideal. The arguments appear to establish profound metaphysical truths but are (for reasons that we will come to in the case of the Cosmological Idea) fallacious. (For this reason, Kant calls them 'dialectical'.) Despite this, the fallacies are, in some sense, ones into which we inevitably fall: a 'natural and unavoidable illusion' (A422=B450). A visual illusion (such as the seeing of black dots at the interstices of a white grid on a black background) is an inherent product of our (correctly functioning) sensory apparatus. Moreover, even when we know this to be an illusion, we cannot help seeing it. Similarly, the illusion concerning the dialectical arguments, which Kant calls 'the Transcendental Illusion', is an inherent product of our (correctly functioning) conceptual apparatus; when we know that the arguments are fallacious, still we cannot help seeing them as correct. Kant's explanation as to why it is that this illusion arises is rather obscure; but the basic idea is that our possession of Transcendental Ideas performs the essential regulative function of forcing us to acknowledge that any determination of conditions is bound to be incomplete, and so motivate us to determine further conditions.

Of the three families of dialectical arguments, only one will concern us here: the Antinomies—those concerning the Cosmological Idea(s). According to Kant, there are four Antinomies, corresponding to the four kinds of Categories (quantity, quality, relation, and modality), though the correspondence is, again, exceptionally tenuous.[10] Each Category produces a kind of condition, and so a corresponding kind of unconditioned *u*.

Now, what is characteristic of the dialectical arguments in the Antinomies is that they come in pairs, each pair establishing—or appearing to establish—the conclusion that *u* has certain contradictory properties. The statements of these contradictories Kant calls the *Thesis* and the *Antithesis*. The argument for the Antithesis turns on the fact that it is always possible to apply the operation corresponding to the condition again. In his words, *u* is 'too small' for the concept which generates it.

10 The arguments are all versions of arguments to be found in the Leibniz/Clarke debate. See Al Azm (1972).

The argument for the Thesis turns on the fact that it is not possible to apply it again. As Kant puts it, *u* is 'too large' for the concept which generates it (A486=B513).

The contradictory pairs in each case are as follows (A427=B455 ff.):[11]

First Antinomy

- *Thesis*: The world has a beginning in time and is also limited as regards space.
- *Antithesis*: The world has no beginning and no limits in space; it is infinite as regards both.

Second Antinomy

- *Thesis*: Every composite substance in the world is made up of simple parts, and nothing anywhere exists, save the simple or what is composed of it.
- *Antithesis*: No composite thing in the world is made up of simple parts, and nowhere exists in the world anything simple.

Third Antinomy

- *Thesis*: Causality in accordance with the laws of nature is not the only causality from which the appearance of the world can one and all be derived. To explain these appearances, it is necessary to assume that there is another causality, that is, freedom.
- *Antithesis*: There is no freedom; everything in the world takes place solely in accordance with the laws of nature.

Fourth Antinomy

- *Thesis*: There belongs to the world, either as part of it or as its cause, a being that is absolutely necessary.
- *Antithesis*: An absolutely necessary being nowhere exists in the world, nor does it exist outside the world as its cause.

Kant's Solution

This is not the place to discuss the details of the arguments which are supposed to establish each contradictory pair. Of more importance is what Kant takes to be the upshot of matters. One might have thought that, in virtue of the arguments, Kant would become a dialetheist.

11 In the first Antinomy, *u* is the whole cosmos; in the second, it is the simple. In the third and fourth, it is not immediately clear exactly what *u* is supposed to be, but it is something like the totality of all things caused and the totality of all contingent things, respectively. See Priest (2002), Chapter 6.

He did not. He diagnoses a subtle fallacy in the arguments, which is as follows.[12]

The limit object, *u*, *qua* phenomenological object, does not exist. This makes both of the apparently contradictory claims false. Thus, for example, in the First Antinomy, both of 'the World has a beginning in time' and 'the World has no beginning in time' are false, since the World does not exist (A497=B525 ff). If it existed, these claims would be contradictories; but if it does not exist, they are mere contraries. In contemporary jargon, we might say that we have here a case of presupposition failure.

Fair enough (one might suppose), but where exactly do the arguments given fail, and why? Kant does not explain in detail, but the reason becomes clear when one starts to scrutinise the arguments through Kantian eyes. Take, for example, the argument for the Antithesis of the Second Antinomy. This goes as follows (A435=B463):

> Assume that a composite thing (a substance) is made up of simple parts. Since all external relation, and therefore all composition of substances, is possible only in space, a space must be made up of as many parts as are contained in the composition which occupies it. Space, however, is not made up of simple parts, but of spaces. Every part of the composite must therefore occupy a space. But the absolutely first parts of every composite are simple. The simple therefore occupies a space. Now, since everything real, which occupies a space, contains in itself a manifold of constituents external to one another, and is therefore composite; and since a real composite is not made up of accidents (for accidents could not exist outside one another, in the absence of substance) but of substances, it follows that the simple would be a composite of substances—which is self-contradictory.

As a moment's consideration shows, we are reasoning about simples and applying the Categories to them—in particular, the Category of substance (see, for example, the last sentence). Now, we do not meet simples in experience. They are therefore noumena. The application is therefore illicit: Categories apply only to phenomena. So the reasoning is illegitimate.

The point generalises to all of the Antinomies. Each one is about an unconditioned thing, a noumenon. Yet in the course of the arguments, we apply the Categories of substance, causation, necessity—and, in the case of the First Antinomy, the forms of space and time, to boot. These things cannot be meaningfully applied.

12 Actually, Kant gives two solutions to the paradoxes. The first is supposed to apply to all of them. The second, which is actually inconsistent with the first, is supposed to apply only to the third and fourth (see Priest 2002, 6.7). I will discuss only the first here, since it is the more general.

The Instability of Kant's Solution

Kant's resolution of the contradiction, then, depends crucially on the distinction between phenomena and noumena and on the fact that the Categories apply only to the former. But this resolution of the contradiction is unstable, precisely because of Kant's own views about the Categories. Let us see why.

That the Categories can be applied only to phenomena entails that there can be no knowledge of noumena. As Kant explains (Bxxv f.):

> that we have no concepts of understanding, and consequently no elements for knowledge of things, save in so far as intuition can be given corresponding to these concepts; and that we can therefore have no knowledge of any object as thing in itself, but only in so far as it is an object of sensible intuition, that is, an appearance—all this is proved in the analytical part of the *Critique*. Thus it does indeed follow that all possible speculative knowledge of reason is limited to mere objects of experience.

This appears to put Kant in a very strange situation. For here he is, after all, writing a large book at least purporting to inform us about, inter alia, noumena. But he doesn't know what he is talking about! Kant tries to soften the blow. The passage I have just quoted goes on:

> But our further contention must also be duly borne in mind, namely, that though we cannot know these objects as things in themselves, we must yet be in a position to at least think them as things in themselves; otherwise we should be landed in the absurd conclusion that there can be appearances without anything that appears.

Hence, though we cannot know anything about things in themselves, noumenal objects, we can at least think things about them.

But the matter cannot be resolved so easily. To say that we cannot know anything about noumena is, whilst true, rather misleading; for it suggests that the impossibility of having knowledge is due merely to our lack of epistemic access. The impossibility of knowledge arises for a much more profound reason, however: a lack of conceptual access. The reason that we cannot have knowledge of noumena is precisely that we cannot even make statements about them: any (meaningful) statement about them would have to apply the Categories and so is impossible.

However, as is quite evident, this fact is belied by Kant's own discourse, which itself makes numerous assertions about noumena, applying various Categories. For just one example, Kant talks of noumena causing our sensations (e.g. A288=B345):

> Understanding accordingly limits sensibility, but does not thereby extend its own sphere. In the process of warning the latter that it must not presume to claim applicability to things-in-themselves but only to appearances, it does indeed think for itself an object in itself, but only as transcendental object, which is the cause of and not itself appearance.

And this is but the tip of the iceberg. When Kant says that noumena may be supposed to exist (A253=B309), he deploys the Category of existence; when he says that they are not in time, he deploys the Category of negation. Even the statement that the Categories cannot be applied to noumena deploys the Categories of possibility and negation.

Kant is well aware of the contradiction involved here and is very uncomfortable about it. This is clearest in the chapter of the *Critique* called 'The Ground of the Distinction of All Objects in General into Phenomena and Noumena', in which he tries to avoid the contradiction by distinguishing between an illegitimate positive notion of noumenon and a legitimate negative, or limiting, notion. This does not help: according to Kant, the negative notion is there to place a limit on the area in which we can apply the Categories and so make judgements (A255=B311). But to say that there are (or even may be) things about which we cannot judge is precisely to make a judgement about them; specifically, it quantifies over them and applies the Category of plurality. The "legitimate" notion is therefore just as illegitimate as the legitimate one.

So unsuccessful was this chapter of the *Critique* that Kant completely redrafted it for the second edition, but without doing anything to remove the fundamental contradiction. As Kemp Smith puts it:[13]

> But beyond thus placing in still bolder contrast the two counter-assertions, on the one hand that the Categories must not be taken by us as other than merely subjective thought functions, and on the other that a limiting concept is indispensably necessary, Kant makes no attempt in the new passages to meet the difficulties involved. With the assertion that the Categories as such, and therefore by implication, those of reality and existence, are inapplicable to things in themselves, he combines, without any apparent consciousness of conflict, the contention that things in themselves must none the less be postulated as actually existing.

Kant's solution to the Antinomies of Pure Reason is therefore distinctly problematic, in his own terms—to say the least. Which brings us, at last, to Hegel.

13 Kemp Smith (1923), pp. 413f.

Hegel's Critique of Kant

According to Hume's empiricism, both knowledge and meaning must be derived from sensory experience. Kant rejected this: knowledge may be *a priori*; and the Categories of reason do not derive in any way from experience but are imposed upon it. However, Kant still gave experience a privileged position in relation to meaning. For, though the Categories might not be derived from experience, they have applicability, as we have seen, only when schematised, that is, only when taken as the forms of *possible* experience.

Hegel rejected this vestige of empiricism. Neither experience nor its possibility has any privileged position with respect to knowledge or meaning. Hence, though the distinction between things perceivable by the senses (phenomena) and things not so perceivable (noumena) makes perfectly good sense for Hegel, the former are not categorically distinct from the latter. For example, it is just as possible to know things about noumena as it is to know things about phenomena; it may even be easier. As Hegel puts it in the *Lesser Logic*:[14]

> The Thing-in-itself… expresses the object when we leave out of sight all that consciousness makes of it, all its emotional aspects, and all specific thoughts of it. It is easy to see what is left—utter abstraction, total emptiness, only described still as in an 'outer world'… Hence one can only read with surprise the perpetual remark that we do not know the Thing-in-itself. On the contrary there is nothing we can know so easily.

Hegel also observes that Kant's very claim that we cannot make epistemically authoritative judgements about noumena is self-inconsistent:[15]

> It argues an utter want of consistency to say, on the one hand, that understanding only knows phenomena, and, on the other, assert the absolute character of this knowledge, by statements such as 'Cognition can go no further'… No one knows, or even feels, that anything is a limit or a defect until he is at the same time above and beyond it.

For Hegel, then, nothing substantial can hang on the distinction between phenomena and noumena. In particular, the essential differences between the two realms to which Kant appeals in order to defuse the Antinomies cannot be maintained.

Hegel drew the appropriate conclusion from this. Since there are perfectly sound (according to Hegel) arguments to the effect that the World (that is, the unconditioned object of each Antinomy) has contradictory

14 Quotations from this are taken from Wallace (1975). Here, at p. 72.
15 Wallace (1975), p. 91.

properties, it does have contradictory properties. Thus, commenting on the Antinomies and Kant's supposed solution to them, he says:[16]

> In the attempt which Reason makes to comprehend the unconditioned nature of the World, it falls into what are called Antinomies. In other words, it maintains two opposite propositions about the same object, and in such a way that each of them has to be maintained with equal necessity. From this it follows that the body of cosmical fact, the specific statements descriptive of which run into contradiction, cannot be a self-subsistent reality, but only an appearance. The explanation offered by Kant alleges that the contradiction does not affect the object in its proper essence, but attaches only to the Reason which seeks to comprehend it.
>
> In this way the suggestion was broached that the contradiction is occasioned by the subject-matter itself, or by the intrinsic quality of the Categories. And to offer the idea that the contradiction introduced into the world of Reason by the Categories of the Understanding is inevitable and essential was to make one of the most important steps in the progress of Modern Philosophy. But the more important the issue thus raised, the more trivial the solution. Its only motive was an excessive tenderness for the things of the world. The blemish of contradiction, it seems, could not be allowed to mar the essence of the world; but there could be no objection to attaching it to the thinking Reason, to the essence of mind. Probably nobody will feel disposed to deny that the phenomenal world presents contradictions to the observing mind; meaning by "phenomenal" the world as it presents itself to the senses and understanding, to the subjective mind. But if a comparison is instituted between the essence of world and the essence of mind, it does seem strange to hear how calmly and confidently the modest dogma has been advanced by one, and repeated by others, that thought or Reason, and not the World, is the seat of contradiction.

Thus, Kant's evasion of the contradictions is not on.

And as Hegel goes on to explain in the next paragraph, he thinks that the Kantian Antinomies are just some amongst many. *All* our concepts, and not just the unconditioneds of the Antinomies, are embroiled in contradiction.

Fichte's Dialectic

Moreover, these contradictions play a central role in a systematic development of the Categories—not just Kant's 12 but the other 70 that he

16 Wallace (1975), p. 77f.

missed. Here, Hegel was influenced not by Kant but by Fichte. So in this section, let us turn to Fichte.

In his *Wissenschaftslehre*, Fichte, like Hegel, started from Kant and, like Hegel, criticised the Kantian postulation of the thing-in-itself.[17] This left only the other part of the Kantian ontology: the transcendental ego. The nature of the ego, or self, is to think; but there is nothing to think about except itself; and it is impossible to think something unless there is something else to contrast it with. (So at least thought Fichte—and Spinoza: *omnis determinatio est negatio*.) Hence, the self had to create something different, the non-self, against which it could conceive itself. It therefore generates a contradiction. Specifically, the non-self must also be self, since nothing else exists. As Fichte puts it,[18]

> … insofar as the not-self is posited [in the self], the self is not posited in the self

but

> … insofar as the not-self is to be posited [in the self], the self must be posited therein.

Thus, the self is both posited and not posited, and the posited is both self and not-self. Or, more pithily, as Fichte puts it a few lines later,

> self = not-self and not-self = self.

The self (thesis), by its cunning postulation of the not-self (antithesis), comes to understand what it is, viz. both, and the two reside together (synthesis). The synthesis may now, in its role as a new thesis, generate a new antithesis, giving rise to a new synthesis, and so on.

Hegel criticised Fichte. But his criticisms were, essentially, twofold: first, that Fichte had not elevated the transcendental ego into something grander, *Geist*; and second, that he had misunderstood the nature and significance of the synthesis.[19] This aside, Hegel took over Fichte's dialectic wholesale and, particularly for present purposes, the contradictory nature of the alienated state of the self. As Hegel himself put it—though hardly pellucidly:[20]

> … in being self-conscious [self-consciousness] is independent, but still in this independence it has a negative relation to what is outside

17 See Taylor (1975), pp. 36, 77.
18 Heath and Lachs (1982), p. 106.
19 Haldane (1892), p. 499.
20 Haldane (1892), pp. 549–50.

> self-consciousness. This is infinite subjectivity, which appears at one time as the critique of thought in the case of Kant, and at another time, in the case of Fichte, as the tendency or impulse towards the concrete. Absolute, pure, infinite form is expressed as self-consciousness, the Ego.
>
> … Self-consciousness thus… recognizes its positive relation as its negative, and its negative as its positive,—or, in other words, recognizes these opposite activities as the same, i.e., it recognizes pure Thought or Being as self-identity, and this again as separation.

Hegel's Dialectic

In Hegel's hands, Fichte's dialectic morphs into something much grander. At the prompting of Schlegel in his *Lectures on Transcendental Philosophy*, the transcendental ego becomes *Geist*, a sort of cosmic mind. It too, needs to understand what it is, and it needs a concept adequate to the task. It starts with the most basic of concepts, *being*. It then works its way through a series of more and more adequate concepts, till it arrives at *absolute idea*, the idea of the absolute, by which time, that is what it is.

The whole process is one in thought. However, *Geist* is essentially embodied—in people, their social institutions, and even in nature. Hence, the conceptual progression is mirrored in a corresponding progression of these things. It is the conceptual progression which is fundamental, however, and which will concern us here.

This is described by Hegel in his *Logic*. Hegel distinguishes between two notions of logic, which he calls *subjective* and *objective*. The subjective logic is the Aristotelian logic of his day and is described in the *Logic* where it deals with the progression of the 12 concepts which are Kant's Categories. Objective logic, which is much more important for Hegel, is the structure of the whole progression: the dialectic itself.

The concepts in the progression show a simple pattern. They are structured as a hierarchy of triples, so that each Category (except those at the tips of the hierarchy) has three sub-Categories. (There is one exception: there are four sub-Categories of *judgement*. This is somewhat ironical, since these sub-Categories or at least their sub-Categories are essentially Kant's Categories.) The triples are also structured. The second of each triad is a category opposing the first. Hegel calls the second the *negation* of the first. And, in the simplest cases at least, 'negation' is logician's negation.

By consideration of the contradiction between the first two Categories of the triad, we arrive at the third Category. This is often referred to by Hegel as the *negation of the negation*. What exactly this means is somewhat moot. What is clear is that the third Category is supposed to be, in some sense, the dialectical union of the first and second. Hegel often

says that the first and second Categories are *aufgehoben*, or sublated, as it is sometimes translated, in the third. This is a dark term of Hegelian art which is virtually impossible to translate into English, since it means both *to remove* and *to preserve*—and Hegel means both of these things at once.[21] In the most straightforward cases, the third Category is the Category of things whose being in the first Category just is their being in the second and which are therefore in both (since they must be in either one or the other).

Hegel is never very clear about the relationship of the third member of a triad to the first member of the next triad he considers. Sometimes it seems to be identity, sometimes it seems to be sublation, and sometimes it is just not clear what it is supposed to be. But fortunately, we do not need to sort this out here.

However, just to give a feel for the whole thing, let us consider the first phase of the dialectic. As already observed, this starts with the concept *being*. But something that *is*, and about which there is nothing more to be said, is no different from nothing. Hence, the second Category is *non-being*. The third Category is something that both is and is not, that is, *becoming*. As Hegel puts it,[22]

> Becoming is the unseparateness of Being and Nothing, not the unity which abstracts from Being and Nothing; rather, Becoming as the unity of Being and Nothing is this determinate unity in which there *is* Being as well as Nothing.

Why *becoming*? This is because of Hegel's account of motion—and, more generally, change—which we looked at briefly in the section "Hegel the Dialetheist". Something that is in a state of change (*becoming*) is in contradictory state. It is what/where it is, but it is also what/where it is not—what/where it *was* and what/where it *will be*.[23]

The next concept is *determinate being*, since something in a state of becoming has some determinacy to its being, unlike something that simply *is*. As Hegel puts it,:[24]

> Determinate Being issues from Becoming; it is the simple oneness of Being and Nothing. From this simplicity it derives its form as something immediate. Becoming, which mediated it, is left behind; it has transcended itself, and Determinate Being therefore appears as something primary and as something from which a beginning is

21 Barry Smith suggested to me that the best translation of *aufheben* is *transcend*—adding that it is the Hegelian equivalent of having one's cake and eating it.

22 Johnston and Struthers (1929), Vol. 1, p. 118.

23 See Priest (2006b), Chapter 12.

24 Johnston and Struthers (1929), Vol. 1, p. 121f.

> being made. First, then, it is one-sidedly determined as Being; the other determination it contains, that of Nothing, will also develop itself in it, in opposition to the other.

And with *determinate being*, the next triplet in the cycle kicks off.[25]

It should be stressed that the fact that categories are *aufgehoben* does not make the contradictions in them disappear. We have new and more adequate categories, certainly. But we still have and operate with the old ones. You cannot do philosophy just be talking about the absolute. You have to employ categories such as *becoming*, *essence*, *appearance*, *quantity*, and *quality*.[26]

Conclusion

There is much more, of course, to be said about Hegel, Kant, and the dependence of Hegel on Kant, but here we have tracked one of the most central connections. Kant's Antinomies provided arguments to the effect that certain noumenal objects have contradictory properties. He, himself, took the arguments to be unsound—though they are subjectively unavoidable, in the sense that the contradictions must arise in thought. His analysis of why the arguments are unsound depends upon his account of the distinction between noumena and phenomena and the claim that the Categories apply only to the latter. But his own text appears to give the lie to the claim. Hegel recognised this, refused to make a categorial distinction between phenomena and noumena, and so accepted the antinomical arguments as establishing that there were contradictions, not just in thought but in how things actually are—reality itself.

Under the prompting of Fichte, he proposed a dialectical progression of Categories ever more and more adequate to conceptualise reality. This starts with *being*, ends with *absolute idea*, and wends its way through another 80 categories, including Kant's 12. The progression zigzags though triples of triples, ever pushed on by contradictions arising and being *aufgehoben*, that is, being subsumed under more adequate Categories.

How much truth there is in Hegel's view is, of course, another, quite different, matter. But whatever one says about that, Hegel is clearly the zenith of dialetheic thinking between Aristotle and contemporary dialetheism.[27]

25 For a logical model of this, see Priest (201+).

26 Indeed, arguably, the category of the absolute idea *just is* the dialectical journey through these categories.

27 A version of this paper was given at the conference *Logic in Kant's Wake*, McMaster University, May 2016. Thanks go to many people in the audience for helpful comments, but especially to Frederick Beiser and Michael Foster.

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3 Hegel's Conception of Thinking in His *Logics*

Clinton Tolley

Introduction: The Fundamental Determinations of Thinking

What does Hegel think the science of logic is about? It is commonplace to suppose that Hegel's view of logic does not seem to be very close to more recent conceptions of the discipline.[1] For one thing, there is the considerable breadth of topics that Hegel seems to accord to the domain of logic. Even a quick glance at the tables of contents of his 1812–16 *Wissenschaft der Logik* ('WL'; 3rd edition 1832) and his shorter 1817 *Encyklopädie Wissenschaft der Logik* ('EL'; 3rd edition 1830) reveals that Hegel takes logic to include topics like substantiality (EL §150), causality (EL §153), atomism (WL 5:184), repulsion and attraction (WL 5:190), mechanism (EL §195; WL 6:409), chemism (EL §200; WL 6:428), teleology (EL §204; WL 6:436), life (EL §216; WL 6:469), willing (EL §233), and the idea of the good (WL 6:541).[2] None of these topics are typically included in contemporary textbooks on logic. For another, at various points Hegel seems to identify the subject matter of logic with that of *theology*, claiming perhaps most memorably that logic 'is *the presentation of God as he is in his eternal essence before the creation* [Erschaffung] *of nature and a finite spirit*' (WL 5:44). This commitment will surely seem to push Hegel even further away from most contemporary conceptions of logic. In fact, this alignment has been recognized as striking even by many of his most sympathetic followers, with his first biographer, Karl Rosenkranz, for example, imagining readers exclaiming, 'God and logic – what a baroque synthesis!' (Rosenkranz 1858: 37). Among more recent sympathetic treatments of Hegel's views of logic, a common response has been instead simply to downplay this alignment or to even omit reference to it altogether.[3]

1 Compare Taylor 1975: 206; Redding 2014: 281–2. The distance from the traditional pre-Hegelian conception of logic was noted already by Bolzano (cf. Bolzano 1851).

2 I will cite Hegel's published works according to the edition, volume number, and pagination of the Suhrkamp Edition of Hegel's *Werke in 20 Bänden*, eds. Moldenhauer and Michel. All translations are my own, in consultation with the recent Cambridge Edition translations of the WL and EL.

3 Compare Burbidge 2004; Pippin 2017, and Stern 2017.

My goal in what follows is to bring new light to what motivates Hegel toward both of these commitments (the 'over-enrichment' of logic, the 'divinization' of its subject matter) in order to help render more comprehensible Hegel's views on logic more generally. The main pathway I will take will be to articulate how Hegel himself takes his own conception to flow quite naturally out of deeper reflection on a fairly traditional conception of logic that was broadly advocated among Hegel's predecessors, including Kant, and also among Hegel's contemporaries. On this conception, logic should be understood as the science of '*thinking* [Denken]'. As I will show below, Hegel's own understanding of the history of the development of this philosophy of logic is what leads him, first, to his ostensibly over-enriched conception of logic, since he thinks this tradition itself shows that all of the aforementioned 'determinations' are required to present the essence of thinking itself, and so they must all be counted as 'logical'. More specifically, it is only once we have in view concepts pertaining to teleology, life, the good, the will, and so on that we can ever hope to have fully comprehended thinking in its highest possibility – namely, in the form of the 'absolute knowing [Wissen]', or absolute '*science* [Wissenschaft]' that Hegel (along with many others still today) takes to be the goal of thinking. Yet because thinking in its absolute form would consist in the complete and total 'agreement' or 'harmony' of the whole of what is thought with the whole of what there is – i.e. it would be the whole '*truth* [Wahrheit]' – Hegel concludes that we ought to recognize that thinking, and with it, the subject matter of logic itself, has shown itself to have the shape of something divine.[4]

My path here will largely follow Hegel's own introduction to his views on thinking (and with it, logic), as they are presented in the early sections of the later editions of the EL.[5] In a part of the text entitled 'Preliminary

4 In taking Hegel's theologically inflected claims about the logic quite seriously, my interpretation here departs in crucial ways from the currently most prominent recent strategy in the interpretation of Hegel's logic, inspired by Robert Pippin, Terry Pinkard, Sally Sedgwick, and others, which has been to look primarily to Kant's views on logic, and in particular his conception of its relation to human self-consciousness ('apperception'), as a template for our understanding of Hegel's (cf. especially Pippin 1989; Pinkard 2000 and 2002; Sedgwick 2012; and more recently Pippin 2014). Against this, I will argue below that, if anything, it is instead Kant's own views on the *divine* understanding which should be thought of as the template for what Hegel has in mind in depicting logic as the science of 'the divine concept' (compare Plevrakis 2017 and Tolley 2018). This emphasis on the divine form of thinking, and the emergent contrast with all forms of human consciousness, will also set my reading apart from otherwise more metaphysically minded readers such as Taylor 1975 and Houlgate 2006.

5 The need for a more introductory exposition of the conception of thinking Hegel is working with, so as to better prepare the reader for the subsequent discussions within the *Logic* proper, was something that Hegel seems to have more fully appreciated only after the first (1817) edition of the *Encyclopedia* was published (= '1817a'), which contained only a very brief introduction that had little beyond the presentation of the

Conception [Vorbegriff]', Hegel provides something of a historical origin story for his own conception of logic, in order to help the reader catch on to what should be had in mind when we claim (rightly, Hegel thinks) that logic is 'the science of thinking' (EL §19 Anm 8:67). Hegel's account proceeds by discussing a series of 'positions [Stellungen]' that have been taken up on the relation of 'thought [Gedanke]' to 'objectivity [Objektivität]', leading up to his own (EL §§26–78). In the next several sections, I will retrace Hegel's own telling of this history, focusing in particular on the way in which Hegel sees this tradition as wrestling (unsatisfactorily) with the right way to cast the relationship between thinking and two concepts taken to be essential to logic – namely, 'truth [Wahrheit]' and 'science [Wissenschaft]'. I will highlight Hegel's reasons for holding that a more careful reflection on these two concepts should lead in the direction of the enriched and ultimately theologized conception of the subject matter of logic itself. From here I will turn briefly to the main text of Hegel's *Logics* themselves in order to outline how the lessons Hegel takes from this history can be seen to shape his own overarching threefold division of logic itself into 'the doctrine of being', 'the doctrine of essence', and 'the doctrine of the concept', with each providing a further set of determinations needed to articulate thinking in its highest (or 'truest') form. I will conclude by providing a preliminary comparative analysis of key ways in which Hegel's philosophy of logic can now be seen to differ from, but also overlap with, several of the views of logic that have also emerged in the wake of Kant, but have become more commonly embraced today than Hegel's own.

Logic as Immediately 'Objective', and then as Merely 'Subjective': 'Metaphysics' and 'Empiricism'

In his developmental account of the 'given positions' on thinking that Hegel thinks one will confront, when looking to the then recent history of philosophy for guidance about the nature of logic, the variety of positions Hegel considers are grouped into three stages, with the middle stage itself being further divided in two. The complete list of Hegel's headings for these four positions is as follows: (1) 'metaphysics', (2a) 'empiricism', (2b)

official division of logic itself. In the first edition of the EL, Hegel begins the '*Vorbegriff*' with only a one-section preliminary treatment of logic and thinking (cf. Hegel 1817a §12), before jumping immediately to what he later calls the 'closer look and division' of logic (cf. Hegel 1817a §§13–17), and then giving a relatively brief overview of the history of philosophy of thinking prior to his logic, covering both the standpoint of 'metaphysics' and that of 'the critical philosophy' (Hegel 1817a §§18–36). It is only in the second (1827) edition that Hegel adds an initial treatment of what might be called 'the phenomenology of thinking about thinking' (EL §§20–25) along with a much fuller elaboration of the history of philosophy of thinking (EL §§26–78). For further discussion of the 'Vorbegriff', see Nuzzo 2010 and Stern 2017.

'critical philosophy' and (3) 'immediate knowing'. Because the first two positions (1 and 2a) neatly mirror one another, I will take them up together in this section. In the following section ("Logic as About What Is Objective for Subjectivity: 'The Critical Philosophy'"), I will turn to Hegel's treatment of Kant's conception of thinking (2b), before moving (in "Logic as About the Thoroughgoing Harmony of Subject and Substance: 'Immediate Knowing' and the Transition to Hegel's Own Conception") to the position entitled 'immediate knowing' (3) that Hegel associates most with Jacobi. This will all help chart the path to a still further position, over and above any of these four positions – namely, Hegel's own position, which will be our topic in the second half of the section "Logic as About the Thoroughgoing Harmony of Subject and Substance: 'Immediate Knowing' and the Transition to Hegel's Own Conception" and in the conclusion.

One final preliminary note: while these sections from the 'Vorbegriff' contain much that is of interest concerning Hegel's interpretations of previous philosophers – especially Hegel's views on Kant, who occupies the lion's share of Hegel's spotlight[6] – our main focus throughout will be limited to the task of using these sections to make clearer what Hegel himself means by 'thinking'.[7] I will also focus primarily on the exposition of Hegel's own presentation and how this clarifies his own views, rather than on assessing either the adequacy of his historical reconstruction or his own critical remarks.

The First Position: 'Metaphysics'

Even though Hegel associates this first position with a view more dominant in 'the *previous* metaphysics, prior to the Kantian philosophy' (EL §27 8:93), it is clear that he thinks that this position is still present and active among his contemporaries. In fact, Hegel claims that this position consists in a 'belief' that 'the daily doings and strivings of consciousness lives in', and so also is embraced by 'all philosophy in its beginnings' and is even upheld in 'all the sciences' (EL §26 8:93). This is the simple belief that, 'the truth' is 'cognized [erkannt]' in '*thinking over* [Nachdenken]', that 'what objects truly are' is directly 'brought before consciousness [vor das Bewußtsein]' in and through thinking them over (ibid.). In effect, this position presumes that 'thinking goes directly to objects', and simply and without any alteration 'reproduces [reproduziert] out of itself

6 The section on 'critical philosophy' takes up by far the largest part of the 'Vorbegriff'. For an analysis of Hegel's interpretation of Kant in these sections, compare Sedgwick (2012) and Ameriks (1985).

7 Indeed, this is Hegel's own official motivation behind his analysis of 'the given positions [Stellungen] of thinking toward objectivity': 'to elucidate and lead us closer to the significance and the standpoint which is here given to logic' (EL §25 8:91) – i.e. Hegel's own standpoint.

the content of sensations and intuitions as a content of thought', and thereby 'finds satisfaction in the like as the truth' (ibid.). This makes the first position '*naïve* [unbefangen]', because it is 'without consciousness of the opposition of thinking in and against itself' (ibid.) – i.e. it doesn't include any accounting for (and perhaps does not even notice) the familiar distinction between acts of thinking and their objects or what they are about; it does not characterize one as being subjective and the other being objective. A fortiori, then, it does not provide any account as to how these two *relata* (moments, aspects) could ever come together in one thing (e.g. consciousness), and so also does not address in any satisfactory way the possibility that they could come apart, e.g. in cases of thinking that is incomplete, confused, false, and so on.

Beyond failing to draw any distinction in kind between the activity of thinking and the object thought about, Hegel notes that this position also makes the 'presupposition' that what might be called the 'content' of thinking – i.e. what is being thought – is of the same kind as being itself, as what is. That is, this position 'regards thought-determinations as the *fundamental determinations of things*' – i.e. that simply 'because it is *thought* [gedacht], that which *is* will be cognized *in itself*' (EL §27 8:94). Insofar as it had been (and, especially since Kant, remains) common to view the most basic contents of thinking to be *concepts* ('universals'), and to take predicative *judging* to be the most elementary way in which concepts are used in thinking, this fundamental presupposition thereby amounts to assuming that the subject-predicate structure that is manifest in the content of judging is itself valid of things. In other words, the features that serve to mark the nature of thinking and the content thought are also taken to be features of the things thought about, taken to be true of what is, and not just of our way of thinking about what is.

Now, even if this assumption itself were to turn out to be true, Hegel points out that the assumption has been made 'without investigating whether the form of judgment *could* be the form of truth' (EL §28 Anm 8:94; my ital.). Hegel himself thinks that the skeptical tradition over the years has provided a good number of reasons to doubt the validity of this assumption, though Hegel's own presentation of these reasons here is admittedly quite compressed.[8] For one thing, this first position recognizes (either explicitly or at least implicitly) that there are multiple distinct, yet equally true, judgments; this, however, seems to entail that no *one* individual predicate 'shows itself to be adequate [angemessen] to the fullness of representation' of the whole truth about what is; each predicate 'is for itself only a limited content' (EL §29 8:96). What is more, the position also (implicitly or explicitly) assumes that there are

8 For some further discussion, see Inwood (1983: 155f).

multiple, distinct yet equally true, judgments about the very same individual object; yet the further truth that all of these judgments are about the same object, and that all of the predicates are thereby 'bound up *with one another* [miteinander; my ital.] *in one* subject', is not itself a content of any one of these (first-order) judgments – rather, in thinking about what is, the predicates can only be 'taken up *over and against one another* [gegeneinander] from the *outside*' (EL §29 8:97). Yet since thinking (judging) does not itself show up as something distinct from (over and against) the objects thought about, and so as something that itself can be judged about, this first position cannot express certain basic truths about thinking itself. The sum-total of the things it takes to have the form of the true will ultimately be '*one*-sided on account of its form and to that extent false'; in short: 'the form of judgment is unsuitable [ungeschickt] to express...the true' (EL §31 Anm 8:98). This itself stands in direct contradiction to the fundamental assumption of the position in question, since, as Hegel notes, it is a common presupposition of this position (and, again, remains so), not only that some individual judgments *can* be true, but that, for any two 'opposing' judgments ('assertions'), 'one *must* be true and the other false' (EL §32 8:98; my ital.).

The Second Position: 'Empiricism'

The transition to the next 'position' on thinking and objectivity – what Hegel initially calls 'empiricism' – comes, Hegel thinks, from an attempt to respond to a general skepticism about the validity of the form of judgment to adequately express what is true. Rather than take the form of truth from thinking as *judging*, 'empiricism' embraces instead the 'great principle that what is true must be in actuality and *be there* [da sein] *for perception*' (EL §38 Anm 8:108; my ital.). The form of truth now is taken to consist in 'seeing [sehen]' the object, along with a new emphasis on the 'subjective side' of 'knowing [wissen] *oneself* to be *present* [präsent]' in the seeing (ibid.), and a new focus on '*immediate presence* [Gegenwart]' for 'consciousness [Bewußtsein]' of the object itself – the combination of which yields '*certainty* [Gewißheit]' (EL §38 8:108).

However this might fare as a method for securing a kind of certainty for what is immediately present to consciousness, it is not clear what room it leaves for the science of *thinking*, understood as the bringing of things to consciousness by way of concepts and judgments, insofar as concepts and judgments are not themselves 'seen' in what is immediately present to perception. In any case, at least in its initial historical form, Hegel thinks that this sort of empiricism does not actually limit its account of what is true simply to 'what is outwardly and inwardly present' to consciousness in perception (EL §37 8:107). This is because empiricism (again, at least in its historical form) cannot resist 'elevat[ing] the content belonging to perception, feeling, and intuition to the *form of universal representations, propositions, and laws*, etc'

(EL §38 8:108). This leads to the moment of 'Humean skepticism', which points out (rightly, Hegel thinks) that, 'insofar as perception is to remain the foundation [Grundlage] of what is to count as truth, universality and necessity appear to be something *unwarranted* [Unberechtiges]', since these features (universality, necessity) are not themselves immediately present or given in any one perception (EL §39 8:111). Rather, these features – and any other form or 'determination' from thought, from outside of perception – come to be seen as, at best, something 'subjective', something added by additional acts of the subject to what is present in perception.

Now, strictly speaking, according to its own principle, empiricism should thereby count thinking and its determinations, one and all, as 'untrue' – including (however problematically) whatever thinking might be a condition for the possibility of articulating the philosophical position of empiricism itself. For Hegel, this would be a rejection, first, of the traditional assumption that logic, as the science of thinking, will also be a science of truth itself, since thinking and the truth are now being sharply separated from one another. What is more, it would also be, in effect, to reject the idea that logic itself should count as a science at all, since its subject matter (thinking) cannot itself come before consciousness in immediate perception, which implies that empiricism should recognize no truths about thinking (so understood).

Logic as About What Is Objective for Subjectivity: 'The Critical Philosophy'

Hegel sees the next position in thought – Kant's 'critical philosophy' – as growing out of an attempt to reconcile both of the previous two 'principles' – that thinking (concepts, judging) is needed for the truth to come before consciousness, but also that the only things which come immediately before consciousness are the objects of perception (sensory 'appearances'). On Hegel's retelling, however, Kant's philosophy itself ultimately moves through three distinct stages in its own 'critical' reconception of thinking itself: from an initial embrace of thinking as judging, as the activity of what Kant calls discursive '*understanding* [Verstand]', in contrast to what is simply given in perception; to a recognition that thinking also can take a more 'dialectical' form, in the movement of '*reason* [Vernunft]' beyond the relation between subject and predicate in a single judgment, toward the unification of objects and judgments in relation to their principles; and finally to a 'speculative' proposal of a kind of thinking that is itself an objective, creative activity, one performed by the absolute or *divine* understanding.

Thinking as Understanding

As Hegel sees it, Kant initially aims to achieve a synthesis of 'metaphysics' and 'empiricism' by retaining the traditional conception of thinking as

judging, but then restricting the scope of the objective validity claimed for the elementary determinations of thinking (Kant's 'pure concepts', 'categories') to the application of these determinations to what Kant thinks is in fact immediately present to consciousness – namely, 'appearances' – and to appearances alone. The doctrine of thinking per se, and the understanding as the capacity for thinking (judging), is what constitutes, for Kant, the traditional logic. The doctrine of thinking (understanding) in its application to appearances is given in Kant's new 'transcendental' logic. To be sure, in this application, thinking does go 'beyond' what is 'given' in any one perception, since it deploys universal representations (concepts) to thereby 'determine' what is given, whereas what is immediately given is something singular. Nevertheless, through this process, Kant claims that the mind thereby achieves, not merely the 'perception' of singular sensory contents (mere appearances), but the 'experience [Erfahrung]' of substances and causes and other objects falling under the categorial determinations of understanding. In Hegel's words: 'through the categories, mere perception is elevated to the level of objectivity, to the level of *experience*' (EL §43 8:119); 'thought-determinations constitute [ausmachen] the *objectivity* of the cognition of experience' (EL §40 Anm 8:113).[9]

Though Kant hopes to show that thinking can and does allow us to 'determine' a kind of objectivity in the constitution of experience – and hence, demonstrate that the principles of traditional logic itself have at least some kind of objective validity – Hegel argues that Kant still acknowledges that, from another point of view, these thought-determinations themselves might be seen to 'belong to *subjectivity*' alone. This is so in two senses: first, Kant himself insists that the categories are subjective with respect to their 'origin', and purports to provide a 'metaphysical deduction' of these thought determinations from the 'subjective activity' of thinking, due to the exercise of our understanding as our capacity to judge (rather than coming to consciousness by being given from outside of acts of our understanding). Second, the categories ultimately turn out to be merely subjective with respect to the domain of their demonstrable validity, since these determinations cannot be demonstrated to have correct application to any 'thing in itself', but only to appearances, which are themselves only representations in the mind (EL §41 8:113–14). What is more, Hegel takes Kant himself to clearly recognize – and even celebrate – both of these limitations; in fact, Hegel takes Kant to openly proclaim that any thinking which arises through the application of concepts in judgment by our understanding 'is incapable of cognizing things in themselves' (EL §44 8:120).[10]

9 For more on the significance of the distinction between perception and experience for the interpretation of Kant's own conception of cognition and objectivity, see Tolley 2017b.

10 There is also the further question of whether Hegel thinks Kant is actually successful in overcoming the deeper Humean worry about the 'application' of the categories

Thinking as Reason

Over and against this position, however, Hegel also thinks we find in Kant's own transcendental logic a basis for a second conception of thinking, one that overcomes the restriction to the understanding and its acts of concept-application in the formation of judgments about appearances in experience. This is because Kant himself ultimately accepts that thinking – not least Kant's own thinking, in the critical philosophy itself – *is* able to 'have insight into [einsehen] what is conditioned about these cognitions of experience', insofar as thinking is able to cognize the *conditions* for the possibility of experience itself (e.g. those presented in Kant's own 'Analytic of Principles'), conditions which are not themselves further conditioned by experience and (crucially) which are not themselves further appearances. The capacity for *this* sort of thinking is what Kant associates, not with our understanding and its acts of judging, but rather with our *reason* and its acts of inference and explanation, in which our judgments become ordered according to relations of consequence, on the basis of which one expresses a principle and which a theorem, which one follows from which, and so on.

Now, insofar as Kant also takes reason to be also the capacity which searches for ultimate or absolute principles, reason can also be characterized as 'the capacity for the *unconditioned* [Unbedingte]', at least with respect to its aim (EL §45 8:121). Yet once it is able to take up the point of view of reason, Hegel argues that thinking must ultimately 'explain [erklären] cognitions of experience as something *untrue*, as appearances', and must 'assume the unconditioned for the absolute and *the true*' (EL §45 8:121; my ital.). This is because thinking qua reason assumes that the true nature of objects lies not in their appearances (as 'empiricism' would have it), nor in anything else merely subjective or contained in consciousness, whether in what is immediately presented in perception or in what is judged or cognized in experience (as Kant's 'Analytic of the Understanding' would have it). Rather, as Kant's own 'Dialectic' indicates, reason takes the essence or truth of objects to lie in the complete conditions which 'explain' why these subjective items are the way that they are – and so not *just* in those conditions that lie in

(thought-determinations) *even to appearances* (to what is immediately present in perception), given their radical singularity, particularity, etc, – in Hegel's words, whether it even makes sense that we can '*think* perceptions' at all (EL §50 10:130). Kant famously tries to overcome just this worry in his 'transcendental deduction' of the validity of the categories at least with respect to appearances, drawing on the earlier findings of the Aesthetic, that appearances already have universal and necessary 'forms' (space, time) – though Hegel's own assessment of Kant's arguments for the universality and necessity of space and time as forms of appearance has received less treatment. For discussion of Hegel's criticisms of Kant's transcendental deduction more generally, see Ameriks 1985, Bristow 2007, McDowell 2009, Sedgwick 2012.

our own mental capacities, which (by Kant's own lights) provide only a partial reason or ground for experience being the way that it is, but also, ultimately, in those conditions that provide the grounds for why our mental capacities themselves are the way that they are.[11]

Famously, however, Kant himself does not take thinking as reason to be able to demonstrate the objective validity of its own 'ideas' of such unconditioned principles, precisely because they are ideas of objects which lie beyond all possible experience. In Hegel's diagnosis, however, Kant reaches this conclusion only because he assumes that 'to *cognize* [erkennen] means nothing other than to know [wissen] an object according to its *determinate* content' (EL §46 8:123), where the only 'determinate contents' that thinking has available to itself are those categories given to it by the understanding. This implies that the only way that thinking *could* achieve cognition of its objects would be by way of an application of those very same 'determinations' that we saw above constitute the predicates in judgment – i.e. by means of an 'application of the categories to the unconditioned' (EL §46 Anm 8:124; cf. EL §48 Anm 8:127). This assumption, Hegel thinks, is what pushes reason directly into an 'antinomy': on the one hand, reason must make use of 'determinate' contents (categories) to judge about its objects; on the other hand, these objects are defined as being essentially 'undetermined' in the specific sense of being 'unconditioned' by anything in appearances themselves.

Because Kant takes himself to have shown that appearances themselves provide the only concrete content for the categories, the thoughts that reason purports to form, by means of these same categories, of objects that transcend appearances altogether, will inevitably seem 'empty' – referring us to what can be pointed to only as a 'something = X'. In Hegel's words, the thoughts that reason thinks in relation to *its* objects are no better than an 'empty identity': reason's thinking is ultimately 'merely empty indeterminate thinking'; 'it thinks *nothing* [nichts]' (EL §48 Anm 8:127). Thus, despite the fact that reason seemed at first, and is officially, for Kant, a 'higher' form of thinking than mere understanding, thinking by reason is ultimately such that '*determinateness* remains something *external*' to it, with the result being that, as reason, 'thinking is in itself merely an *indeterminate unity* and the *activity* of this *indeterminate unity*' (EL §52 8:137).

Because of this, the thinking of reason, too, cannot be in accord with the truth. Thinking as understanding fails to accord with the truth as thought by reason, because the basic contents (categories) of thinking are 'incapable of being determinations of the absolute', such that 'the

11 For a recent analysis of Hegel's positive assessment of Kant's own treatment of reason in the Dialectic, see especially Kreines 2015—though Kreines stops short of taking up Hegel's own positive assessment of the prospects of reason being able to provide a grounding of the metaphysics of the human mind itself (and its experience).

understanding or cognition by means of the categories is incapable of cognizing *things in themselves*' (EL §44 8:120). Yet thinking as reason also itself fails to accord with the truth, insofar as reason inevitably leads thinking either into contradictions and antinomies or into empty identities.[12] As long as 'the Kantian philosophy…leaves the categories and the method of ordinary cognizing completely uncontested' (EL §60 Anm 8:144), it will therefore remain in the dialectical moment of contradiction and nothingness.[13]

Thinking as Intuitive Understanding

Even so, Hegel sees Kant as implicitly recognizing a *still higher* power of thinking in the very idea of 'the thing-in-itself', thought however indeterminately by reason. On the one hand, this very idea is *itself* 'merely *the product* of thinking, more specifically, of thinking that has progressed to pure abstraction' (EL §44 Anm 8:120–1). More specifically, Hegel thinks that 'the thing-in-itself…expresses *the object* insofar as one abstracts from *everything that it is for consciousness*, from all determinations of feeling as well as from all determinate thoughts of it' (ibid.; my ital.). Yet with this abstraction, *thinking itself* has 'progressed' to 'the beyond, the negative of representation, of feeling, of determinate thinking, etc' (EL §44 Anm 8:121) – i.e. beyond not just understanding but also reason construed as limited in its thinking by the 'determinate' categories of understanding – and so, not just to an empty nothing but to what Kant himself would call a positive conception of a noumenon (object of *nous*).

As Hegel sees it, this still higher idea of thinking arises in the course of two further reflections. First, Hegel notes Kant's belief (articulated in the second *Critique*) that the activity of reason itself is also 'practical', in the sense of making actual (causing) things to be which are not yet so. What is more, though reason is faced with antinomy when it thinks about objects it takes to be real but would have to be 'given' to it from without (but cannot be given, due to the limitations of our sensibility), reason is nevertheless able to think consistently of objects that it *itself* will *make*

12 In fact, Hegel thinks Kant radically underestimates the pervasiveness of the dilemma that thinking qua reason will face; on Hegel's analysis, an antinomy will obtain for reason with respect to the application of categories 'in *all* objects of all genera, in *all* representations, concepts, and ideas', and therefore indicates a 'property' that arises in relation to the thinking of reason *as such*, a 'property' of thinking called 'the *dialectical* moment of what is logical' (EL §48 Anm 8:128). For helpful discussion of some of Hegel's motivations for claiming to uncover a more radicalized form of Kant's antinomies, see again Kreines 2016.

13 On the details of Hegel's discussions here (and elsewhere) of Kant's account in the Dialectic of the limits of cognition and the categories, see especially Ameriks 1985; compare Longuenesse 2007, Bristow 2007, and Sedgwick 2012.

real or actual, as their cause (i.e. as 'will'), by way of its ideas of what '*ought to happen*' (EL §53 8:138). But then, in its practical-causal form, reason's thinking *is* an 'activity that is *objectively determining*' (ibid.), insofar as its thinking itself gives its ideas 'worldly existence, external objectivity' (EL §54 8:138).

Even practical reason, however, still remains 'external' to objectivity in the following sense: though it is a kind of thinking that achieves objectivity through its own causality, it does so in relation to products or effects which need not be identical to itself. That is, practical reason does not (or at least not always) 'make actual' more practical reason; rather, it causes *nature* to be configured in a certain way (consonant with its idea of how it should be). This implies that, even when it is effective, practical-rational thinking still stands at some remove from the objectivity it produces, and reason itself, and its causal power, remains something subjective in this sense.

It is with this second reflection that Hegel shifts our attention to what he sees as a third, 'speculative' stage in Kant's thinking, one which articulates a concept of thinking that lies 'beyond' reason (so construed) altogether, whether theoretical or practical. Hegel sees Kant's later speculation (in the third *Critique*) concerning what he calls an 'intuitive understanding' as eventually bringing into focus an idea of a thinking that would *not* be external to its effects in the same way, but would instead itself *be* the external effect *as well as* the cause, and so itself *be* what is objective. Kant is lead to this higher conception of thinking by reflection on the kind of thinking that 'is to be experienced in the products of art and in organic nature' (EL §55 8:139; my ital.). In these cases, Kant thinks we encounter existences which are (or seem to be) effects of ideas about how nature ought to be, but existences whose causes are not something external to themselves; rather the actuality of these beings is in some sense the cause *of itself*, insofar as the actuality of the activity is itself the 'end' of the activity itself; the actual doing is itself the goal or purpose, it is done 'for itself', and it (the activity) is itself what ought to be. Here, as Hegel sees it, Kant has finally hit upon the idea of a thinking that itself *is* the objective reality experienced, *is* both cause and effect. This gives Kant the 'distinction' of attaining what Hegel calls the fully '*speculative*' idea of thinking that transcends the 'dialectical' thinking of reason spelled out above (EL §55 Anm 8:139–40).

Yet if Kant himself spends more time in the third *Critique* articulating how this form of thinking is actualized in the course of specifically human and biological activity (in art, in 'organized' nature), Hegel also thinks that, by the conclusion of the book (cf. §§76–78 of the third *Critique*), Kant takes one last, final, crucial *further* step in the philosophy of thinking, by forming the 'idea' that the actualization of just this form of thinking is what is ultimately responsible, not just for this or that work of art or living body or any other finite part of the natural world, but for

existence as a whole. With this, Kant forms the unlimited, 'encompassing [umfassende] idea' of 'the postulated *harmony* [Harmonie] of *nature* or necessity with the *end* of freedom, in the final *end* of the world thought of as *realized*' (EL §55 Anm 8:140; my ital.). Crucially, however, this is not a conception of existence as a whole as simply caused by thinking, where the thinking as cause remains external to what exists itself (as in the thinking that characterizes practical reason). Rather, actuality itself just *is* the ongoing activity of a higher thinking, self-actualizing, whose purpose or end lies entirely in itself.

Now, the traditional name for a 'power' for thinking described in this 'encompassing idea', one that is capable of being this sort of cosmic-level activity – being itself the 'third' term that perfectly unifies in itself the good as idea and what there is (the world) as what is actual – can only be something *divine*:

> the idea in its entire unlimitedness would be that the universality determined by reason, the absolute final end, *the good*, would be actualized [verwirklicht] in the world, and indeed through a third, the power [Macht] positing this final end itself and is realizing it – *God*....
>
> (EL §59 8:142)

What is more, because it is therefore 'the essence, the substance, the universal power, and the determination of the end for the world', it is only this thinking that will be the 'absolute truth' of everything (EL §50 Anm 8:131). For 'while being belongs to the world, this being is merely a semblance [Schein], not the true being, not absolute truth; this is instead beyond that appearance [Erscheinung], in God alone, that God alone is true being [das wahrhafte Sein]' (EL §50 Anm 8:132).

As Hegel sees it, then, Kant's own progressive analysis of thinking, when taken to its full conclusion, leads us to reconceive of the highest form of thinking along explicitly divine, panentheistic lines. Nevertheless, Hegel takes Kant himself to fundamentally misunderstand the full significance of this higher conception of thinking as speculative. This is because Kant holds that we humans can only relate to this speculative form of thinking as itself a 'harmony...that merely *ought* to be, i.e., that at once does *not* have *reality* – as *something believed* [Geglaubtes], to which pertains only subjective certainty, not truth, i.e., *not* the objectivity corresponding to this idea' (EL §60 8:143).[14] At the same time, however,

14 In fact, Hegel notes that this is so, even with respect to the 'limited' cases, insofar as, strictly speaking, Kant doesn't think we can demonstrate the objective validity of the concept of self-determining purposiveness with respect to *anything* in nature, including ourselves, but can only take this concept as a 'principle of assessment belonging to *our* understanding', and so ultimately 'something subjective' (cf. EL §58 8:141). Hegel

Hegel means to highlight the fact that, even according to Kant himself, at least the *idea* of speculative thinking is something whose reality *thinking itself* can 'know [wissen]', and not just 'believe' in. This, Hegel thinks, will provide sufficient opening for an argument that the *thinking represented in this idea* can *also* be 'known' – again, *by thinking itself*.

Logic as About the Thoroughgoing Harmony of Subject and Substance: 'Immediate Knowing' and the Transition to Hegel's Own Conception

Trying to work out these last two thoughts – that thinking enjoys a real relation at least to its own speculative 'idea' of the highest thinking as divine (since it can form this idea), and that thinking thereby might enjoy a real relation to the object of this idea, i.e. this divine thinking itself – is what Hegel thinks drives the philosophy of thinking (and hence the philosophy of logic) toward its next and penultimate position, prior to arriving at Hegel's own. This position takes its starting point from the fact that it is only this highest form of thinking (God's), rather than thinking as understanding or even as reason, that should be counted as 'absolutely *true*'. One key reason for this is that it is only in this thinking that there will be a perfect 'harmony' (*adaequatio*) between thought and being, since what there is just *is* the actualizing of this divine thinking itself. In 'the absolute inseparability of the thought of God from his being', Hegel thinks we have now moved to conceiving of thinking as having achieved an 'immediate *knowing* [Wissen]' (EL §51 Anm 8:137).

Intuitive Understanding and the Truth

In the WL, Hegel spells out in greater detail how this transition to thinking as immediate knowing is supposed to work, by drawing out a conflict within Kant's own 'definition' of truth – indeed, a conflict that echoes the one that we saw arise in the first 'position' of 'metaphysics'. On the one hand, Kant officially embraces the traditional 'definition' of truth as

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also notes that this same estimate is given (and perhaps even more obviously so) with respect to our relation to the speculative idea of thinking when it is deployed at the cosmic scale: here Kant again emphasizes the absence of objective validity (though also the subjective usefulness) of the concept of an intellect whose activity would serve as the supersensible ground for all of nature (cf. 5:469f).

There is a question as to whether Hegel means to claim that Kant himself holds that speculative thinking transcends reason altogether; if so, this would seem to saddle Kant with a position on reason that would be incompatible with Kant's claim (emphasized in the *Groundwork* and elsewhere) that God, too, is a member of the community of specifically rational beings. There is also a question of whether Kant himself would accept that an intuitive understanding would still count as 'thinking' at all, in his sense of the term (cf. B71).

a relation of 'correspondence or agreement [Übereinstimmung]' between thinking and its object (cf. B82–3). On the other hand, Kant's idealism seems to render all thinking qua understanding and reason – including the thoughts of reason and of things in themselves – as 'untrue', since incapable of agreeing with things in themselves:

> If we recall this definition [of truth as agreement of cognition with its object] together with the fundamental thesis of transcendental idealism, namely that *cognition of reason* is incapable of grasping *things in themselves*, that *reality* lies *absolutely* outside the *concept*, it is then at once evident that such a *reason*, one which is *incapable of setting itself in agreement with* its subject matter, and the things *in themselves*, such as *are not in agreement with* the concept of reason – a concept that does not agree with reality and a reality that does not agree with the concept – that these are *untrue representations*.
>
> (WL 6:266; my ital.)

In fact, as we have seen, Hegel thinks that Kant, too, implicitly recognizes that the only kind of thinking that could even possibly enjoy an absolute agreement or harmony with its object is not that of our understanding or even that of our reason, but the thinking performed by the intuiting understanding:

> If Kant had measured the *idea of an intuitive understanding* against that first definition of truth, he would have treated that idea which expresses the required agreement, not as a figment of thought, but rather *as truth*.
>
> (6.266; my ital.)

Note again that Hegel's point is not just that the intuitive understanding is capable of attaining the truth, or cognizing it as an object, but rather that its thinking itself simply *is* the truth.[15]

Hegel's criticism, then, is ultimately that, by treating thinking qua speculative as merely an 'idea', Kant treats *the truth itself* always only as something we humans must 'believe' ought to exist, and hence as

15 Hegel makes a similar point a few pages earlier: 'It will always be a source of wonder how the Kantian philosophy did cognize that the relation of thought to sensuous existence (the relation at which it stopped) is only a relation of mere appearance, and also well recognized and asserted in the *idea* in general a higher unity of those two terms, as for example in *the idea of an intuitive understanding*, and yet remained standing at that relative relation and at the claim that the concept is and remains utterly separate from reality – thus asserting as *truth* what it declared to be finite cognition, and explaining away as extravagant and illegitimate figments of thought what it recognized *as truth* and had specifically defined as such' (6.264; my ital.).

something we relate to only by our thinking of it 'in idea'. Kant never adequately takes up the truth itself as an 'objectivity' that *already does* exist in reality, yet insofar as he claims that there are truths – and claims, moreover, to know some of them – then Hegel thinks Kant is ultimately committed to there being a real, actual object (i.e., the truth itself) corresponding to our idea of truth – and hence, to our idea of the thinking performed by the intuitive understanding. Kant's own reflections lead him only to form the *idea* of a 'harmony' of subjective activity (causality) and objectivity (effect) which itself exists objectively; he takes the *object* of this idea – this panentheistic activity 'in itself' – to lie beyond our own finite consciousness or representations. In so doing, however, Hegel sees Kant as placing the truth itself beyond our consciousness (qua understanding, reason), as something which cannot be known in or through consciousness itself, since the truth itself can be *nothing other than* this divine activity.

Rethinking the 'Immediacy' of the Highest Thinking

Returning now to the final section of the EL's history of philosophy of thinking, Hegel then highlights one particular attempt after Kant, to try to take up the challenge of more directly articulating the shape or form of the activity that would be 'speculative' in the sense articulated above (i.e. would itself be an objective harmony (agreement, unity, 'identity') of subjective and objective). This is the 'speculative' conception of 'immediate knowing' articulated by Jacobi in his 1785/9 *Letters on Spinoza*.

In one sense, the turn to Jacobi is surprising, because, as Hegel sees it, Jacobi counsels that we simply *reject thinking* itself – understood along official Kantian lines as the activity of consciousness that determines objects through categories – as the manner in which the truth is to be cognized (cf. EL §§61–2), in order to affirm a more '*immediate* knowing [unmittelbare Wissen]' (EL §62 Anm 8:148). It is a *knowing* because, in it, the mind itself achieves the 'harmony or agreement' between something subjective and something objective that is constitutive of having the *truth* 'in mind'. It is not a *thinking*, however – at least in the sense familiar from Kant – because the consciousness of this agreement is 'immediate', and occurs by way of a consciousness of a simple '*representation*', rather than anything discursive, predicative, inferential, etc. Thinking, by contrast, is always 'the activity of *the particular* [das Besondere]' (EL §61 8:148), which implicitly involves differentiation of one thing from another by means of negation, and so involves 'mediation [Vermittlung]' in Hegel's sense.

In its emphasis on immediacy, and in its efforts to effect a complete '*exclusion* [Ausschließung]' of mediation, Hegel recognizes that attempts like Jacobi's will (rightly) sound like 'a falling back [Zurückfallen] into the metaphysical understanding' we met with above in the discussion of the first 'position' concerning thinking (EL §65 8:155). Nevertheless,

Hegel recognizes that the position at least *intends* the immediacy in question *not* to be naive at all, as it intends something 'higher' than both thinking qua judging (understanding) and thinking qua reason – and in fact, intends nothing short of the 'intellectual intuiting of God' (EL §63 Anm 8:151). What is more, it intends this both in the sense of at least representing the intellectual intuition that God has of what God knows, and also in the sense of intuitively representing God himself, where this is done by the divine itself, 'in' and 'through' us.

Still, Hegel himself does not see anything in the way that the position (at least in Jacobi's version) spells out this allegedly higher immediacy of 'intuiting' that would determinately differentiate it from the initial naive immediacy of the mere having of an object in a 'representation', insofar as Jacobi doesn't seem to allow even consciousness of the representation, or subjective 'certainty' of its presence (à la empiricism), or any further determinations to obtain. Nor does Hegel think Jacobi *could* articulate how this immediacy could be 'higher' than the simple immediacy from the first position *without* incorporating any of the further intellectual aspects that were taken to characterize thinking by the later positions in the history of philosophy.

Even so, Hegel's complaints against Jacobi's specific way of formulating the nature of 'immediate knowing' should not be read as a wholesale rejection of the idea itself – nor should Hegel be taken himself to reject the possibility that 'immediate knowing' might nevertheless turn out to be a form of *thinking* after all. In fact, Hegel argues against Jacobi that the relevant 'higher' mental activity should not be thought of as enjoying *less* mediacy (and so a fortiori would not be *devoid* of mediacy) but should in fact be thought to incorporate a *more* thoroughgoing – in fact, 'absolute' – mediation – and so one that incorporates the previous forms of mediation constitutive of understanding (predication, judgment) and reason (inference, systematic ordering) but then supersedes them (rather than simply negating them).

This is so, even if the higher thinking in question might give the impression, for example, of being entirely 'spontaneous'. At this point Hegel turns to several examples – some from Kant's third *Critique*, some which draw upon aspects of human life that do not receive extended treatment in any of Kant's *Critiques* – to consider what we undergo in the partial or imperfect realizations of intuitive understanding that we ourselves might be thought to achieve. Hegel's aim is to highlight several dimensions of mediacy that remain present not just in the organic and aesthetic examples of thinking as intuiting understanding that Kant himself had begun to sketch, but also in the thinking that constitutes scientific (e.g. mathematical) practice:

> [I]t is one of the most common experiences that truths, which one knows very well to be the result of the most complicated [verwickeltsten] and highly mediated considerations, present themselves

> [sich präsentieren] *immediately* in the consciousness of someone conversant [geläufig] with such cognition. The mathematician, like everyone else trained [Unterrichtete] in a science, has solutions immediately present [gegenwärtig] to which a very complicated analysis has led; every educated [gebildete] person has immediately present in their knowing a set of universal viewpoints and principles that have come forth only from repeated reflection and long life-experience. The facility [Geläufigkeit] we have achieved in any kind of knowing, also in art, in technical skill, consist precisely in such acquaintances [Kenntnisse]....
>
> (EL §66 8:156)

Here Hegel is describing moments in which a solution to a problem seems to come to consciousness in a flash, despite having only been made available to the relevant individual due to their acquisition of this 'facility' through 'training', 'education', 'repetition', and so on:

> In all these cases the immediacy of knowing does not only not exclude its mediation, but rather they are so connected that immediate knowing is even the product and result of knowing that has been mediated.
>
> (EL §66 8:156)

Hegel then goes on to emphasize that a similar 'mediation' – in the sense of a dependence on prior 'training', 'reflection', 'life-experience', etc. – obtains even in the kind of knowing we might enjoy in 'religion' and 'ethical life' – and even in philosophy itself ('even for Platonic recollection'). In all such cases, '*education* [Erziehung], development [Entwicklung] is essentially required to bring to consciousness what is contained therein'; these cases of knowing 'are absolutely conditioned by the *mediation* that is called variously "development", "education", "formation [Bildung]" (EL §67 8:157). 'It is thoughtlessness', Hegel insists, 'not to know that, with the conceded necessity of an education, the essentiality of mediation is thereby asserted' (EL §67 Anm 8:158).

One of Hegel's main points here is that the ostensibly 'immediate knowing' enjoyed by an individual 'mathematician' or 'artist' – i.e. in the flash of their apprehension of the harmony between their concept and its object, which is enjoyed in the seemingly effortless appearing of the truth in their consciousness – is itself actually made possible by many earlier acts of thinking, acts which are in a fairly straightforward sense 'external' to the moment of seemingly immediate apprehension. What is more, this moment is mediated not just by the necessity of earlier acts of thinking by the *individual,* which are required to have achieved certain capacities or expertise, but also by the acts of thinking by *other* individuals, i.e. thinking performed by the community responsible for the

education and training of the individual. Especially the second activity of thinking is one that in no way could be seen as entirely 'my' own but rather necessarily includes the activity of the broader social world of other subjects (other 'I's) who put each individual (each 'me') through training, education, and so on.

Hence, what can seem initially as something that arises immediately to my mind, thanks wholly to 'my' own freedom and spontaneity in thinking in that moment, shows itself to be, in several senses, given *to me* from without – even if not necessarily given from outside of *thinking* as such. Even – and perhaps especially – in its most 'scientific' form, 'my' thinking bears within itself the marks of having been 'developed' by the thinking of others; 'my' own thinking includes a dimension of activity that is still subjective but beyond 'my' own doing, and so also stands over and against me as something 'objective'. Or, to more fully put this point in the terms of the third part of Hegel's *Encyclopedia*, the *Philosophy of Spirit*: the thinking performed by 'my' ('subjective') 'spirit' – and perhaps especially when it is most 'true' (most scientific, most ingenious, most expert) – is made possible only by the thinking performed by the 'objective' spirit of the family, community, and history into which I am born and from which I will depart upon my death.

Thinking as Absolutely Self-Mediating

Faced with the limits of Kant's doctrine of understanding and reason, Jacobi had assumed that the only path left to this adequacy would be to reject or exclude all mediacy from truth itself. Hegel's reflections on the sociality and historicality of even the most 'scientific' moments of human knowing are meant to demonstrate that increase in mediacy (education, training, etc.) does not, in fact, imply a decrease in adequacy. But while the appeal to the mediation present via the sociality and historicality that pertains to human knowing, even in its ostensibly more 'immediate' forms, can help to point up the mistaken presupposition in Jacobi's conception of the higher form of knowing, it is important that Hegel's own account of the progressive development of the history of philosophy of thinking does not conclude here.

One indication of this is Hegel's explicit rejection, at this point, of the idea that the '*consensus gentium*' could be a final 'criterion of truth' (EL §71 8:160). Even if it is necessary to recognize that the prior thinking of the other 'I's that constitute 'my' community is part of what makes possible 'my' thinking and therefore any 'agreement' between concept and object that obtains in 'my' consciousness, this will remain always only *part* of the story. For one thing, genuine knowing requires not just dependence upon – and in this sense, agreement with – the thinking of others (via education, training, etc) but also the agreement between all of

these thinkings and the *object* of the thought itself. For another, the very possibility of the social-historical development of thinking itself within objective spirit, along with the initial arising of what Hegel famously calls the 'second nature', through the historical-communal realization of reason, has its further own 'presupposition' – namely, the existence of the 'first' *nature* out of which human (subjective and objective) spirit in general arises (cf. EG §381 10:17). Only if social-historical thinking qua objective spirit was all that there is to think about (the only object), or if social-historical thinking were somehow itself what was responsible for all that there is, including itself and first nature both (and so was in this sense 'presuppositionless'), could this social-historical form of thinking itself be adequate to being the absolute truth.

The final step to Hegel's own position, then, will be to combine this lesson (that immediacy of knowing is not incompatible with very rich forms of mediacy) with the earlier thesis, anticipated in Kant, that the only thinking that will be absolutely adequate to what is being thought – and so will have the absolute form of 'the true' – will be divine thinking. This will lead Hegel to claim, first, that rather than divine knowing being akin to what Jacobi had described as an absolutely un-mediated intuition, this highest form of thinking will include all the mediation required to go beyond and perfect whatever partial forms of intuitive understanding that objective human spirit is able to achieve. And in order to achieve perfect harmony with what is being mediated, Hegel will claim, secondly, that divine knowing must be absolutely *self*-mediating. As he briefly puts this thought at the conclusion of the 'Vorbegriff', what we are ultimately aiming for is a conception of God as 'known [gewußt] as *mediating himself* in himself *with himself* [als sich in sich selbst mit sich vermittelnd]' (EL §74 8:163). Rather than excluding mediation, the absolute form of thinking will not only include all relevant mediation but will itself be what is mediating, what is being mediated, and that in which such mediation will take place.

To be sure, this only provides us with a 'preliminary conception [Vorbegriff]' of what thinking must ultimately be conceived as, or 'determined' to be, as the subject matter of logic. In order to adequately think of thinking in this form, Hegel thinks we will need to develop our concept of thinking from the simplest determinations that we will need to predicate of thinking, upward until thinking in its truth is truly comprehended. In the logic itself, then, we will begin by thinking of thinking as simply 'being', but then also having 'quantity' and 'measure' – and then to successively include more complicated ones – such as that of having an 'essence', being something which grounds 'appearances', having a kind of 'substantiality' and 'causality' – finally, onto ones that begin to become increasingly adequate to the truth about thinking in particular – such as that of having the shape of a 'concept', 'judgment', and 'inference', being itself an 'object' of a concept, being 'alive', involving 'cognizing' and

'willing' – and then, finally, to thinking of thinking as itself being Hegel will call 'the *absolute idea*' or 'divine concept'.[16]

This progressive development of 'logical determinations of thought [Denkbestimmungen]' is what comprises the body of the text of the *Logics* proper, which is itself organized into the 'doctrine of being', the 'doctrine of essence', and then 'the doctrine of the concept and of the idea' – all of which are moments in the overarching 'doctrine of thought [Gedanke]' (EL §83 8:179). The final determination of thinking is as 'an object into which all [these] determinations have gone together', which finally presents thinking as itself both 'the *absolute* and *total truth*' and 'as the self-thinking idea [sich selbst denkende Idee]' (EL §236 8:388). And with this, we will have reached something of a 'metaphysical definition of God' (EL §85 8:181) – at least as to God's own 'essence', 'prior to his creation of nature and of finite spirit' (WL 5:44).[17]

Conclusion: Hegel's Conception of Logic in Dialogue with Other Post-Kantian Positions

Logic as the Science of Truth and the Science of Science

This should suffice to give at least an outline of the context and motivations for Hegel's 'theologized' or 'divinized' conception of the subject matter of logic, as well as at least some initial indications as to why Hegel thinks that such a seemingly 'enriched' list of 'determinations' will be necessary to present what this thinking itself is, if logic is to adequately articulate its subject matter. Logic is the science of thinking, but thinking is essentially defined by its relation to the truth; logic itself can thus be understood as a science of 'the true'.[18] Thinking that is itself 'the truth' must be in absolute harmony with its object; absolute harmony will obtain only in divine thinking – and indeed, only in this when construed in something of a panentheistic manner.[19] To incorporate two well-known

16 For more on these transitions, see Bowman 2017, Quante 2017, Ng 2017, Zambrana 2017, and Kreines 2017. Compare as well Kreines (2015) for a different, non-theologized account of how best to understand the 'self-mediation' of the absolute idea; for some points of criticism, see Tolley 2017a.

17 The 'definitions' of the divine specifically as to how it manifests as nature and as spirit are not topics for the science of logic itself, as they are too 'concrete' (cf. 6:257, and see below).

18 Hegel claims in his lectures that, though it has traditionally been seen as 'the science of thinking', it would be as true to say that 'the task of logic would be grasped in the question "what is truth?"' (Hegel 1817b: 3). In the EL itself, Hegel even goes so far as to identify 'what is logical [das Logische]' with 'the absolute form of the truth', claiming 'even more than that, [it] *is* the pure truth itself' (EL §19 Anm 8:68; my ital.).

19 For more discussion of the relation between Hegel's views and panentheism, see Williams 2017.

phrases from Hegel's *Phenomenology*, this thinking, as 'what is true [das Wahre]', '*is* the whole' (PG §20 3:24), so that 'what is true' is not just something in the thinking (knowing) 'subject' but is also the 'substance' that is thought (known) (PG §17 3:23). Hence, any science of thinking which purports for thinking to be able to true will by necessity have to characterize thinking itself in such a way so as to show how this is possible. And in order to sufficiently characterize (or 'determine') thinking in this way, as this sort of thing, Hegel thinks that logic will need to develop just that series of concepts Hegel presents in his *Logic*.

If we continue to broaden our perspective to include not just the *Logics* themselves, but also the *Phenomenology*, we can also better appreciate that Hegel takes this reconception of logic be of a piece with the reconception he proposes there for what is involved in truly 'scientific' thinking and knowing – indeed, his reconception of what is constitutive of '*science* [Wissenschaft]' itself. In the *Phenomenology* Hegel purports to have demonstrated that, strictly speaking, 'knowing [Wissen] is actual [wirklich] *only* as science or as system' (PG §24 3:27; my ital.); the 'result' is the appearance of 'the *concept of science*' itself (WL 5:42). More specifically, the whole text itself 'presents the coming-to-be of *science in general* or *knowing*', beginning from the point of view of 'consciousness' (PG §27 3:31), and this exposition of the 'appearance' of science for consciousness itself provides what Hegel calls a 'deduction' of the validity of the 'concept of pure science' itself (WL 5:43). Even so, the *Phenomenology* does not yet itself present this concept 'in its *true* shape' (PG §38 3:40); this task is said to be left to *logic* (cf. PG §37). Because it is the science of thinking as absolute knowing and truth, logic can therefore equally be understood as the presentation of the 'determinations' of the 'true shape' of *science itself* – indeed, as the true science of science.

Against Subjectivism, Against Objectivisms

When taken out of context, sentences proclaiming Hegel's divinization of thinking, science, and truth can surely suggest that there might be little if any points of overlap with other post-Kantian conceptions of logic.[20]

20 Though it is not always emphasized (and is in fact often explicitly de-emphasized) among his recent readers (compare, however, Plevrakis 2017), the divinized conception of logic is in fact something Hegel affirms quite frequently throughout his writings and lectures. Hegel begins the *Encyclopedia* as a whole, for example, by claiming that philosophy 'has its objects in common with religion' because 'both have the *truth* for their object, and indeed in the highest sense – in the sense that *God* and God *alone* is the truth' (EL §1 8:41). And this same point is then repeated, with respect to logic in particular, in the very first section of the EL itself: 'logical determinations in general can be regarded as the definitions of the absolute, as *metaphysical definitions of God*' (EL §85 8:181). In his 1817 lectures on logic, as well as in the first (1817) edition of the *Encyclopedia*, Hegel explicitly aligns logic with 'speculative theology' (cf. Hegel 1817a: §17 Anm; Hegel 1817b: 8). See as well the end of the WL, where Hegel

In conclusion, however, I would like to provide the beginnings of a comparative analysis that tries to highlight points of continuity between Hegel's position and several other positions on the nature of logic that were developed in the wake of Kant in the 19th and 20th centuries and have gained a more widespread acceptance than Hegel's own. I will focus on the following three conceptions: the *mathematical-objectivist* conception of logic, put forward by Russell and others; the *semantical-objectivist* conception, put forward by Bolzano, Frege, and Husserl, among others; and the *pragmatist-intersubjectivist* conception, put forward most influentially by Robert Brandom. I will say more about what I mean to be associating with these labels in the course of the comparisons.

A first thing that Hegel's conception of logic shares with these others is that logic is not restricted in its focus to reporting what has been true of already-existent human mental activity, nor does it focus primarily on something that is possessed by any one individual human mind. In both of these respects, Hegel agrees with these other perspectives in affirming that the subject matter of logic should be kept distinct from that of individual psychology.

Yet even if Hegel's conception of logic is not psychologistic in this sense, Hegel's conception does take the determination of what is logical to involve reference to an activity that is associated with *subjects* – namely, *thinking* – even if Hegel does not mean to claim that the primary subject of this thinking is any individual human being. In this essential reference to subjectivity as such, Hegel would seem to agree both with the semantical and pragmatist conceptions of logic, though this pushes Hegel (and the others, incidentally) away from the mathematical conception. For their part, the semantical-objectivist takes logic to be essentially about a sphere of items that, though they are not properties or states of any individual subject's psychology, are nevertheless essentially the kinds of things that relate subjects to objects – for Frege, 'thought [Gedanke]' or 'sense [Sinn]'; for Husserl, 'meaning [Bedeutung]'; for Bolzano, 'concepts and propositions *an sich*'. The pragmatist-intersubjectivist might seem to incorporate even more of subjectivity into logic: Brandom, for example, takes what is logical to be not primarily an ideal, static, eternal realm of meaning-relations between subjects and objects, but instead a set of rules for activity by subjects in an essentially intersubjective context, along with the interrelation among the statuses that come along with following or failing to follow these rules.

Only the mathematical-objectivist insists that logical properties and logical laws are not properties and laws that pertain in any special way to subjects or their mental activity at all. On this conception, logic is concerned solely with very specific sorts of very abstract or universal objects (truth-values, functions, sequences of these), their properties (identity,

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describes what has been presented as 'the science of the divine [göttliche] concept' (6:672). Compare also Tolley 2018.

difference), and the relations between them (tautologicality, satisfaction, validity, etc.), typically pursued as the semantical correlates of a suitably formalized language and usefully modeled within set-theory. What makes these objects, properties, and relations 'logical' is that they exist or apply to the most universal domain; the laws and principles that govern these items (e.g. the law of identity, contradiction) are valid of everything; everything has at least logical properties or falls under logical categories; logical modality has the widest scope (what is logically possible is absolutely possible; what is logically impossible is absolutely impossible).[21]

In other words, according to the mathematical-objectivist, logic is no longer specifically 'about' thinking at all. Instead, logic is essentially about the most universal (and in this sense: 'formal') properties and relations that obtain between anything whatsoever (identity, difference, self-identity, etc). To be sure, logic does come into some relation with thinking: since logic is about the most general properties, relations, and laws that obtain with respect to anything which can *be*, they will also obtain with respect to anything which can *be thought about*. Its laws also hold of all acts and contents, considered as mathematical entities in their own right (e.g. as members of sets of thoughts, etc.). Nevertheless, at least officially, the sphere of logic is not in any way constrained by the sphere of what can be thought about; if there is any dependence, it will go in the opposite direction.

From the Hegelian point of view, traditional mathematical-objectivism will look most like the first position of thinking ('metaphysics'), insofar as the mathematical-objectivist is largely unconcerned to specify any role for thinking itself in the basic articulation of what they call specifically 'logical' (formal) properties, relations, laws, etc. Beyond the assertion of the existence of such items, and the implicit claim that they can be thought of and known, there is little attempt to explain how or why such correlation between thinking and the objects of logic should obtain or even be possible, let alone knowable – nor is there an attempt to provide an analysis of other 'epistemological' concepts, such as that of 'science'. None of these concepts are themselves taken to be among the basic concepts of logic proper.

Despite these differences, however, there is a further respect in which Hegel's conception *does* overlap with these universalist-objectivist commitments – and in this way actually pushes Hegel away from the other two conceptions. Given the afore-listed table of contents of Hegel's own *Logics*, it might come as a surprise to some readers to learn that Hegel himself *also* means for his own logic to be 'universal' – and indeed 'formal' – in a parallel respect. This is because Hegel, too, agrees that

21 For the laws and properties view, compare Russell 1918; for the properties view, compare Tarski 1986 and Sher 1991; for the modality view, compare Williamson 2013.

logic should not occupy itself with anything that is peculiar to one specific kind of concrete reality – including specifically human subjectivity and its activity. The two main kinds of concrete reality that Hegel identifies are 'nature' and 'spirit [Geist]', which are themselves divided up into several moments or aspects (or 'shapes'): nature divides into mechanical, physical (dynamical, chemical), and organic shapes; spirit divides into subjective (roughly: the consciousness, self-consciousness, and reason of individuals), objective (roughly: the family, corporation, state, history), and then absolute shapes (art, religion, philosophy itself). At key points in his *Logics*, Hegel takes pains to emphasize that logic should not concern itself with how its subject matter is realized in any concrete shape of nature or spirit, but only with the 'scaffolding [Gerüst]' that is common to both:

> Concerning the subject-matter [of logic] itself, we should note, first of all, that each of the shapes of intuition, representation, and the like belong to self-conscious spirit, and so are *not* as such to be considered in the logical science. The pure determinations of being, essence, and concept surely constitute the foundation [Grundlage] and the inner simple scaffold [Gerüst] of the forms of spirit. Spirit as intuiting just as much as sensory consciousness is in the determinacy of immediate being, just as spirit as representing and also perceiving consciousness has raised itself to the step of essence or reflection. These concrete shapes, however, *belong in the logical science just as little as* the concrete forms which the logical determinations in nature assume, and which would be space and time, then filled space and time, then inorganic nature, and organic nature.
>
> (WL 6:257; my ital.)

In fact, in notes from his lectures, Hegel claims that 'all the other philosophical sciences, the philosophy of nature and the philosophy of spirit' should be thought of instead as 'applied [angewandte] logic' (cf. EL §24 Z2 8:84). By contrast, Hegel thinks that his own list of logical 'forms' are a part of 'pure' logic, and therefore satisfy something close to the unrestricted universality thesis of the mathematical-objectivist: since these are the forms of 'what is absolute', these forms characterize (at least in some sense) absolutely everything. This can be seen throughout the *Logics*, but perhaps reemerges especially clearly throughout the EL, with Hegel describing the successive 'logical determinations' as determinations of 'the absolute', such that *everything*, for example, is (has being), has an essence, has a concept.[22] Hegel signals his kinship with

22 In the first sections of the EL proper, concerning the first 'logical determination' of 'being [sein]', and generally concerning 'logical determinations in general', Hegel claims that they 'can be viewed as definitions of the absolute' (EL §85), such that 'the

universalist-objectivism in his acceptance of the Anaxagorean thought that 'understanding and reason [Verstand, Vernunft] are *in the world*' (EL §24 Anm 8:81).

Conversely, though their continued reference to subjectivity might seem to draw the semantic-objectivist and pragmatist-intersubjectivist closer to Hegel, it should be noted that the subjectivity *they* mean to refer to is not itself 'absolute' in Hegel's sense. Because of this, their reference to subjectivity actually seems to push them further away from Hegel, precisely to the extent to which, unlike Hegel, neither means to embrace a kind of universalism with respect to *their* logical forms. Bolzano, Frege, and Husserl all mean to sharply separate the specifically logical 'forms' (of 'propositions *an sich*', of 'thought', of 'meaning') from the most general metaphysical or ontological forms of being.[23] Brandom, too, means for the normative principles and statuses to pertain first and foremost only to intersubjective inferential activity, and, in fact, in some sense might never refer to anything beyond this activity.[24] This, however, leaves it open that *not everything* there is will fall within the domain of 'what is logical', or essentially incorporates what is logical in its very being. On both accounts, the domain of logic is therefore not absolutely universal.

In any case, one of the most salient contrasts that will have already been felt to distance all three of these conceptions from Hegel's own is Hegel's thesis that it is necessary to ascribe a kind of active *causality* to thinking, one that would seem to go well beyond anything that any of these three positions would ascribe to what they take to be 'what is logical'. For Hegel (again, echoing Anaxagoras), the thinking in question is a 'principle [Prinzip]' of the world (WL 5:44), as part of what produces and thereby 'rules [regiert]' the world (12:23). Of course, it is this last commitment that lies behind Hegel's most infamous claim about what is logical – namely, that the subject matter of logic and *theology* coincide and that logic 'is *the presentation of God as he is in his eternal essence before the creation* [Erschaffung] *of nature and a finite spirit*' (WL 5:44).

It can be wondered, however, just how different in principle this is from what we might say if contemporary science purported to achieve a 'grand unified theory' that could 'explain' the emergence of the universe itself, and we were to ask: what kind of thing (ontological category) are the basic elements that structure this explanation? For it to be a theory

absolute is being' (EL §86 Anm), 'the absolute is essence' (EL §112 Anm), 'the absolute is identical with itself' (EL §115 Anm) – and then also glosses this commitment as entailing: '*everything* is differentiated' (EL §117 Anm), '*everything* is a concept', '*everything* is a judgment' (EL §181 Anm), and so on.

23 This is so, even if Husserl and possibly Frege, too, seem to embrace a kind of correlationism between the logical forms and the forms of being (compare Husserl 1900).

24 This is meant to pick up on Brandom's general proposal to 'explain away' both sense (conceptual content) and reference through inferential goodness (cf. Brandom 2000).

and an explanation (to be *science*), Hegel will insist that its elements must be thoughts. Yet if there were to be a genuine science of the coming to be of the universe – and with it, nature, finite spirit, and everything else; the coming to be of being itself – and if therefore there were truths about this coming-to-be, then, Hegel will insist, only a thinking that is *in perfect agreement with* this originary coming-to-be will itself be of the right shape to be true.

The main issue, of course, will be: what would this perfect agreement itself consist in? – which is itself a version of the general question that Hegel takes to animate logic itself: what would it mean for science to be true? Whether or not Hegel's own answer to this question is ultimately a convincing one, my hope is that the foregoing suffices to motivate Hegel's insistence that the question itself is one of deep interest for logic in particular, as it has traditionally been conceived. I hope also to have shown, more generally, how Hegel's reflections on this question, and his resulting reconception of the domain of *das Logische*, draws direct motivations from more familiar Kantian advances in the philosophy of thinking. I hope, finally, to have at least begun to sketch the extent to which Hegel's own view, suitably recontextualized, might nevertheless be seen to overlap with more recent post-Kantian developments in philosophy of logic on several fundamental points – despite first appearances to the contrary.[25]

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25 I would like to thank Sandra Lapointe, Anthony Bruno, Michael Forster, Marcela García, Michael Hardimon, Jim Kreines, Karen Ng, Lydia Patton, Terry Pinkard, Robert Pippin, Michael Pittman, Nick Stang, Eric Watkins, Chris Yeomans, Richard Zach, Rocío Zambrana, and audiences at the University of Chicago, UC San Diego, University of Montreal, University of Cambridge, University of Toronto, University of Pittsburgh, Georgetown University, and McMaster University for very helpful discussion and feedback on earlier versions of this material.

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4 Bolzano on Logic in Mathematics and Beyond[1]

Sandra Lapointe

Bolzano's Logical and Mathematical Work at a Glance

According to standard narratives, the origins of formal logic as we know it are to be found within the push toward logicism, axiomatisation and the foundations of set theory for which Frege's foundational project in mathematics often serves as muster. Frege, however, was by no means the first logician of the 19th century to seek to provide a new logical foundation to mathematical knowledge. At least one other author was driven by concerns, insights, ambitions and philosophical acumen that were as remarkable as Frege's. This author's efforts too resulted in a full-scale logical system whose conceptual resources, while they do not have the elegance and simplicity of Frege's "concept-script", are nonetheless as rich as those of first-order predicate calculus and powerful enough to generate Russell's paradox.[2] This author is Bernard Bolzano.

Bernard Placidus Johann Nepomuk Bolzano was born on 5 October 1781 in Prague. He was the son of an Italian father and of a German-Speaking Czech mother. His early schooling was unexceptional: private tutors, Lyceum. In the second half of the 1790s, he studied philosophy and mathematics at the Charles-Ferdinand University. He began his theology studies in the fall of 1800 and simultaneously wrote his first mathematical treatise, the *Considerations on Some Objects of Elementary Geometry*. When he was completing his doctorate in 1804, he applied for two positions at Charles University in Prague: one in Mathematics, the other one in the "Sciences of the Catholic Religion". He obtained the second. He was hastily ordained and took up his function in 1805. His professional career would be punctuated by sickness – he suffered from respiratory illness – and controversy: Bolzano's liberal views on public matters and politics would serve him ill in a context dominated by conservatism in Austria. In 1819, he was accused of "heresy" and subjected

1 This chapter is based on material that was first published as part of an entry in the *Internet Encyclopedia of Philosophy*. It has been revised and enhanced. I'd like to thank Sean Morris for a careful reading and helpful comments.

2 Cf. Simons (1997) and Lapointe (2011, Chapter 3).

to an investigation that would last five years after which he was forced to retire and banned from publication and public office. From then on, he devoted himself entirely to his philosophical work. The result is monumental.[3]

Bolzano is known to have been an important contributor to early modern mathematics. However, his *Considerations on Some Objects of Elementary Geometry* (1804) received virtually no attention at the time they were published and the commentators who have appraised his early mathematical work concur to say that Bolzano's positions may not have more than historical interest (cf. Russ 2004 and Sebestik 1992[4]; see also Waldegg 2001). Part of the reason for this lukewarm appraisal rests on the fact that Bolzano's investigations into geometry fail to anticipate modern axiomatic approaches to the discipline and did not belong to the trend that would culminate with the birth of non-Euclidean geometries, the existence of which Bolzano's contemporary Johann Carl Friedrich Gauss (1777–1855) claimed to have discovered and whose first samples were found in the works of Nikolai Lobatchevski (1792–1856) and Janos Bolyai (1802–60), whom Bolzano did not read.[5]

By contrast, Bolzano is renowned for his anticipation of significant results in the arithmetisation of analysis. This is important: Bolzano's advances here cannot be dissociated from his conclusions regarding the need for a logical reform. Three booklets that appeared in 1816–17 have drawn the attention of historians of mathematics and logic, one of which, the *Pure Analytic Proof…* was re-edited twice at the turn of the 20th century, in 1894 and 1905 (Rusnock 2000, 56–86; 158–98). At the time of their publication, however, they attracted hardly any notice. Only one review is known (see Schubring 1993, 43–53). And even though, according to Grattan-Guiness (1970), there is a possibility that Cauchy plagiarised the *Pure Analytical Proof…* in his *Cours d'Analyse*, this hypothesis is disputed by Freudenthal (1971) and Sebestik (1992, 107ff).

Over the course of the 1820s, Bolzano returned to the philosophical and methodological investigations he had initiated earlier in the *Contributions to a Better Founded Exposition of Mathematics* (1810), a book in two volumes on logical methodology which seems to have received little attention. At the end of the 1830s, after he had worked out the basis for the logical system he deploys in full in his main logical treatise, i.e. the *Theory of Science* (1837), Bolzano returned once more to the philosophy of mathematics and spent the last years of his life working on the

3 Cf. Lapointe (2014a).

4 As Sebestik explains (1992, 35 note), Bolzano, however, never put into question the results to which he had come (Bolzano 1804) in his attempt to prove Euclid's parallel postulate.

5 See Sebestik (1992, 33–72) for a discussion of Bolzano's contribution to geometry; see also Russ (2004, 13–23).

Theory of Magnitudes (*Grössenlehre*). The latter remained unpublished until after his death, and only excerpts appeared in print in the 19th century, most notably the *Paradoxes of the Infinite* (1851). The *Theory of Function* (1930) and the *Pure Theory of Numbers* (1931) were edited by the Czech mathematician Karel Rychlik as part of a commission for the Royal Bohemian Academy of Science. All of these works have now been translated into English by Steve Russ (2004).

By current analytical standards, the scope of Bolzano's philosophical project is considerable. Bolzano delivered a fully worked-out alternative to the logic of his time. Bolzano understood the main obstacle to the development of mathematics in his time to be the lack of proper logical resources – post-Cartesian theories of conceptual analysis and syllogistic inference being unfit for the purpose of modelling deductive reasoning in arithmetic and geometry. Bolzano correctly saw the logic of his contemporaries in the first decades of the 19th century as solidly anchored in the new Kantian epistemology and not, as many have assumed, in some stale modern version of traditional Aristotelian syllogistic.[6] While Bolzano's efforts toward the reform of logic were by no means solely motivated by the needs of mathematical practice – he considered the need to provide ethics and the philosophy of religion with a solid logical and epistemological foundation to be prevalent – it is nonetheless anchored in a general concern for methodology and the theory of knowledge in the context of a theory of deductive systems, i.e. the "purely conceptual disciplines". In such disciplines – mathematics, ethics, pure physics, etc. – truths are related as "grounds to consequences, a notion that can in turn be modelled as part of a theory of deductive theories.

Bolzano's efforts toward developing this new logic fell into two phases: the first extends throughout the 1800s and 1810s and includes the period over the course of which Bolzano produced and published the *Contributions…* (1810) as well as the bulk of his strictly mathematical work (1816, 1817a, 1817b). The second period covers the 1820s and culminated in the publication of the *Theory of Science* (1837), a four-volume treatise extending over 2400 pages that includes everything from a calculus of classes, a theory of conceptual analysis and a semi-formal account of logical consequence to a theory of linguistic interpretation and an account epistemic justification and objective proof. In the *Contributions*, Bolzano's undertaking remained largely programmatic. By the time he had started writing the *Theory of Science*, he had abandoned the main doctrines at play in his earlier work. Between the *Elements of Logic* (1812) and *On the Mathematical Method* (1841), Bolzano rejected his theory of the "multiple copula", his views on the content and structure of concepts, as well as the detail of his positions on modality and

6 See my Introduction, infra.

inference.[7] Nonetheless, it is fair to say that the leitmotiv of Bolzano's mature epistemology already came through in the early 1810s. Bolzano's foremost motivations to call for a logical reform was his fundamental disagreement with the idea that the proper foundation of mathematical knowledge rests in the "Kantian Theory of Construction of Concepts through Intuitions", a criticism to which he devoted an "Appendix" to the *Contributions…* of 1810.[8] As Bolzano saw it, an adequate account of deductive reasoning – in mathematics or anywhere else – must exclude appeal to non-conceptual inferential steps, be they putative "pure intuitions" or any other proxy for logic. In this, Bolzano can be seen to have anticipated an important aspect of later criticisms of Kant, including Bertrand Russell's.[9]

Bolzano's Logical Reform

Even while accounting for the fact that he is credited with anticipating some of 20th century's major philosophical-logical innovations, e.g. antipsychologistic semantic realism, the definition of analyticity and logical truth as invariance under substitution and the Tarskian definition of logical consequence using similar substitutional resources, Bolzano's reform of logic was considerably more radical than what it would be natural to assume in light of the standard narrative. There are two reasons for this. First, standard narratives in the history of logic are almost always based on an approach to historiography that involves deliberately projecting contemporary norms and standards onto past theoretical achievement and to assess philosophical view in light of these. However, Bolzano's main theoretical innovations – I discuss his views on analyticity, deducibility and ground-consequence below – draw on conceptual resources that are exceptionally germane to contemporary analytical philosophers. Because the standard approaches tend to track past anticipations of contemporary ideas and to emphasise theoretical features that speak directly to analytical readers' own current interests, it is bound to picture Bolzano as a kindred spirit whose views do not need to be contextualised to be properly understood.

Second, standard narratives in the history of logic and analytical philosophy are eminently deficient when it comes to understanding the context in which logic developed in the century before and after the publication of the *Critique of Pure Reason*. In particular, they tend to overlook the importance of Kant and the work that was done within his "school" in shaping the discipline over the course of the 19th century.

7 Cf. Lapointe (2011, 18–90).

8 The text has been translated into English by Rusnock (2000, 198–204) and Russ (2004, 132–7).

9 Cf. Russell (1903, §§ 4, 5, 423, 433–4).

As a result, they fail to do justice to what turned out to be the context of Bolzano's logical reform and Bolzano's actual philosophical reasons to effect the shifts in question. The resources Bolzano had to deploy to achieve his results were specifically designed to address problems he found with the doctrines of his contemporaries. In order to appraise the magnitude of Bolzano's proposed change, and do justice to his contribution to the discipline, his views also need to be understood as arising from this context.

One important feature of Bolzano's theory that has been almost constantly overlooked is the significance of his rejection of his predecessors' account of conceptual relations in terms of inclusion in setting the stage for his own views on conceptual and propositional content. The standard account of conceptual relations was couched on a number of doctrines inherited at various stages, for instance, (i) the idea that definition is a matter of (Porphyrian) division and results in conceptual hierarchies in which (ii) the canon of inverse proportion between comprehension (*Inhalt*) and extension (*Umfang*) holds.[10] What is especially relevant here is how (i) and (ii) tie into theories of deduction at the time. The doctrine that concepts' structure is decompositional is the basis for the doctrine according to which logical deduction proceeds on the basis of the principles of identity and/or contradiction and/or sufficient reason and/or excluded middle, i.e. the principles that were meant to do the heavy deductive lifting in the theories of Leibniz, Wolff and even the early Kant.[11]

The connection between decompositional analysis and the theory of deduction in post-Cartesian theories was such that it would have been impossible to dismiss the first without creating deep conceptual difficulties for the second (and vice versa). At least part of the reason why Bolzano was led to develop a new account of analyticity, inference and validity (more on this below) was precisely his rejection of the mainstream account of conceptual analysis in terms of inclusion and decomposition – a criticism that extends over dozens of more or less contiguous pages in the *Theory of Science*. The significance of Bolzano's rejection of decompositional analysis – this included an explicit rejection of both (i) and (ii)[12] – is inescapable and it should be properly contextualised.

10 See Lapointe (2011, Chapter 2).

11 Cf. Heis (*infra*) for a description of various ways of conceiving of the relation between these principles in and after Kant. As Heis explains we must be cautious not to infer, from the fact that Kant thinks that all *analytic judgements* derive from the principle of contradiction, that all *logical knowledge* derives from the principle of contradiction. But this is precisely what is at stake here: given that there are, in Kant and in Bolzano's views, non-empirical (or, as Bolzano sees it, purely conceptual) truths that are not analytic, which principles need to be at work to justify them? And are these principles themselves logical (Bolzano says yes) or otherwise (i.e. "transcendental," in Kant's view)?

12 Cf. Lapointe (2011, Chapter 2).

In the *Theory of Science*, Bolzano's theory of propositions and ideas provides the footing for a series of investigations into the epistemology and pragmatics of the deductive disciplines – including a theory of scientific demonstration. This theory is based on two crucial innovations. First, Bolzano commits to a form of antipsychologistic, non-hypostatic realism about propositions of which we find another version in Lotze.[13] On Bolzano's account, properties such as analyticity and deducibility (*Ableitbarkeit*) are defined not for subjective thoughts or sentences but for what Bolzano conceives to be the "objective content" (*objektiver Inhalt*) of the former and the "meaning" (*Sinn*, *Bedeutung*) of the latter. He calls these entities, whose role is to be the primitive bearers of semantic properties 'propositions in themselves' (*Sätze and sich*). Second, properties such as analyticity and deducibility are defined on the basis of a substitutional method. Bolzano's strategy is to define "classes" of propositions on the basis of a fixed vocabulary that they share or, put in his own terms, to use substitution as a procedure to "generate" classes of propositions that differ only by virtue of the fact that certain (non-logical) parts are exchangeable (*veränderlich*).

This approach allows Bolzano to define some interesting types of semantic regularities. For instance, propositions have a certain "degree of validity" (*Grad der Gültigkeit*). This property comes to the fore when we consider the set of all propositions that differ only by virtue of some determinate arbitrarily exchangeable components, and ponder the ratio of true to false propositions in that set.[14] Bolzano would agree to say, e.g. that, if we take 'Caius' to be arbitrarily exchangeable in the proposition 'Caius, who lives in Saguenay is a descendent of Nicolas Audet', the degree of validity of the latter the is higher than that of 'Caius, who lives in Saguenay is a descendent of the current Belgian King'. This is documented by a quick survey of genealogical data for the region where I was born.

As Bolzano sees it, analyticity is just like validity a property defined on the basis of the substitutional method. Of course, Bolzano's theory of analyticity is an eminently favoured topic in the literature (cf. Bar-Hillel 1950; Etchemendy 1988, 2008; Künne 2006; Lapointe 2000, 2008, 2014; Morscher 2003; Neeman 1970; Proust 1981, 1989; Textor 2000, 2001). This should be no surprise. Not only is analyticity a topic that has been central to analytical philosophy in the 20th century, but Bolzano's theory speaks directly to what analytical philosophers identified to be the main problems of previous conceptions. On the one hand, Bolzano's definition applies to all grammatical constructions, including those that are not of the subject-predicate form. This was notoriously not the

13 Although it's unclear that Lotze was aware of Bolzano's work in any detail, the conceptual points of convergence are striking. Stang's presentation of Lotze (*infra*) is eminently propitious to a comparison with Bolzano.

14 Cf. Bolzano (1837, §147); see Lapointe (2011, Chapters 4 and 5) for a discussion.

case for Leibniz's and Kant's, for instance, who both defined analyticity precisely in terms of the inclusion of the predicate-concept in the subject-concept in categorical judgements. A judgement or proposition is analytic on their account if the analysis or decomposition of the subject-concept into its parts reveals the predicate-concept as one of them. In this respect, the advantage of Bolzano's approach is that it affords for a maximally broad range of applications: 'analytical' can be predicated or denied of conditionals, disjunctions, conjunctions, etc., and, in general, of propositions that present a syntactic complexity that is foreign to post-Cartesian categorial logics.

Bolzanian Analyticity

As Bolzano sees it, a proposition is analytic when its truth remains constant under substitution of some of its component parts. To be precise, on Bolzano's account, a proposition S is analytic with respect to some variable components i, j… if and only if

i i, j, … can be varied so as to yield at least one "objectual" substitution instance of S
ii All substitution instances of S have the same truth-value as S

A substitution instance is "objectual", on Bolzano's account, if the concept designated by the subject "has an object" (at least one), i.e. if it has an extension (*Umfang*).[15] On this account, propositions can be analytically true or analytically false. When they are analytical, they are not absolutely so either: propositions are always analytic relative to some exchangeable parts.

Admittedly, there are "problems" with Bolzano's conception of analyticity, at least if we assume that the notion is supposed to provide an insight into the property, e.g. Carnap and Quine sought to define under the same name.[16] For Bolzanian analyticity does not provide an account of what it means for a proposition to be true *by virtue of meaning* alone and to be knowable as such. Arguably, however, Bolzano did not think that his definition of what he calls 'analyticity' was meant to do that. The problem Bolzano understood Kant to be grappling with was not the problem of truth by virtue of meaning but a different one: to provide a systematic account of quantificational statements that express generality. There is ground to debate whether this constitutes a gross misunderstanding of Kant's views, or whether Bolzano is here picking up on the fact that Kant himself does not offer a treatment of generality that is

15 In Bolzano's view, there are no empty extensions.
16 Cf. Lapointe (2014b).

both (i) adequate in that it does more than append quantificational particles to categorical statements and (ii) clearly separate from an account of analyticity.[17] Strictly speaking, however, a statement ascribing Bolzanian analyticity to a given propositional form, say,

'X who is a man is mortal' is analytic with respect to X

if it is true, is true because every objectual substitution instance of 'X who is a man is mortal' is also true.

Bolzano's definition of what he calls 'analytic' propositions offers a fairly clear description of the property we associate today with substitutional quantification. If a proposition, say, 'Caius who is a man is mortal', is Bolzano-analytically true with respect to 'Caius', then all substitution instances of the corresponding propositional form 'X who is a man is mortal' are true.[18] The main conclusion to be drawn at this point is the following: a proper contextualisation of Bolzano's views on analyticity should show that there is no simple way to project back contemporary logical and semantic concerns and concepts onto Bolzano's theories. The question Bolzano sought to answer when he developed his views on what he calls 'analyticity' was something like "What does it mean to say that a statement, say,

Caius who is a man, is mortal

is generally true?" This question he answers with roughly the following: it means that all instances of the corresponding propositional form

X who is a man, is mortal

are true. Given the centrality of quantificational issues and the limitation of syllogistic theories in this respect, a reform of logic that would include a systematic treatment of generality was undeniably urgent and crucial

17 Kant says little about the relation between the two notions of universality and necessity, what he does say can at best allow us to infer that he did not distinguish the two notions properly (cf. Lapointe 2000, Chapter 1). While this will sound odd to contemporary readers, the project of testing the career of the two notions in the 17th and 18th centuries would throw light on a neglected topic that is taken for granted.

18 Bolzano primarily deals not with sentences and words but with their objective "meaning" (*Sinn*), i.e. objective ideas and propositions in themselves. Yet in Bolzano's theory, there is (at least) one objective idea for every object, and in this sense, Bolzano assumes that there is in principle a "name" for every object. For this reason, though Bolzano's approach to quantification is substitutional, he is not liable to the reproach following which his interpretation of the universal quantifier cannot account for every state of the world: the resources he assumes he has at his disposal are in principle as rich as necessary to provide a complete description of the domain the theory is about.

in context. Bolzano's analysis of generality – even under a retrospective misnomer – was crucial to articulating the logical syntax that underpins his entire logic. Bolzano not only saw a great deal of significant, articulate structure within concepts and propositions and offered an elaborate theory of such structure; he also had an account of quantifiers and logical connectives – including a non-extensional definition of deducibility, which is meant to account for conditionality – that goes much beyond anything that was available at the time.[19]

Importantly, Bolzano did not fail to distinguish the problem of generality and that of truth by virtue of meaning, and he does have a remarkably sophisticated and systematic account of the latter as well. Bolzano deals with truth by virtue of meaning as part of his theory of "a priori" or "conceptual" knowledge. The most important feature of Bolzano's theory in this respect is the fact that he understood conceptual, i.e. a priori knowledge to be invariably deductive. Strictly speaking, in Bolzano's view, there is no such thing as truth by virtue of meaning outside of a deductive order, a point which was quite unclear before him but which will be echoed in a number of his successors.[20]

Deducibility

Given the last remark, then, it would be right to conclude that Bolzano's account of deduction is key to his epistemology. Bolzano, however, has not one but two at once competing and complementary accounts of logical deduction: "deducibility" (*Ableitbarkeit*) and ground-consequence (*Abfolge*). Just like Bolzanian analyticity, and in spite of its great similitude with Tarskian logical consequence in the resources it deploys to model entailment, Bolzanian deducibility is not meant to define a relation by virtue of which truth is preserved a priori and as a matter of necessity from premises to conclusion. The reason why Bolzanian deducibility does not, on its own, provide such an account is that in Bolzano's theory, the task of providing an account of what it means to (i) preserve truth from premises to conclusion (ii) as a matter of necessity, is precisely "split" between deducibility and ground-consequence. The notion of deducibility presents a semantic account of (i), i.e. an account of truth-preservation that is neither trivial nor careless (see Siebel 1996, 2002, 2003; van Benthem 1985, 2003). On the other hand, Bolzano's attempt to do justice to (ii) results in a definition of objective "ground-consequence" that constitutes the basis of his account of a priori knowledge and mathematical explanation – a theory some have sought to vindicate[21]

19 Cf. Lapointe (2011, 54ff).

20 Cf. Lapointe (2009).

21 Many have translated Bolzano's term 'Abfolge' into English with 'grounding' – including me at various places. Terminological similarities with theories of "grounding" in the

(see Mancosu 1999; Tatzel 2002, 2003; Dubucs and Lapointe 2006). Both notions are immensely original and interesting. While the division of labour between them runs counter to what we assume makes the contemporary, post-Tarskian notion of logical consequence interesting in the first place, namely the fact that it provides a semantic account of what it means for the truth of the conclusion to be necessarily entailed by the truth of the premises, there is nonetheless great value in understanding Bolzano's strategy in context. If nothing else, it is an occasion to re-evaluate our own assumptions concerning the relationship between truth-preservation, entailment and logicality.[22]

Although Bolzano did not say this explicitly, Bolzano's definition of deducibility made it possible for the first time to systematically identify inference forms – propositions in which at least one part is considered to be exchangeable – that preserve truth from premises to conclusion without being constrained by a particular regimentation of grammar. Post-Cartesian syllogistic theory, by contrast, models inference within a language that is both considerably regimented and resource-poor – it does not add much to medieval Aristotelian syllogistic in this respect. Traditional syllogistic definitions of validity generally concur to suppose that there is only a finite number of possible forms of inference, namely 256. This number comes up as a result of various assumptions, including the supposition that:

- inferences ought to be modelled as having exactly two premises and one conclusion and – in the case of classical syllogistic ;
- the form of the latter is invariably *subject-copula*-predicat; and
- there are only four variants of the *subject-copula-predicate* form, namely the forms that are traditionally referred to as a, e, i and o.
- any given inference contains at most three different terms.

While later syllogistic theories were developed to account for inferences that contain hypothetical and disjunctive premises, definitions of validity for such inference forms were premised on the same type of syntactic limitations.

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context of contemporary work on fundamentality and dependence in metaphysics could lead to mistaken assumptions as to the nature of Bolzano's own concern. Bolzanian ground-consequence is not a metaphysical notion and should not to be confused with contemporary mereological and non-mereological accounts of "grounding". The purpose of Bolzanian ground-consequence is to define the order in which propositions are related in axiomatic systems, not to provide a model for relations of dependence between facts, events, properties and so on.

22 Etchemendy (1990), is an example of such questioning. See also Rusnock and Burke (2010) for a critical perspective.

From Bolzano's perspective, syllogistic theory marks out a syntax whose expressive resources are too limited to account for the richness of inferential practices in the sciences and beyond. Bolzano generalises the problem: as he sees it Aristotle's crucial intuition as regards the notion of validity is that it ought not to be bound to any particular syntax.[23] He writes:

> Aristotle began with such a broad definition of the word syllogism that one is astonished that he could have subsequently restricted the concept of this kind of inference so severely. He writes (in Anal Pr. I, 1) 'syllogism is a discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so.' This definition obviously fits every inference, not only with two, but also with three and more premises, and not only simple inferences but complex ones as well.
>
> (1837 §262: 535)

As Bolzano sees it, then, in order to do justice to Aristotle's insight, we need to first take stock of the fact that inference forms come in all sizes and packages. If that is the case, however, syllogistic theory's approach – which consists in identifying which inferential forms among those that are syntactically canonical always preserve truth from premise to conclusion – becomes intractable.

Whether or not Bolzano was aware of it, he has at least one other reason to be dissatisfied with classical syllogistic definitions of validity and it is connected with his views on the nature and role of definition in theorising. Knowing which inferential forms among a predefined set satisfy the intuitive Aristotelian definition of deduction does not, by Bolzano's account, provide a genuine "determination" (*Bestimmung*) of the notion of good or valid deduction, that is, a definition that allows us to properly operationalise the concept within the logical theory. By Bolzano's account, merely knowing what falls in the extension of a concept – in this case the class of all putatively valid syllogistic inferential forms – does not amount to having a proper understanding or determination of that concept. Although Bolzano does not say so himself, determination is a form of conceptual exercise whose end-purpose is to serve the explanatory goal of a theory. Since definitions are partial grounds for conclusion we draw about the corresponding objects, when we define concepts, i.e. when we identify the components and structure of what ultimately counts as a principle or axiom within a deductive order, we

23 This would have been even more obvious to Bolzano in light of the beginning of the second book of the *Prior Analytics*, which is devoted to the relationship between premises and conclusion as regards their truth-value.

are in fact pinning down its role in that theory. I refrain from using the term 'inferential role' in this context in reference to Bolzano's notion of definition so as to avoid anachronism and hasty associations. But given Bolzano's views on a priori deductive knowledge, he has something of the sort in mind: concepts (and by extension their determination) are always part of deductive structures, and we use them to draw conclusions within a theory.[24]

Bolzano's definition of "deducibility" is based on the same conceptual resources as his definition of analyticity. As Bolzano presents it, deducibility is a property defined for sets of propositional forms, namely sets of propositional forms in which some play the role of premises and others the role of conclusion.

I describe deducibility as a property of sets of "propositional forms", where Bolzano describes deducibility as a property of classes or "species" of propositions *with respect to some exchangeable components*. Here again, the relativisation of deducibility to some variable component is what allows it to mark generality, i.e. semantic invariance under substitution. As Bolzano puts it, the set of propositions T is deducible from the set of propositions S with respect to constituent-ideas i, j,... if and only if

i i, j, ... can be varied so as to yield at least one true substitution instance of S and T (compatibility)
ii whenever S is true, T is also true

Bolzano's discussion of deducibility is quite detailed – it extends over three dozen sections – and insightful. In particular, his definition yields a series of theorems that contribute to further delineating the notion he has in mind, the most significant of which are the following:

- It is not the case that if T are deducible from S, then S are deductible from T... (asymmetry).[25]
- If T are deducible from S and R are deducible from T, then R are deducible from S (transitivity).
- In addition, assuming that the propositions included in the set S are compatible in the sense specified by the aforementioned condition (i), i.e. they share at least one variable that make them all true at the same time, then S is deducible from S (reflexivity).

24 I explain this in more detail in Lapointe (2009).

25 This is different from anti-symmetry. Deducibility is not anti-symmetric, since, in the case in which T and S are logically analytic, if T are deducible from S, then S are deducible from T.

There are striking similarities between Bolzano's notion of deducibility and post-Tarskian model theoretic accounts of logical consequence. But one obvious difference between Bolzanian deducibility and post-Tarskian logical consequence is the fact that Bolzanian deducibility indiscriminately includes *formally* valid arguments as well as materially valid arguments. In other terms, all arguments in which the conclusion is a logical consequence of the premises in the contemporary, post-Tarskian sense are instances of Bolzanian deducibility. For instance,

Caius is rational

is Bolzano-deducible with respect to 'Caius', 'man' and 'rational' from

Caius is a man
Men are mortal

But Bolzanian deducibility also allows for cases of merely "materially" valid consequence. For instance,

Caius is mortal

is Bolzano-deducible with respect to 'Caius' from

Caius is a man

Assuredly, the distinction between arguments of the former kind and arguments of the latter is an important one, and providing an account of this distinction is at the core of logical theory. Part of the reason for this is that it is assumed that logic distinctively seeks to account for "formal" and "a priori" features of languages and deductive systems. Provided one has a satisfactory account of logical form, in order to know that the conclusion follows from the premises of a formally valid argument one only needs to consider its structure or form, the presumption being that one can do so *a priori*: no other kind of knowledge is required than logical knowledge to recognise logical validity. In the materially valid argument, however, in order to infer from the premise to the conclusion, one needs more than logical knowledge: one also needs to understand the signification of the terms, e.g. 'man' and 'mortal' in the previous example, since in order to know that Caius is mortal one also needs to know, in addition to the fact that Caius is a man, that all men are mortal. But this is not something one knows a priori.

One consequence of the failure to provide an account of the difference between logical and material consequence is that we are left unable to account for the fact that we seem able to extend our knowledge through

putatively pure logical cognitions. If we know, for instance, that all instances of *modus ponens* are logically valid, we can infer from two propositions whose truth we've recognised:

> If Caius is a man, then he is mortal
> Caius is a man

A new proposition:

> Caius is mortal

whose truth we might not have previously known. But Bolzano's account of deducibility does not allow one to extend one's knowledge in this way. In order to know that an argument is an instance of Bolzano-deducibility, one already needs to know for every substitution, both that the premises are true *and* that the conclusion is true. What we have, then, is a systematic way to model the formal features – variant and invariant – of arguments that preserve truth "universally", i.e. for every substitution instance. But the substitutional method does not give us the means to know a priori which arguments preserve truth from premises to conclusion and to leverage this knowledge to acquire new knowledge.

Most arguments that count as logically valid in the post-Tarskian sense would also count as instances of Bolzanian deducibility. Most, but not all: Bolzano's account of the conditions under which an argument counts as an instance of deducibility is in fact both broader – as we've seen it includes instances of merely material consequence – and stricter than what we find in standard model-theoretic practice. In order for a conclusion to be Bolzano-deducible from a given set of premises, there must be at least one substitution that makes both the premises and the conclusion true at once – what Bolzano calls the "compatibility" (*Verträglichkeit*) condition – a requirement that is not reflected in classical conceptions of logical consequence. As a result, Bolzano's programme converges with some contemporary attempts at a definition of non-classical notions of logical consequence in at least two different respects.

First, given the compatibility condition, while a logical truth may follow from any (set of) true premises, *nothing* as opposed to everything is deducible from a contradiction. In other terms, the compatibility condition invalidates the *ex contradictio quod libet* or explosion principle: no substitution of 'p' in ' 'q' is deducible from 'p and non-p' ' can fulfil the compatibility constraint. In other words, no interpretation of 'p' in 'p and non-p' can yield a true variant since there is therefore no idea in a proposition of this form – a logical falsehood – that can be varied so as to make it true. If that is the case, trivially, the premise and the conclusion can never all be true at once. This has at least two further upshots. First, the compatibility constraint invalidates the law of contraposition.

Whenever at least one of the propositions in the antecedent is analytically true – when all its substitution instances are true – we cannot infer from:

T are deducible from S

to

¬S are deducible from ¬T

since ¬S entails a contradiction. For instance, while

Caius is a physician who specialises in the eyes.

is deducible with respect to 'ophthalmologist' from

Every ophthalmologist is an ophthalmologist

and

Caius is an ophthalmologist

It is not the case that

It is not the case that every ophthalmologist is an ophthalmologist
It is not the case that Caius is an ophthalmologist

is deducible with respect to the same component from:

It is not the case that Caius is a physician who specialises in the eyes

Second, the compatibility condition makes Bolzano's logic non-monotonic. Classical logical consequence is monotonic: if a conclusion follows from some premises, then it follows from these premises and any other premise. However, by Bolzano's account, whenever the premise added contains a contradiction or contains information that contradicts the information already at hand, the conclusion no longer follows.

Ground-Consequence

The contemporary reader of Bolzano is likely to expect an account of validity and inference to do more than what Bolzanian deducibility does, namely to identify inference forms such that true substitutional variants of the premises co-occur with true substitutional variants of the conclusion. Bolzanian deducibility does not, however, epitomise our

intuitions as to what is presumably distinctive of knowledge we acquire in "formal" or "conceptual" disciplines such as mathematics and pure physics, and this is a problem Bolzano was well aware of. Bolzano considered mathematical knowledge to be "purely conceptual", and in his eye, this implied that it is both "necessary" and "a priori", two qualities for whose explanation substitutionally defined notions are not equipped. Hence for Bolzano the problem of providing an epistemological foundation to mathematics is ultimately that of defining the logical relations by virtue of which true propositions are ordered within an axiomatic structure. To provide an account of mathematical knowledge and do justice to what makes it a priori and necessary one must therefore, by Bolzano's account, be in a position to offer a theory of deductive theories, that is, a theory of system in which true propositions relate as "objective grounds" to "objective consequence". Bolzano called "ground-consequence" (*Abfolge*) the relation he took to define this relation.

As Bolzano conceived of it, to say that mathematical knowledge is a matter of ground-consequence is to say that mathematical proposition are justified by establishing their status as "objective consequences" of "objective grounds". Ultimately, then, propositions be it in mathematics, in ethics or in logic are a priori and known as such to the extent that they acquire their justification objectively from "primitive truths" (*Grundsätze*), which by their very nature do not themselves require a justification. Objective, and a priori justification is thus defined only for truth that relate as objective ground-consequence.[26]

One important feature of Bolzano's account of objective justification is that it is meant to do justice to the idea that in order to know that a proposition is true necessarily we need to know *why* it is true. In this sense at least, Bolzanian ground-consequence is meant to have explanatory import. My knowing that 'p' grounds 'q' has explanatory value: q, because p. Bolzanian ground-consequence is designed so as to capture roughly what, according to Bolzano, the truly scientific mind ought to mean when in the conduct of a scientific inquiry, she uses the phrase '…, because….' in response to the question 'why …?' What's special about arithmetic and geometry is the fact that since the propositions they contain are invariably true – Bolzano was a child of his time in assuming that the "fundamental propositions" in deductive orders need to be true – and "purely conceptual", grasping any such proposition as the conclusion of a ground-consequence inference invariably warrants knowledge that is a priori in the sense that it does not involve "extra-logical" resources. The challenge for Bolzano was thus to define the relation of ground-consequence in such a way as to have the resources to model every inferential step along the way.

26 I discuss this and what follows in more details in Lapointe (2011, 72–90).

When Bolzano speaks of ground-consequence, what he has in mind is invariably *immediate* ground-consequence. Mediate ground-consequence thus is a derivative notion. We can define mediate ground-consequence as the transitive closure of the more primitive notion of immediate ground-consequence: p$\phi$is the mediate consequence of $\phi_1, \ldots, \phi_n$, if and only if there is a chain of immediate consequences starting with $\phi_1, \ldots, \phi_n$ and ending with p. p$\phi$is the immediate consequence of $\phi_1, \ldots, \phi_n$ if there are no intermediate logical steps between $\phi_1, \ldots, \phi_n$ and p. Importantly, ground-consequence is anti-reflexive. p cannot be its own ground, whether mediate or immediate. The anti-reflexive character of grounding can be inferred from its asymmetry. If grounding were reflexive, then p could be grounded in itself. But given that if p grounds q, it is not the case that q grounds p, this would imply a contradiction: by substitution p could at once ground itself and not ground itself. Finally, Bolzanian ground-consequence is unique: for every true proposition that is not primitive, there is a unique tree structure that relates it to the primitive truths from which it can be deduced. Deducibility, by contrast, is not unique. Likewise, unlike ground-consequence, which holds only between truths Bolzanian-deducibility can hold between false substitutional instances as well as true ones.

What precedes is sufficient to establish that ground-consequence is not a special case of deducibility in Bolzano's account, and vice versa. On the one hand, not all cases of deducibility are cases of ground-consequence. For instance, while

> (T1) It is warmer in the summer than in the winter

is deducible (with respect to 'summer' and 'winter') from

> (T2) Thermometers, if they function properly, are higher in the summer than in the winter

(T1) is not an objective consequence of (T2) in Bolzano's sense. Bolzano assumes that the objective ground of a proposition needs to play a role in explaining its truth. But the reason for (T2), i.e. the reason why thermometers are higher in the summer, is precisely (T1), i.e. that it is warmer then, so that, in the previous example, the order of ground-consequence is the reverse of the deducibility relation.

Logical consequence as we conceive of it today is meant to combine the main features of Bolzanian deducibility and ground-consequence. Bolzano thought that instances of deducibility that are also instances of ground-consequence were interesting enough to deserve their own name: 'formal ground-consequence'. In an inference that fits both deducibility and ground-consequence, true interpretations of the premises always coincide with true interpretation of the conclusion AND the conclusion

follows *necessarily* from the premises: its truth is objectively justified by the truth of the primitive propositions from which it is derived. But the notion of formal ground-consequence is theoretically inert: it is not an additional resource of Bolzano's logic but a designation for types of inferences that happen to fall under two different rubrics.

Conclusion

When past philosophers put forward doctrines and theories that resonate with those that make for the matter of contemporary philosophical discussion, the temptation is great to ignore the context in which they emerged and to emphasise similarities, possibly more in an effort to validate recent developments than in order to do justice to the development of the discipline. Given the extent of Bolzano's anticipatory spirit, his theories offer an eminently fertile ground for such an approach. Bolzano's contribution to the philosophy of logic and semantics bears on problems prima facie similar to those that later defined early analytical philosophy, such as for instance the pursuit of logical foundations for axiomatic disciplines and deductive theories. The resources he deploys, while they can't be said to have the same elegance and simplicity as those that have shaped the fields after the turn of the 20th century, are complex and sophisticated enough to be contemplated with awe. The same holds for his contribution to the pragmatic and cognitive basis of mathematical practice. Far from ignoring epistemic and practical constraints, Bolzano discusses them in detail. As a mathematician, Bolzano was attuned to philosophical concerns that escaped the attention of most of his contemporaries and many of his successors.

Much can be learned by comparing our theories with those of Bolzano, of course. But this does not exhaust the range of purpose history of philosophy may serve. One important task the historian of philosophy may set for herself is to track the factors that have contributed to doctrinal and theoretical transformation *in context* so as to understand the dynamics of disciplinary development. This requires that past doctrines and theories, whether or not they appear to present similarities with those to which contemporary philosophers subscribe and which they value be contextualised and understood as the result of agreement, criticism, reinterpretation, and/or rejection of other doctrines and theories that shaped the author's effective background.

What's relevant with respect to our discussion of Bolzano is the fact that the context in which his views on logic emerged and developed is prone to being mischaracterised or oversimplified. On the one hand, the idea that Bolzano's logic should be understood as a "development" of Leibniz's has the same sort of import as the claim that Kripke's views were shaped by Russell's and Wittgenstein's. It feeds into a superficially

traditionalist narrative whose purpose is not to do justice to the way in which doctrines and theories effectively transformed as a result of philosophical activity. The purpose of such claims is often broadly genealogical, part of an attempt to position oneself in some putatively worthy lineage. The same holds for the claim that Bolzano's philosophy is "anti-Kantian".[27]

In order to appreciate the connection between Bolzano's views and the post-Kantian context, a rather substantial overhaul of our assumptions concerning the development of logic over the 17th and 18th centuries would be required. For instance, the view that logical theories, including Kant's, were then still "broadly Aristotelian" and that logic was the more or less stagnant repository of doctrines that had not evolved since the middle ages does not survive proper contextualisation. The benefit of doing justice to logical theories in view of their context however is not merely historical. What's interesting in observing that Bolzano's views on conceptual analysis are the result of his rejection of the Leibnizo-Kantian decompositional approach is that it emphasises the fact that philosophical concepts and doctrines are not abstract and immutable but rather historical constructs that cannot be dissociated from the broader theoretical and metaepistemological frame to which they are attached. The same holds for the observation that the doctrines Bolzano associates with "analyticity" and "deducibility", rather than providing an account of truth by virtue of meaning, seek to offer a substitutional account of generality and truth-preservation in a context where such an account was still at best pedestrian.

The lesson here is not trivial. Each time we identify the way in which critical engagement with specific texts and corpora in a given context leads to doctrinal and theoretical transformations, we acquire metacognitive insight into our own practice. Disciplinary history teaches us as much about the past as it teaches about the mechanisms that underpin our own philosophies.

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27 See Lapointe and Armstrong (2014).

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5 Laws of Thought and Laws of Logic after Kant

Lydia Patton

Road Map

In the mid-1800s, George Boole developed a theory of logic as an instrument for representing the structure of mathematical problems. For Boole, the laws of logic are laws of thought, but they are not (merely) psychological laws, for all that. Boole's approach draws on a dialogue within the post-Kantian "New Analytic" tradition, then current in the United Kingdom.[1]

Some contemporary inferentialists argue "against the view that there are facts of matters of logic that obtain independently of us, our linguistic conventions and inferential practices" (Resnik 1999, 181). Boole, along with those in the post-Kantian tradition who influenced him, the New Analytic, took the position that the laws of logic are laws of thought. Logical laws govern inferences. In that case, there can be facts of the matter about logic, namely facts about the laws of thought that are valid principles of inference in certain domains. One of the key insights of Boole's method is that the epistemic status of such laws can be established by studying logic's application to solving problems in mathematics.

Boole's approach is one origin of the contemporary discipline of model theory, which has branches in philosophy, logic, and mathematics. Model theory analyzes the notion of a given proposition being true under an interpretation (Hodges, forthcoming). Alfred Tarski's "The Concept of Truth in Formal Languages" (1933/1983) is a seminal paper in this tradition. As Hodges notes, Boole's work is a significant precursor to the model-theoretic approach. In particular, Boole pays close attention to the extent to which logic can represent problems in algebra so that, within a given logical interpretation, results in algebra can come out true – that is, desired solutions can be found. Boole's approach takes its cue from critical responses to Kant and from the British reception of

1 Realists about logic take the position that logic is a domain of truths independent of any particular subject matter and of our inferential practices and subjective constitution. Inferentialists need not take any stance about the correspondence of logical propositions to reality or truth. Rather, logical propositions, relationships, and terms acquire their meaning from the use to which they are put in inferences. Peregrin 2014, and Resnik 1999, among many others, define inferentialism in these terms.

the post-Kantian logicians Wilhelm Traugott Krug, Wilhelm Esser, and Jakob Friedrich Fries.

Logic as Art; Logic as Science

In 1826, Richard Whately published the *Elements of Logic*.[2] Before Whately, much of British logic was in the Lockean tradition, seeing logic as the "art" of thinking about the truth, not as a "science" that discovers novel truths. Levi Hedge's 1818 *Elements of Logick* "well illustrates the prevailing view of logic in the Anglophone world before Whately" (Heis 2012, 102). Levi Hedge argues that logic "traces" the development of thought from perception to judgment. Like an artwork, then, logic attempts to give a rendering, tracing, or picture of judgments of truth. But Hedge believes that logic cannot itself prove truths, much less discover them.

Richard Whately responds, to Hedge and to others, that logic is a science as well as an art. For Whately, logic provides "an analysis of the process of the mind in reasoning" and to that extent is strictly a science (Whately 1870, §1, 1). However, logic also concerns itself with "practical rules" for "guarding against erroneous deductions", and, to that extent, logic is an art (§1, 1). Whately stipulates that "a *science* is conversant about speculative *knowledge only*, and *art* is the application of *knowledge* to *practice*" (§1, 1). Whately maintains that the scientific element of logic consists of speculative knowledge about the reasoning process, while an equally significant element of logic consists of applying that speculative knowledge to reasoning in practice.

The Scottish philosopher William Hamilton[3] wrote a substantial review of Whately's *Elements* in which Hamilton took the Anglophone logicians to task for neglecting "contemporary German logics". The debate over whether logic is an art or a science presumes that logic is either screened off from the content of science (art), or is itself an independent tool for the discovery of psychological or metaphysical truths (science). Hamilton defends a different position: that logic can consist of a set of truths, but that they are *formal*, not substantial truths.

According to Hamilton, the Anglophone tradition at the time had no analogue of Kantian formal logic, which is why Hedge, Whately, and others were stuck. As Heis (2012) summarizes Hamilton's account,

> we can more adequately purify logic of intrusions from psychology and metaphysics and more convincingly disabuse ourselves of the conviction that logic is an "instrument of scientific discovery" by

2 For the logical context of Hamilton's work, I draw on Jeremy Heis's excellent essay "Attempts to Rethink Logic" (Heis 2012).

3 Not to be confused with the Irish mathematician William Rowan Hamilton.

> accepting Kant's idea that logic is *formal*.[4] Hamilton's lectures on logic, delivered in 1837–8 using the German Kantian logics written by Krug and Esser (Krug 1806, Esser 1823) thus introduced into Britain the Kantian idea that logic is formal.[5]

If logic is formal, Kant argued, then logic can be a "canon" of rules of inference that have validity over a certain domain. But logic does not, itself, expand the domain of our knowledge: logic is not what Kant calls an "organon" or what Hamilton calls an "instrument of scientific discovery".[6] The laws of thought are normative, formal rules describing "how we *ought* to think", rather than descriptive, psychological laws telling us "how we *do* think".[7]

While Hamilton criticizes Kant's reasoning about 'regulative ideas' and Kant's account of judgment using the categories (*Kategorienlehre*), he adopts Kant's notion of logic as a formal science and Kant's divorce of logic from psychology.[8] Hamilton combines the idea that he had borrowed from common sense philosophy, that thought presupposes principles of thinking, with Kantian formal logic.[9] In the review, Hamilton writes,

> Logic they [the Kantian logicians] all discriminated from psychology, metaphysic, &c. as a *rational*, not a real, – as a *formal*, not a material science. – The few who held the adequate object of logic to

4 Hamilton, "Recent Publications," 139.

5 Heis (2012), 103. Heis cites "Hamilton, *Logic*... from the 1874 3rd ed. (Original edition, 1860)".

6 "General logic for Kant contains the 'absolutely necessary rules of thinking, without which no use of the understanding takes place' (A52/B76). The understanding – which Kant distinguishes from 'sensibility' – is the faculty of 'thinking,' or 'cognition through concepts' (A50/B74; Ak 9:91). Unlike Wolff, Kant claims a *pure* logic 'has no empirical principles, thus it draws nothing from psychology' (A54/B78). The principles of psychology tell how we *do* think; the principles of pure general logic, how we *ought* to think (Ak 9:14). The principles of logic do not of themselves imply metaphysical principles; Kant rejects Wolff and Baumgarten's proof of the principle of sufficient reason from the principle of contradiction (Ak 4:270). Though logic is a canon, a set of rules, it is not an organon, a method for expanding our knowledge (Ak 9:13)" (Heis 2012, 98).

7 Ak 9:14; Heis 2012, 98.

8 See Durand-Richard (2000), §2.3, for more details and historical background on the material in this section.

9 "For [Hamilton], the form of thought is the kind and manner of thinking an object (I 13) or the relation of the subject to the object (I 73). He distinguishes logic from psychology (against Whately) as the science of the *product*, not the *process*, of thinking. Since the forms of thinking studied by logic are *necessary*, there must be *laws* of thought: the principles of identity, contradiction, and excluded middle (I 17, II 246). He distinguishes physical laws from 'formal laws of thought,' which thinkers *ought* to – though they do not always – follow (I 78)" Heis 2012, 103–4.

> be *things in general*, held this, however, under the qualification, that things in general were considered by logic only as they stood under the general forms of thought imposed on them by the intellect, – *quatenus secundis intentionibus substabant*. – Those who maintained this object to be the *higher processes of thought*, (three, two, or one,) carefully explained, that the intellectual operations were not, in their own nature, proposed to the logician, – that belonged to the psychologist, – but only in so far as they were *dirigible*, or the subject of laws.[10]

Hamilton identifies the key contribution of "intellectual operations" as not their nature or particular content but their lawlikeness. That is why formal logic can be a kind of a science, as well as an art. It doesn't merely retrace the justification for a particular inference; it also provides laws that are valid for inferences in other domains. Hamilton's initial response to the debate about whether logic is an art or a science is to argue that logic is a formal practice, describing normative laws of reasoning, which can be the basis of inferences beyond the initial domain in which they are analyzed. In this sense, logic is a science, but it is a merely formal one. According to Hamilton,

> Logic is a formal science; it takes no consideration of real existence, or of its relations, but is occupied solely about that existence and those relations which arise through, and are regulated by, the conditions of thought itself. Of the truth or falsehood of propositions, in themselves, it knows nothing, and takes no account: all in logic may be held true that is not conceived as contradictory. In reasoning, logic guarantees neither the premises nor the conclusion, but merely the *consequence* of the latter from the former; for a syllogism is nothing more than the explicit assertion of the truth of one proposition, *on the hypothesis* of other propositions being true in which that one is implicitly contained.[11]

Mill and the New Analytic

As is well known, in 1865, John Stuart Mill published *An Examination of Sir William Hamilton's Philosophy*. In this astonishingly long work – it has two volumes, and volume one is 650 pages – which went through several subsequent editions, Mill subjects Hamilton's work to searching criticism. One of the central points of Mill's criticism is Hamilton's

10 Hamilton 1866/1833, 137. The notion that operations must be *lawlike* will be central to Boole's theory as well.

11 Hamilton 1866/1833, 144.

exclusion of "whatever relates to Belief and Disbelief, or to the pursuit of truth as such" from logic and his restriction of logic to "that very limited portion of its total province, which has reference to the conditions, not of Truth, but of Consistency".[12] Mill disagrees: for him, logic must be a science of truth, or it is not a science at all. Mill objects to Hamilton's theory on practical grounds, as well. He argues that Hamilton does not provide a rigorous way to distinguish, in practice, between the formal and the material elements of logical inferences (Mill 1882, 25 and passim). As a result, Hamilton's attempt to defend logic as a purely formal science fails.

A number of figures within the New Analytic tradition responded to Mill's criticisms, including Francis Bowen (1874), Henry Mansel (1866), James McCosh (1869), and William Thomson, whose response appeared in the many editions of his *Outlines of the Laws of Thought*. Mansel and Thomson, in particular, stress the Kantian notion that thought is a free product of the mind.[13] Mill had objected that Hamilton could not identify in a reliable way what was formal and what was material in a given logical inference. Thomson responds that we can identify logical laws because they are the freely chosen tools we use to investigate the phenomena. Since we choose the logical tools, which Thomson describes as a priori "rules" (see below), we can make a distinction between what is formal and a priori and what is material in any domain we investigate using logical reasoning.

Thomson's *Outlines*[14] begins with an explicit statement that logical reasoning is prior to logical laws, which is why logic is a science. Logical laws do not express psychological laws or metaphysical truths. Instead, logic is a science *of* scientific knowledge, because it encodes the rational process of coming to have scientific knowledge.

It's said, in language reminiscent of a Platonic dialogue,

> Poems must have been written before Horace could compose an 'Art of Poetry,' which required the analysis and judicious criticism of works already in existence. Men poured out burning speeches and kindled their own emotions in the hearer's breast, before an Art of Rhetoric could be constructed.[15]
>
> And wherever our knowledge of the laws of any process has become more complete and accurate; as in astronomy, by the

12 Mill 1882, 25.

13 Mansel (1866) links his argument to a theological one defending the freedom of the will, with which Mill is hardly likely to have been impressed.

14 I cite from the first edition, 1849. While of course only the later editions respond to Mill, Boole's early work responds to the earlier editions of Thomson (see the next section). Citations modernize Thomson's spelling to reduce the irritation of the reader.

15 Thomson 1849, 1.

> substitution of the Copernican for the Ptolemaic system; in history, by a wiser estimate than our fathers had the means of forming, of modern civilization and its tendencies; in chemistry, by such discoveries as the atomic theory and the wonders of electro-magnetism; our progress has been made, not by mere poring in the closet over the rules already known, to revise and correct them by their own light, but by coming back again and again to the process as it went on in nature, to apply our rules to facts, and see how far they contradicted or fell short of explaining them.[16]

To borrow an example from Hans Reichenbach, when we choose to use a meter stick to measure a table, we can establish how many meters long the table is. The measurement yields statements about the properties of the table. One such statement might be "This table is seven meters long". The word "meter" in that statement is a feature not of the table itself but of the standard we used to measure the table. While the statement of the properties of the table mixes formal and material content, we can nonetheless identify in practice what is formal and a priori. For instance, we know that we chose to employ the 'meter' as a standard of measurement. How many meters the table measures is a material property of the table, while the standard of measurement used is an a priori decision.

Logic is a science and not an art, because logic uses an experimental method to uncover the rational justification of scientific knowledge. We use logical "rules" to explain the facts as they emerge and to explain natural processes. By trial and error, we discover to what extent the rules can account for the facts and where we have gone wrong.

Thomson's experimental method evades Hamilton's difficulty of trying to find some principled way of distinguishing between the matter and the form of logical inferences. Moreover, it allows for the possibility that logic can be a formal science: by accounting for all the inferences in natural science. Logic in Thomson's theory is also an art – but it is not a merely aesthetic art of "tracing" inferences, as Levi Hedge had argued. Thomson's logic is the "art" of finding the justification for scientific inferences that result in knowledge. Work in the New Analytic tradition undermines the distinction that others had tried to make between logic as art and logic as science.

George Boole, *An Investigation of the Laws of Thought*

In 1854, five years after the publication of Thomson's *Outlines*, George Boole published *An Investigation of the Laws of Thought*. Boole's *Investigation* responds explicitly to the New Analytic tradition. However, Boole goes well beyond that tradition: in proposing a distinctive method

16 Thomson 1849, 2.

for logic, in linking algebra and logic, and in specifying a particular domain for the justification of the laws of logical inference.

Boole was spurred to take on questions of logic by the priority dispute between De Morgan and Hamilton over the quantification of the predicate. During this dispute, Hamilton argued that logic and mathematics should be separated, because philosophy "answers the question 'Why?', whereas mathematics is credulous in its premises" (Gray 2014, 99). As Boole notes, Hamilton even argues that the study of mathematics is "at once dangerous and useless" (Boole 1847, 11). Boole responds that, while "Of Sir W. Hamilton it is impossible to speak otherwise than with that respect which is due to genius and learning", he disagrees (Boole 1847, 12).

The disagreement is embodied in Boole's fluid employment of the methods of algebra in logical reasoning. Hamilton argues that logic, formal reasoning, must be separated from science as a doctrine of truth and reality. Insofar as Hamiltonian logic is successful, it must *correspond* to truth and reality – but logic itself is not an organon, it is a canon. Boole counters that, in the case of mathematics, logic can play the role Thomson assigns it. Logic can capture the justification for the inferences that result in scientific knowledge. When restricted to the domain of mathematics, logic can depict the reasons why inferences are justified and, to that extent, can be a doctrine of truth that yields real solutions to problems. While this may not amount to full Kantian objectivity, it nonetheless connects logic to mathematical science.

Boole's negative appraisal of Hamilton's position on the relationship between mathematics and logic could be taken, and often is taken, as a negative estimation of Hamilton's work generally. This chapter will encourage a reading on which Boole's critical reading is a step taken *within* the New Analytic tradition, to solve a problem for that approach: how are we to distinguish between the formal and the material content of logical inferences, and how are we to give a foundation for the laws of logic as laws of thought? Boole's project, conceived early on, was to show that applying logical and mathematical (algebraic) reasoning in a restricted domain could yield demonstrations of the validity and scope of logical laws as necessary laws of thought.

Here, we can distinguish two problem structures: first, issues in the foundations of mathematics, including the relationship between arithmetic and algebra, and the study and application of differential equations; second, the derivation of the laws of logic from the laws of the operation of the human mind.

Boole's account of algebra and of logic is intended to solve both problems.[17] Boole's early study of differential equations and complex

17 In working through Boole's contributions in these areas, my work is made much easier by the contributions of a recent volume on Boole (Gasser 2000), especially the essays by Durand-Richard and Panteki, and by recent work by Heis (2012) and Gray (2014).

numbers convinced him that there were holes in the foundations of the study of both.[18] Moreover, Boole's mentors, including Duncan Gregory and his predecessors such as George Peacock, were preoccupied with the question of how to give a foundation for calculation with "impossible" quantities. The question was partly occasioned by John Playfair's introduction of Laplace's *Mécanique celeste* and partly by the difficulties encountered when calculating with complex numbers and differential equations.[19] One response to such difficulties would be to argue that such quantities are merely tools of reasoning. However,

> the mathematicians of the English Algebraic School did not embrace instrumentalism. On the contrary, they were convinced that practices such as those of analytical algebra are fruitful only because they are founded on reason: efficiency results from the laws of symbolical calculus, which they considered it their job to discover… They thus sought to formulate explicitly the principles of a logical and symbolical calculus adequate for founding algebra (Durand-Richard 2000, 153–4).

It had been noted for some time that the operations of algebra and the study of differential equations can lead to the employment of mathematical and logical signs that seemingly have no meaning. Boole argues that they do have meaning if they are interpreted in the context of a demonstration according to the laws of thought that govern a symbolic calculus.

As van Evra (2000) notes, one wing of Boole reception is critical of the presence of "meaningless" symbols in his logical calculations. The presence of meaningless symbols *in the actual practice of mathematics* was precisely the *problem* that Boole was trying to solve in his earliest work. As van Evra observes correctly, Boole's aim was to show that there was a general method for logic that sprang from necessary laws of thought about a given domain of elective symbols. Once that general method and the laws of logic are justified, the laws governing inference could serve as a foundation for calculation even with meaningless symbols, because the symbols would be given a contextual definition within the confines of any given proof.

It is unjust, then, to fault Boole for the presence of meaningless symbols in his work. It is not as if Boole conceived of a general method, and then that method ran aground because it resulted in the presence of meaningless algebraic symbols. Boole was aware that mathematical practice in the English algebraic school had resulted in proofs involving

18 See Panteki 2000 for an excellent analysis, and of course, see Boole 1841 and Boole 1844.

19 Durand-Richard 2000, 152–3 and *passim*.

meaningless or even impossible quantities. His symbol language was intended to provide a secure way to deal with precisely those quantities. The presence of meaningless symbols in his work is a feature, not a bug.

In one of his earliest works of 1847, *The Mathematical Analysis of Logic*, Boole argues for a number of related theses. He splits from Hamilton, going so far as to argue that logic is a doctrine of truth, but he retains the Hamiltonian idea that logic does not deal with the real causes of things (p. 13 and *passim*). Logic is associated not with metaphysics but with mathematics. Election (the choosing of a variable, for instance), selection (selecting among the members of a class), and classification are mental acts or operations which are governed by laws. If those acts were different, the laws – and logic – would be different. Distributive and commutative laws, the syllogistic, and categorical and hypothetical judgments are expressible in elective symbols. The doctrine of elective symbols is independent of quantitative *origin,* though it may be expressed quantitatively. Boole agrees with those who thought formal logic should have a content autonomous of the general doctrine of magnitude, but he thought, explicitly, that that content was expressed mathematically.

By re-expressing mathematical equations in an elective symbol language, Boole provided a way to perform calculations in a distinct system, one that was governed by necessary laws of thought. Between 1847, when he wrote *The Mathematical Analysis of Logic*, and 1854, when he wrote *An Investigation of the Laws of Thought*, Boole became increasingly familiar with the work of the New Analytic.[20]

In the 1854 work, we find a more sophisticated account of how the laws of logic are necessary laws of thought. Boole begins with a method quite close to that defended by Hamilton, by Thomson, and later by Jevons and others:[21] to trace the development of science on the basis of principles taken as axioms. Boole focuses, however, on the role of mathematical thinking in the development of the sciences. Mathematical reasoning may consist in "rearranging" truths to show which are fundamental and which are derived. But such a rearrangement is by no means empty or merely negative.

> All sciences consist of general truths, but of those truths some only are primary and fundamental, others are secondary and derived.

20 In the Preface to *An Investigation*, Boole remarks, "That portion of this work which relates to Logic presupposes in its reader a knowledge of the most important terms of the science, as usually treated, and of its general object. On these points there is no better guide than Archbishop Whately's *Elements of Logic*, or Mr. Thomson's *Outlines of the Laws of Thought*" (Boole 1854, iii).

21 These include, in the German tradition, Adolf Trendelenburg, Hermann Cohen, and Ernst Cassirer, as well as, of course, Ludwig Boltzmann, Heinrich Hertz, David Hilbert, and the axiomatic tradition generally. See Patton 2009 and Patton 2014, including references to further work there.

> The laws of elliptic motion, discovered by Kepler, are general truths in astronomy, but they are not its fundamental truths. And it is so also in the purely mathematical sciences. An almost boundless diversity of theorems, which are known, and an infinite possibility of others, as yet unknown, rest together upon the foundation of a few simple axioms; and yet these are all general truths.
>
> (Boole 1854, 5)

Boole goes on to say that logic allows us to provide "uniform processes" from which we can "deduce" the results of science:

> Let us define as fundamental those laws and principles from which all other general truths of science may be deduced, and into which they may all be again resolved. Shall we then err in regarding that as the true science of Logic which, laying down certain elementary laws, confirmed by the very testimony of the mind, permits us thence to deduce, by uniform processes, the entire chain of its secondary consequences, and furnishes, for its practical applications, methods of perfect generality? Let it be considered whether in any science, *viewed either as a system of truth or as the foundation of a practical art*, there can properly be any other test of the completeness and the fundamental character of its laws, than the completeness of its system of derived truths, and the generality of the methods which it serves to establish.[22]

Thomson's method of testing is here taken as a fundamental method in *logic*, as well as in science. Once we have determined the relationships of interdependence in science, and discovered the basic mathematical statements on which the results depend, we can then re-derive those results using logic. That derivation requires us to find some way to compare logic and mathematics and to show that results in one can be reproduced in the other. Boole does not give an ultimate justification for this method. Instead, he argues, we can prove it by practical demonstration, based on the possibility of science itself.[23]

22 Boole 1854, 5, emphasis added.

23 "Whence it is that the ultimate laws of Logic are mathematical in their form; why they are, except in a single point, identical with the general laws of Number; and why in that particular point they differ; – are questions upon which it might not be very remote from presumption to endeavour to pronounce a positive judgment. Probably they lie beyond the reach of our limited faculties. It may, perhaps, be permitted to the mind to attain a knowledge of the laws to which it is itself subject, without its being also given to it to understand their ground and origin, or even, except in a very limited degree, to comprehend their fitness for their end, as compared with other and conceivable systems of law. Such knowledge is, indeed, unnecessary for the ends of science, which properly concerns itself with what is, and seeks not for grounds of preference

> [This book] is designed, in the first place, to investigate the fundamental laws of those operations of the mind by which reasoning is performed. It is unnecessary to enter here into any argument to prove that the operations of the mind are in a certain real sense subject to laws, and that a science of the mind is therefore possible. If these are questions which admit of doubt, that doubt is not to be met by an endeavour to settle the point of dispute a priori, but by directing the attention of the objector to the evidence of actual laws, by referring him to an actual science. And thus the solution of that doubt would belong not to the introduction to this treatise, but to the treatise itself.[24]

Boole argues that if we restrict the *domain* of the laws of the operations of the mind artificially, to the symbols 0 and 1, we can prove that the laws of logic and of mathematics both are valid in that domain.[25]

> Let us conceive, then, of an Algebra in which the symbols x, y, z, &c. admit indifferently of the values 0 and 1, and of these values alone. The laws, the axioms, and the processes, of such an Algebra will be identical in their whole extent with the laws, the axioms, and the processes of an Algebra of Logic. Difference of interpretation alone divide them. Upon this principle the method of the following work is established.[26]

Based on this method, we can show that if a result is derivable in logic, then its counterpart is derivable in algebra. However, the converse is not the case: not every operation in algebra is logical. In particular, algebraic division has no counterpart in logic, as Boole is aware.

In the concluding chapters of *An Investigation*, Chapters V and following, Boole introduces "a fundamentally different topic. It is within these chapters that virtually all of the expressions with which the critics are concerned appear, and it is here that Boole lays out what he calls a 'general method in logic'" (Van Evra 2000, 90; see Boole 1854, 70). As van Evra notes, this method is strikingly innovative. It involves extending the operations of logic to domains other than logic, in order to

or reasons of appointment. These considerations furnish a sufficient answer to all protests against the exhibition of Logic in the form of a Calculus" (Boole 1854, 11).

24 Boole 1854, 3.

25 "Each of the functions serves as an analogue of its arithmetical counterpart, and the laws of logic correspond in like fashion with expressions in mathematics. Boole circumscribes the extent of the similarity by laying particular stress on the law of idempotence, xx =x, which holds universally in the logic, but in standard algebra, only for the values 0 and 1" (Van Evra 2000, 89).

26 Boole 1854, 378.

support results that go beyond logic itself – but also to show that logical operations can illuminate and support extralogical conclusions. Boole argues that

> We may in fact lay aside the logical interpretation of the symbols in [a] given equation; convert them into quantitative symbols, susceptible only of the values 0 and 1; perform upon them as such all the requisite processes of solution; and finally restore to them their logical interpretation.[27]

Logical laws govern sciences that do not belong entirely to logic itself. Extralogical operations, including operations with no logical counterpart like division, can be treated still with the Boolean calculus. But logical reasoning itself also can be expanded by applying it to operations outside the logical domain. The trick is the restriction of the values of a given expression to the "quantitative symbols" 0 and 1.

Concluding Remarks

Boole's work emerged from the difficulties found within the work of the English algebraists, who encountered seemingly impossible or meaningless quantities in their mathematical exploits. Boole borrowed William Thomson's "experimental" approach, arguing that, if the laws of logic are truly the laws of thought, then we should be able to use logic to retrace the demonstration of results within mathematics. Then, we can retranslate those results back into the language of logic and secure not only mathematics but logic itself.

But if logic is considered in its formal aspect, as the doctrine of the laws of thought and their consequences, then what is its content? If it has no content of its own, then we might conclude that logic is not an independent science but only a Lockean art of thinking. We might conclude, as many do, that logic depends on psychological laws and view these laws as contingent.[28]

Boole's approach on this score has much in common with the contemporary inferentialist and model-theoretic approaches in logic. For Boole,

27 Boole 1854, 70, original passage in italics. Van Evra (2000) remarks, "He is suggesting that any logical symbol may be treated as its mathematical counterpart in the manner laid down in Chapter II. Then any available mathematical operation may be used on it, whether that operation is logically interpretable or not. The final (mathematical) expression in the sequence must again be one which corresponds to a logical expression. With the purely mathematical interlude lying between, the sequence may then be treated as the inference of the final (logical) expression from the initial one" (p. 91).

28 See the very illuminating discussion in Kusch 1995, Chapter 1.

establishing the content of logical statements is only a matter of showing how a given symbol works in inference. Boole argues for what he calls the "directive" character of the logical calculus in constructing proofs. Boole's method is to establish a sphere of validity for the laws of logic, which are the laws laid down by the operations of the mind, as expressed in operations on arbitrarily chosen signs. Boolean algebra is based on the idea that, if we assign the values 0 and 1 to algebraic variables, the laws and axioms governing operations on those variables will be identical to the laws and axioms of logic:

> Let us conceive, then, of an Algebra in which the symbols x, y, z, &c. admit indifferently of the values 0 and 1, and of these values alone. The laws, the axioms, and the processes, of such an Algebra will be identical in their whole extent with the laws, the axioms, and the processes of an Algebra of Logic. Difference of interpretation alone divide them. Upon this principle the method of the following work is established (Boole 1854, 37–8).

Boole uses 0 and 1 as values for the arbitrarily chosen symbols to make it clear that there can be no case in which the laws of algebra, under such an interpretation, are distinct from the laws of logic. This, in turn, allows us to argue that algebra can be shown to be governed or interpreted by the laws and operations of logic.

Boole rejected any notion that the symbols of logic are chosen to resemble their objects or their content. Boole is quite clear in Chapters I and II of Boole (1854) that such symbols are arbitrary "signs" and even that classes or sets of objects are chosen by election. For Boole, logic does not track truth because it is a universal language that describes actual thought processes. It is a science because it is a flexible language capable of representing the structure of mathematical problems and because the laws governing logic also govern mathematical inferences.

Boole had a characteristic and innovative method of developing proofs within logic, of relating those proofs to results in mathematics, and he gave a fluid and flexible way to derive the foundations of both sciences. Understanding Boole's achievements requires looking more deeply into the Kantian tradition in logic and epistemology, the German logicians who built on that tradition, and on the reception of both in the English traditions that influenced Boole directly.

The above discussion traces the influence of the "New Analytic", the hidden tradition behind Boole. This tradition, unabashedly Kantian in its origins and motivations, was concerned with the status of the laws of logic. It was also concerned with the Lockean question, popular at the time, of whether logic is an art or a science: whether it has laws and results of its own, or whether it is the art that traces the sources of justification of the true sciences.

The influence of the New Analytic on Boole is deep but also mixed. Boole's approach, of showing the justification of the laws of thought as laws of logic, owes a great deal to the New Analytic. But his approach goes beyond theirs, in drawing an explicit connection between logic and algebra. Boole argues against the idea that logic is purely formal, which was central to the New Analytic approach. He defends the notion that logic has a content, independent of its formal properties as a system of inference. However, that content depends on *using* logic to depict the structure of problems in algebra and differential calculus, a method that Boole develops thoroughly and that became part of the origin story of model theory.

Acknowledgment

I would like to thank Sandra Lapointe for providing the intellectual framework and leadership for this project, for organizing excellent workshops that were the site for substantive collaboration with others working on this project, and for comments on a draft. At the workshops, Scott Edgar, Michael Forster, Gary Ostertag, Consuelo Preti, Nick Stang, Clinton Tolley, and Richard Zach asked questions that provoked reflection on the central questions of the paper and revision to its strategies and composition.

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6 Platonism in Lotze and Frege

Between Psyschologism and Hypostasis

Nicholas F. Stang

Introduction

> [W]e are all convinced in the moment in which we think the content of any truth, that we have not created it for the first time but merely acknowledged it. It was valid before we thought about it and will continue so without regard to any existent of any kind, whether things or us, and whether or not it ever finds application in the actuality of existence, or becomes an object of cognition in the actuality of being thought.
>
> (Lotze, *Logik* §318)

> The thought we expressed in the Pythagorean theorem is timelessly true, true independently of whether anyone takes it to be true. It needs no bearer. It is not true only from the time when it was discovered, just as a planet, even before anyone saw it, was in interaction with other planets.
>
> (Frege, "Der Gedanke")

A towering figure in late 19th- and early 20th-century German philosophy, Hermann Lotze (1817–81) was a major influence in continental Europe as well as England and North America. Not only Rickert, Cohen, and Husserl, but also Bradley, Royce, and James were all importantly influenced by his writings.[1] However, despite a sizeable scholarly literature, and at least one recent high-profile monograph,[2] Lotze no longer commands the kind of attention that inspired Heidegger to describe his *Logik* as the "foundational book of modern logic."[3]

Lotze wrote about all major areas of philosophy (including metaphysics, aesthetics, religion, and history of philosophy), but the work for which he is now primarily known is his *Logik*, first published in 1843 and then in a substantially revised version in 1874 as the first part of

1 For more on Lotze's influence, see Beiser (2013), 128–31, Dahlstrom (1994), 35–37, and Gottfried Gabriel's introduction to Lotze (1989*a*).

2 Beiser (2013), a study of Lotze and another forgotten giant of 19th-century German philosophy, Adolf Trendelenburg.

3 Quoted by Beiser (2013), 130 n. 9.

his *System der Philosophie*. Lotze's work on logic was part of a larger reaction against psychologism, empiricism, and naturalism in late 19th-century German philosophy. In Book III, Lotze makes a famous distinction between existence (*Dasein*) and validity (*Gültigkeit*). The former characterizes the mode of being of mental and physical objects and events, while the latter characterizes the mode of being of propositions: they are valid or invalid, but they do not "exist" as mental or physical objects. Lotze was by no means the only figure in this period concerned to articulate the ways in which contents of acts of judgement are ontologically distinct from mental and physical events. This was a recurring theme of Brentano's school and of the phenomenological movement that grew out of it. But Lotze's way of drawing this distinction in Book III of the *Logik* was massively influential. It became a kind of rallying cry for a generation of philosophers who wanted to reject the crude naturalism that had flourished in Germany after the waning of Hegel's influence. Hans-Johann Glock goes so far as to identify Lotze as the father of the whole German antipsychologistic movement.[4]

Lotze introduces that famous distinction in the course of explaining that he is a follower of Plato, although not a "Platonist" as that term has come to be understood.[5] Early in the reception of Plato, according to Lotze, he was misread as "hypostasizing" the Forms. Properly understood, Plato was merely making the distinction between the content of judgement (atemporal and non-spatial propositions, composed of atemporal and non-spatial concepts/Forms) and our spatially located and temporally extended mental events of judging those contents. Lotze distinguishes between the mistaken hypostatic reading of Plato, on which the Forms (concepts, constituents of truths) are treated as entities in their own right, existing in some kind of Platonic heaven, and the "true" Platonism, in which the doctrine of Forms is only intended to make the distinction between what exists (mental and physical objects and events) and what is valid (propositions/contents of acts of judgement, and, derivatively, the Forms/concepts composing them).

The aim of this essay is to understand Lotze's non-hypostatic Platonism. In order to shed light on Lotze's doctrines, I compare them to those of Gottlob Frege, a near-contemporary whose commitment to Platonism is the subject of a sophisticated scholarly literature. Frege's reputation has undergone the opposite reversal to Lotze's: obscure in his own lifetime until he was "discovered" by Russell and Wittgenstein, Frege is now recognized as one of the main figures of 19th-century philosophy and canonized as a founder of analytic philosophy (at least according

4 Glock (2015), 74.

5 Although I will talk about "Platonism" throughout this paper, I am not making any claims about the historical Plato.

to the standard narrative). Although he was a lifelong opponent of psychologism, it is a matter of controversy when, and to what extent, he endorsed a Platonist view about the ontological status of numbers, concepts, and thoughts.[6] Looking to Frege in order to understand Lotze is natural, given that Frege was already linked with Lotze in the minds of his contemporaries.[7] Bruno Bauch, who invited Frege to publish "Der Gedanke" in his journal, *Beiträge zur Philosophie des deutschen Idealismus*, prefaced it with an essay of his own, "Lotzes Logik und ihre Bedeutung im deutschen Idealismus," in which he describes Frege's work as continuous with the philosophical project of Lotze.[8]

As the earlier quote from "Der Gedanke" suggests, it would be natural to take Frege as a representative of precisely that hypostatic form of Platonism that Lotze rejected. Lotze rejects the existence of thoughts, for thoughts have validity, not existence (*Dasein*), while Frege (at least by 1892) accepts that there exist (there are) thoughts, even though they are not actual (*wirklich*).[9]

In this essay I will argue, against this tempting story, that the difference between Lotze's Platonism and hypostatic Platonism is not a difference in *ontology*—whether there are propositions—but a difference in *meta-ontology*: what there being propositions amounts to. To provide a more precise characterization of this distinction, I borrow some ideas from a reading of Frege developed by Thomas Ricketts and Erich Reck.[10] On the non-hypostatic reading of Frege that Ricketts and Reck develop, the fact that there are thoughts (propositions) is not a fact distinct from the laws of logic; instead, the fact that the thought that *p* exists and is true *just is* the fact that *p*. By contrast, for the hypostatic Platonist, these are distinct but mutually necessarily entailing facts. The existence of propositions is something metaphysically "over and above" the laws of logic. While remaining neutral on whether Ricketts and Reck have interpreted Frege correctly, I argue that they have provided the correct frame for interpreting Lotze, specifically how we can be non-hypostatic Platonists (distinguishing the *Gültigkeit* of propositions from the *Dasein* of mental and physical objects) while accepting, with the

6 For a critical discussion of Frege's Platonism, see Weiner (1990), 176–226.

7 While studying at Göttingen, Frege had attended Lotze's lectures, but they were on the philosophy of religion. See Gabriel's Introduction to Lotze (1989*a*), xiii.

8 Bauch (1918–19), 48. Bauch agrees with Glock's high assessment of Lotze's importance (see earlier): "[D]er moderne Kampf der Logik gegen die Unlogik des Psychologismus [ist] ohne Lotze überhaupt nicht zu verstehen" (44). The relation of Lotze and Frege is the subject of a famous *Auseinandersetzung* between Michael Dummett and Hans Sluga: see Sluga (1975), (1976), (1977), (1980); Dummett (1973), (1976), (1981*a*), (1981*b*), (1982). For a discussion of that debate, see the Gottfried Gabriel's Introduction to Lotze (1989*a*).

9 See §3 for details.

10 See Ricketts (1986) and (1996), Reck (2005) and (2007).

mature Frege, that *there are* propositions, or, as Frege would call them, thoughts (*Gedanken*).[11]

In the section "Validity and Existence in *Logik*, Book III," I explain Lotze's famous distinction between existence and validity in Book III of *Logik*. In the following section, "Lotze's Platonism," I put this famous distinction in the context of Lotze's attempt to distinguish his own position from hypostatic Platonism and consider one way of drawing the distinction: the hypostatic Platonist accepts that there are propositions, whereas Lotze rejects this. In the section "Two Perspectives on Frege's Platonism," I argue that this is an unsatisfactory way of reading Lotze's Platonism and that the Ricketts-Reck reading of Frege is in fact the correct way of thinking about Lotze's Platonism.

Validity and Existence in *Logik*, Book III

Although Lotze originally introduces the existence–validity distinction in the broader context of his rejection of "hypostatic" Platonism, I am going to explain this distinction first and only then turn to examining Lotze's Platonism. I proceed in this way because the existence–validity distinction will give us key conceptual resources for articulating Lotze's Platonic commitments.

First of all, Lotze's distinction is not in fact a dichotomy (existence vs. validity) but a fourfold distinction among kinds of "actuality" (*Wirklichkeit*). This is an odd terminological choice, since at least one of the main categories of actuality, validity, does not *act* (*wirken*) in any sense (it has no causal efficacy). It is also crucial, in the context of an essay like this one, not to confuse Lotze's use of the term with Frege's. Frege uses "actual" in a way that one would expect, given its etymology: the *actual*, for Frege, is, roughly, whatever is in space and time and causally efficacious.[12] This cannot, of course, be what Lotze means by "actuality" because one of the main species of actuality, namely validity, is characterized by being non-spatio-temporal and causally inert. Nor is actuality modal in any important sense. Insofar as there is modality in Lotze's system, it is orthogonal to actuality; within each species of actuality we can distinguish between what is merely possible and what is actual in the modal sense. For instance, existing objects are actually (in the modal sense) a certain way, but possibly different.

11 Gottfried Gabriel, who has done more than anyone to explore the relation of Frege and Lotze, gives a similar, but much briefer account, in his Introduction to Lotze (1989*b*). For more on Frege and Lotze on the ontological status of thoughts/propositions, see Gabriel (1986), (1996), (1998), and (2002).

12 Frege (1884), §26, §85; in Frege (1918–19), he admits that thoughts have a kind of *Wirklichkeit* because they can be indirectly causally efficacious, e.g. an agent can cause a change in the world because he grasps a certain thought.

Lotze explains his terminological choice of "actuality" as follows:

> There is a very general concept of affirmation or positing, which we encounter in various investigations, the indication [*Bezeichnung*] of which languages typically lack an abstract expression of the requisite purity, for they do not in the first place concern themselves with the simplest elements of thought, but with very complex and concrete representational contents [*Vorstellungsinhalte*]. But it would not be wise to invent a technical term to represent it, the meaning of which would always be doubtful, because it could never come naturally to the lips or the thoughts of anyone; the very term "positing" [*Position*], which is frequently used for it, suggests by its etymological form the quite inappropriate connotation of an act, or operation of positing, to the execution of which the affirmation which we wish to indicate then seems to owe its being. We will instead stick to ordinary speech and must choose a word that, recognizably in ordinary *usage* [*Gebrauch*], proves itself at least to approximate to the expression of thought we seek.
>
> (*L* §316, 511)[13]

Lotze eschews the coining of a technical term, for that would be artificial. More interestingly, he eschews the use of the term "positing" (*Position* or *Setzung*), which figures so prominently in the writings of the classical German idealists.[14] He does not want to use "positing" because it connotes that what is posited (*das Gesetzte*) owes its being to the act of its positing or its being posited (*gesetzt sein*), a connotation that was fully endorsed by some idealists (e.g. Fichte). Intriguingly, in the fullest explanation he ever gives of his own use of the term, Kant equates positing with *being* in general: "The concept of positing (*Position*) or setting (*Setzung*) is perfectly simple: it is identical with the concept of being in general (*Sein überhaupt*)."[15] Lotze eschews "positing" because of its idealist connotation of a dependence of the posited upon the positing, but "being" carries with it no such idealist connotation. Nor does it carry any connotation of being spatio-temporal or causally efficacious.

Thus, I propose we read Lotze's fourfold distinction among kinds of actuality as a distinction among kinds of *being*. The passage continues as follows:

> For indicating this thought in German, the word *actuality* (*Wirklichkeit*) will serve. For we call a thing actual (*wirklich*) if it *is*, in

13 *L* stands for Lotze (1989*b*). All translations from Lotze are my own, though I have consulted Bosanquet's translation, Lotze (1884). I have rendered Lotze's use of *Fettdruck* (i.e. extra spacing between characters) as italics.

14 E.g. the ubiquity of the term *setzen* in Fichte's *Wissenschaftslehre*.

15 Kant (1992), 119.

> contrast to another which is *not*; we call an event actual if it *occurs* or has occurred, in contrast to one which does not occur; we call a relation actual if it *obtains*, in contrast to one that does not obtain; and finally, we call a proposition actually true [*wirklich wahr*] if it is *valid* [*gilt*] in contrast to one whose validity [*Geltung*] is still open to question. This linguistic usage is intelligible: it shows that by actuality [*Wirklichkeit*] we always intend an affirmation, the sense of which, however, varies greatly according to which one of these different forms it assumes; it must assume some one of these, and none of them is reducible to the others or contained in it. For we can never make an occurrence out of being, and the actuality which belongs to things, namely *being*, never attaches to events; events never *are*, but *occur*; a proposition neither *is*, like things, nor occurs, like events; in itself [...] its actuality consists in its being *valid* [*gültig*] and its opposite in not being valid.
>
> (*L* §316, 511)[16]

On my reading, Lotze is distinguishing among: (1) *existential* being: the being of a thing; (2) *eventual* being: the occurrence of an event; (3) *relational* being: the holding or obtaining of a relation; and (4) *veritative* being: it being the case that p or $\sim p$.[17] With respect to (4), it is crucial to note that not all propositions are valid. As Lotze writes: the actuality of a proposition "consists in its being *valid* and its opposite not being valid" (see earlier). This is why I have coined the more general category of "veritative" being: being either true (valid) or false (invalid).

Lotze explicitly addresses the relation of his category of actuality to being in the course of explaining why Plato was misinterpreted, even by his own school:

> Plato wanted to teach nothing other than what was discussed above: the validity [*Geltung*] of truths, regardless of whether they are verified, as their way of being, by any objects in the external world. [...] But the Greek language, then and later, lacked an expression for this concept of validity that contains no being: precisely this expression, being, took its place, often unproblematically, but in this case quite fatefully. Every content graspable by thought, when one wanted to consider it as unified in itself and distinct from others, for which the schools later coined the not totally incorrect name of "thought-thing"

16 Lotze expresses validity by the verb *gelten*, but in English "valid" is an adjective which requires completion by the copulative verb "be." English-language readers should thus be aware that some uses of "is" are merely artefacts of translation.

17 In some contexts, Lotze identifies being (*Sein*) specifically with the first form of actuality, i.e. existence. See §4 for a more complete discussion of these passages; in brief, I think that Lotze is identifying the first kind of actuality with the being of *things* (in a sense to be specified), not with being in general.

> [*Gedankending*)], was for the Greeks a being [*Seiendes*], *on* or *ousia*. And when the distinction between an actually valid truth [*wirklichen geltenden Wahrheit*] and an allegedly valid one came into question, the former was also an *ontos on*; the Greek language never knew how to indicate that actuality of mere validity without the constant confusion [*Vermischung*] with the actuality of being; the expression of the Platonic thought also suffered from this confusion.
>
> (*L* §317, 513–4)

I will discuss Lotze's distinction between "hypostatic" and "logical" Platonism in the next section, but for now I want to focus on the linguistic point he makes here. Ancient Greek, he claims, had only one set of expressions for the first and the fourth of Lotze's modes of actuality: the verb "to be" (*eimi*) and the nouns formed from it (e.g. *ousia*, *to on*).[18] So Plato had to express his doctrine of the abstract validity of propositions in terms that were inevitably misread as postulating the existence of propositions as abstract things. Lotze's blaming of the misreading of Plato on the lack of an alternative to words formed from the verb *eimi* ("to be") may suggest that my proposal to think of modes of actuality as modes of being is misguided, but I think that it in fact confirms my reading. Both *Sein* and "being" allow for multiple senses, both the "existential" sense (there are/*es sind*) and the "veritative" sense (in which it is the copulative verb). So long as we are careful not to conflate these two, we will not assume, like Lotze's ancient Greeks, that the *being* of valid propositions is the *existence* of things.

In some contexts, Lotze simplifies this fourfold distinction into a simple distinction between existence (*Dasein*, sometimes *Realität*, sometimes *Sein*) and validity (*Gultigkeit/Geltung*, sometimes objectivity, *Objectivität*).[19] In a way, this is very natural, for arguably there is no deep difference between existential and eventual being: the occurrence of an event just is its existing (assuming there are no non-occurring events). Likewise, from the point of view of later logic, we can assimilate the obtaining of a relation to the truth of a proposition: the relation R obtains among $x_1, \ldots, x_n$ just in case the proposition that $R(x_1, \ldots, x_n)$ is valid. Lotze is thus willing to compromise somewhat on the irreducibility of the four different categories of actuality/being, but never on the irreducibility of the first category (*Dasein*) to the fourth (*Geltung*). Veritative being (including the holding of relations) is simply irreducible to the existence of things and the occurrence of events. I will therefore focus on the distinction between existence and validity.

On Lotze's antipsychologistic view, propositions are not identical to the mental representations by which they are grasped, much less the

18 Cf. Rödl (2012), 31–32.

19 See *L* §3, §319–20. Cf. Dummett (1982), 96, on Lotze's shifting terminology.

written and verbal marks by which they are expressed. Lotze thinks of mental representations as ultimately events, having actuality/being of type (2). Given the irreducibility of the four kinds of actuality/being, a proposition (the content of a belief event) cannot be identical, or reducible, to any set of events, no matter how complex. A proposition is timelessly true or false. Consequently, it cannot be identified with any set of mental or physical events or constituents, which come into and go out of existence:

> Representations, insofar as we have them and grasp them, possess actuality in the sense of an event. They *occur* in us, for as expressions of a representational activity they are never a being at rest [*ein ruhendes Sein*] but a continual becoming; their content, on the other hand, so far as we regard it in abstraction from the representational activity which we direct at it, can no longer be said to occur, though neither again does it exist as things exist. Rather, it is merely valid [*gilt*].
>
> (*L* §316, 512)

A true proposition is true whether or not anyone ever holds it to be true, or even grasps it; all such "holdings" and "graspings" fall in the Lotzean category of events, actuality/being of type (2).

Validity (truth), according to Lotze, is timeless, aperspectival, independent of position, and independent of whether any subject ever thinks that *p*. The predicate "is true" makes no reference to time, speaker, position, etc. A proposition is true or false *simpliciter*. As Lotze writes:

> We are all convinced in the moment in which we think the content of any truth, that we have not created it for the first time but merely acknowledged it. It was valid before we thought about it and will continue so without regard to any existent of any kind, whether things or us, and whether or not it ever finds application in the actuality of existence, or becomes an object of cognition in the actuality of being thought. This is what we all believe with regard to truth when we set out to search for it, and it may be that we lament over its inaccessibility, at least to any form of human knowledge; truth which is never apprehended by us is valid no whit less than that small fraction of it which finds its way into our thoughts.
>
> (*L* §318, 515)

Likewise, since a proposition is either true or false, the content of a proposition has a kind of timeless, aperspectival, subject-independent being: that content *is*, either by being valid or by being invalid. Contents do not "come into being" or "go out of being," for if they did, truths would come into or go out of being, which is excluded by the very nature of

truth. Lotze denies that a genuinely subjective propositional content, one accessible only by one thinker (one sequence of mental representational events), is even possible:

> [I]t is impossible that an individual subject sense or represent something whose content [*Inhalt*] does not have its determinate place in this universal world of the thinkable [*allgemeinen Welt des denkbaren*], possessing its similarities and differences to others one and for all, but remains a peculiarity of this subject, belonging nowhere else, without relation to the whole world.
>
> (*L* §318, 516)

If any thinker thinks about anything, the content of their thought is either true or false, and thus that content has a kind of timeless, subject-independent, aperspectival being. Subjects access these contents; they do not create or generate them. There cannot, even in principle, be "private" content.

The irreducibility of validity to any of the other categories of actuality/being means that truth cannot be defined:

> [Validity] has to be regarded as much as [existence] as a basic concept that rests only on itself, of which everyone knows what he means by it, but which cannot be constructed out of any constituent elements which do not already contain it.
>
> (*L* §316, 513)

Any attempt to define validity would be implicitly circular, for in order to understand it, we would need to deploy our understanding of validity itself. A putative definition of validity would have the form:

(*) *p* is valid iff F(*p*).

Lotze's point is that, to understand the content of (*), you must understand the content of its right-hand side. You understand that content only if you understand that F(*p*) or ~F(*p*), that is, if you understand it as a content that is either valid or invalid, whether or not it is asserted, held to be true, etc. So (*) cannot provide you with an understanding of validity that you otherwise lack; in order to understand (*), you must already understand what validity is. When Lotze writes that "everyone knows what he means by [validity]," I take him to mean everyone who can make judgements. There is no such thing as being able to judge that *p* and then acquiring the capacity to think of one's judgements that they are valid or invalid, because all there is to judging that *p* is judging that *p* is valid. All there is to the obtaining of a proposition (what one judges) is its being valid; so in judging the former, one judges the latter. Consequently, Lotze is committed to both sides of this biconditional having the same content:

(†) the proposition that *p* is valid iff *p*.

When one judges a proposition, one thereby judges it to be valid; there is no gap between judging that *p* and judging that *p* is valid.

Lotze's Platonism

The context of Lotze's fourfold distinction among kinds of actuality/being is his appropriation of Plato. Lotze creatively reads Plato's metaphysics as fundamentally a theory of *truth*. On his reading, Plato's core doctrine is that truth (e.g. the truth about what justice is) is timeless, unchanging, and independent of whether subjects apprehend it. The key role of the theory of Forms, according to Lotze, is to maintain, against the sophists, that the truth about what justice is, is independent of our beliefs, not subject to change, and not sensibly perceptible by us. However, Plato was incorrectly interpreted, including by later members of his own Academy, as asserting that this requires that there be timeless, unchanging, subject-independent "things" that ground these truths—that is, the Forms. Plato's teaching was fundamentally a theory about what we are doing when we judge something to be true, *not* a theory of non-spatio-temporal abstract entities.[20]

Even on Lotze's reading, however, there are certain divergences between Lotze's theory and Plato's. For one, Lotze reasons *from* truths *to* concepts/Forms: if the proposition "*a* is *F*" is valid, then concepts <*a*> and <*F*>, the constituents of this truth, must have some kind of timeless being. The exact status of the constituents of propositions (bearers of validity) within Lotze's fourfold division of the modes of actuality/being is somewhat unclear, since, syntactically, they cannot be said to be valid *sensu stricto*. They are valid in the derivative sense that they (atemporally) refer to (*bedeuten*) objects about which there are valid propositions, namely those propositions in which the relevant objects are subsumed under the concepts. Plato, however, emphasized the constituents of truths, that is, the Forms (concepts), over the truths themselves:

> [One notices] how comparatively rarely general propositions appear [in Plato's writings]; they are not completely lacking, but constitute in individual cases objects of important discussion; Plato had not realized that they are, in this form as propositions, the essential constituents of the ideal world.
>
> (L §321, 521)

Another difference is that Lotze accepts the simple schema (†) from the previous section, which entails that there will be valid propositions about *everything*, and thus that there will be atemporal concepts (having whatever mode of actuality/being such propositional constituents have)

20 *L* §313–321, 505–523.

of everything. Thus, for Lotze, Socrates should not have been perplexed when questioned by the Eleatic stranger about the forms of dirt, hair, etc., in the *Parmenides*.[21] Since dirt is dirt, it follows that the proposition "Dirt is dirt" is valid, so *a fortiori* <*dirt*> has whatever mode of being/actuality propositional constituents have. The generality of Lotze's semantic theory means he must accept concepts of everything that can be the topic of a valid judgement, which, given (†), includes absolutely everything.

The difficult question is how to distinguish Lotze's Platonism from the Platonism he rejects, on which propositions and their constituents are hypostatized as "things." He writes:

> For the Greeks that which is not in space *is* not at all, and when Plato banishes the Ideas to this non-spatial home, this is not an attempt to hypostasize their mere validity into any kind of existential being [*seiender Wirklichkeit*], but rather a clear effort to ward off any such attempt from the outset. [...] Nevertheless although these various utterances point one and all to the fact that Plato only ever asserted the eternal validity of Ideas, but never their existence [*Sein*], he still had no better answer to give to the question, what then are they, than to bring them again under the general concept of *ousia*; thus was opened a door to a misunderstanding, which has since been propagated further, although one never knew how to say, on the hypothesis that blames him for this, exactly what it is that Plato is supposed to have hypostasized his Ideas into.
>
> (*L* §318, 516)

The hypostatic Platonist takes propositions and their constituents (Forms/concepts) to fall under the first mode of actuality/being: they exist as things, albeit as non-spatio-temporal things. This is informative as a characterization of hypostatic Platonism only to the extent that we understand the first mode of actuality/being: existence. But the trouble is that Lotze frequently characterizes this mode in terms of its causal efficacy and spatial location. Further, he explicitly associates it with the ontological category of things. Since the hypostatic Platonist clearly does not think that propositions and concepts are spatio-temporal or causally efficacious, we need a more general characterization of the existence–validity distinction. In other words, we need a more precise characterization of what it would mean to call proposition or concepts *things*.

One tempting option would be to interpret Lotze's characterization of the first mode of actuality/being as existence in light of Frege's quantificational theory of existence and to characterize hypostatic Platonism as what I will call "ontological Platonism" (OP): there are propositions.

21 *Parmenides* 130c4–d9.

On this reading, Lotze fails to be a hypostatic Platonist because he is not ontologically committed to propositions; they are not in his inventory of "what there is." Admittedly, much of Lotze's discussion of validity seems to "quantify over" propositions, but perhaps this apparent quantification can be paraphrased away. On this reading, Lotze's "object language" claims about propositions need to be read as "meta-level" claims about the logical grammar of various terms. For instance, it was confusing for Lotze to say that propositions are either true or false, timelessly and aperspectivally. This point would better have been stated as follows: "is true" cannot be supplemented by a reference to time, speaker, etc. This ontologically conservative Lotze could more perspicuously have expressed his core semantic doctrines by saying that you have not specified the content of the judgement that *p* unless you have specified it so fully that it is true or false that *p* timelessly, nonperspectivally, independently of position, and independently of whether any subject judges that *p*.

If this is the correct reading of Lotze's non-hypostatic Platonism, it marks a clear difference between Lotze and the mature Frege, from roughly 1892 (the year "Über Sinn und Bedeutung" was published) onwards. Frege held that thoughts are the senses (*Sinne*) of complete sentences, but that in indirect discourse (e.g. in belief attributions), expressions refer (*bedeuten*) to the senses they express in direct discourse. For instance, consider these two sentences:

1 One can reach India directly by sailing due west from Spain.
2 Columbus believed that one can reach India directly by sailing due west from Spain.[22]

In (2), the sentence "One can reach India directly by sailing due west from Spain" refers to the sense it expresses in (1). Since it expresses a thought in (1), namely, the thought that one can reach India directly by sailing due west from Spain, in (2) it refers to that same thought. To get from the idea that thoughts can be the referents (*Bedeutungen*) of expressions to the idea that thoughts exist, we need an additional piece of Frege's doctrine, his purely logical conception of an object:

> When we have thus admitted objects without restriction as arguments and values of functions, the question arises, what it is that is here being called an object [*Gegenstand*]. I regard a scholastically correct [*schülgemäße*] definition as impossible, since we have here something that, because of its logical simplicity, does not admit of a logical analysis. It is only possible to indicate what is meant. Here I

22 This example is adapted from Frege (1892*a*), 152–3.

> can only say briefly: an object is anything that is not a function, whose expression therefore has no empty place.
>
> (Frege 1891, 134)[23]

Any expression that does not contain an argument place, which is not "unsaturated" (*ungesättigt*), refers to (*bedeutet*) an object. The *that* clause in (2) has no argument place; it is fully saturated, so it refers to an object. Though it may sound odd to say that thoughts are *objects*, this is a direct consequence of Frege's logical conception of an object and his view of indirect discourse. Given that thoughts are objects, the existence of thoughts is entailed by Frege's view that "Thoughts exist" is equivalent to "There are thoughts." That there are thoughts means simply that there is an argument, an object, relative to which the function *x is a thought* has the value True.[24] Since there are thoughts, and thoughts are objects, there is such an object, and so thoughts exist.

Lotze and Frege agree that whether it is valid (true) that *p* has nothing to do with the psychological acts by which thinkers grasp (or fail to grasp) whether *p*. They agree that logic studies the laws that govern the contents of acts of judging, not those acts themselves. But this reading locates the difference between them in their ontologies (as that term has come to be used after Quine): Frege accepts OP, but Lotze rejects it. In the next two sections, I will argue that the difference between Frege and Lotze is more complex and subtler than this.

Two Perspectives on Frege's Platonism

Although the "ontological" interpretation makes for a clean account of the difference between hypostatic and non-hypostatic Platonism, there are reasons to be dissatisfied. For one, Lotze shows no reservations about "quantifying over" propositions. He says, for instance:

> All representable contents stand in fixed and unalterable relations, and however arbitrarily or accidentally our attention moves from one to the other, and in whatever order one after the other is brought to our awareness, prompted by we know not what, we will always find them in the same relations in which the infinitely and objectively [*sachlich*] multifaceted articulation of the world of ideas [*Ideenwelt*] is given once and for all.
>
> (*L* §346, 572–3)

23 All translations from Frege are my own, though I have consulted Beaney's translation in Frege (1997).

24 See Frege (1891), 138–9; (1892*b*), 173; cf. (1884), §53.

Admittedly, it is always tricky to determine the ontological commitments of statements in ordinary language, especially before Frege's introduction of the quantificational theory of existence and the quantificational notion of an object. But to the extent that Lotze's text gives us any guidance on whether he would be willing to countenance propositions (and their constituents) in his account of "what there is," it gives little comfort to the non-ontological reading of his Platonism.

I want to propose that the difference between Lotze's Platonism and hypostatic Platonism is not *ontological*, that is, it is not a difference about *whether* there are propositions, but *meta*-ontological, that is, it is a difference about *what it is* for there to be propositions. To articulate this meta-ontological reading of Lotze's Platonism, I am going to draw on a reading of Frege developed by Thomas Ricketts and Erich Reck.[25] While I will remain neutral as to whether their reading is correct, I will argue that Ricketts and Reck have (unintentionally) given a quite perceptive characterization of Lotze, in particular, the nature of his non-hypostatic Platonism.

Given our focus on Platonism, perhaps the best way to explore the Ricketts-Reck reading is via Reck's (2005) distinction between two kinds of Platonism about numbers, which he calls Platonism A and Platonism B.[26] Reck's distinction is ultimately a distinction between two ways of understanding the objectivity of numbers and truths about them. Platonism A understands objectivity in metaphysical terms.[27] It takes the notion of *object* to be basic, as well as the notion of an object and its determinate properties being *metaphysically independent* of other objects and facts. Platonism A holds that numbers exist as objects and have determinate properties independently of the judgements that we make about them. Judgements about numbers are objective, according to the A-Platonist, when they succeed in corresponding with, or "matching," the metaphysically objective properties of numbers.

The B-Platonist, by contrast, begins with a conception of objective *judgement*, understood *not* as judgement that corresponds to or matches some metaphysically independent standard (as the A-Platonist does), but as obeying the logical laws internal to judgement itself. For the B-Platonist, the objective existence of numbers is derivative of objective judgement: to say that numbers exist objectively is just to say that the existence of numbers follows from the laws of logic. The objectivity of the laws of logic is explanatorily primitive. The B-Platonist might offer

25 See Ricketts (1986) and (1996); Reck (1997), (2005), and especially (2007). There are differences in their readings, but I will not focus on them here.

26 Reck (1997) characterizes it as the distinction between "metaphysical" and "contextual" Platonism.

27 I use "metaphysical" where Ricketts and Reck tend to use "ontological," for I want to reserve the latter for its post-Quinean meaning of "what there is."

some arguments in favour of a particular regimentation of these laws, or why we cannot do without them in reasoning, but no explanation can be given of *why* they are objective (except by deriving them from more basic laws of logic).[28]

It is crucial for our purposes to understand why Platonism B, according to Reck, is not merely a form of psychologism. The core idea of Platonism B is that *judgement* is primary.[29] Platonism B does not reject psychologism on the grounds that mental states and abstract propositional contents belong to two different metaphysical categories, where these categories are assumed to be intelligible independently of judgement. Psychologism is false, according to the B-Platonist, because the act of judging is not reducible to the occurrence of a mental episode (much less to the occurrence of a neural event).[30] From this perspective, Platonism A is equally misguided because, having accepted a realm of metaphysically distinct abstract objects, perhaps including propositions (thoughts, contents of judgements), we are no closer to understanding what it is to *judge* them. Judging must be taken as explanatorily primitive, and once it is, the abstract objects of Platonism A are an ontological "free lunch": the existence of these objects follows from the laws of objective judging, the laws of logic (assuming that Frege's logicist programme in arithmetic succeeds). Platonism A most naturally goes with a "correspondence" conception of truth, on which truth is a substantive property of a proposition, namely the property of corresponding to reality. Platonism B, however, has no need for any substantive theory of truth. The objectivity or truth of judgement is not a property of a judgement, according to the B-Platonist, for to judge that some proposition p is true *just is* to judge that p. There is no difference in the content of these judgements.[31] Thus, according to the B-Platonist, once we take on board this conception of judgement, we get notion of objectivity/truth, as well as the existence of numbers, "for free."

Although Ricketts and Reck put a fair amount of weight on a correspondence theory of truth as marking the difference between Platonism A and Platonism B, I do not think it is crucial, for one can combine deflationism about truth with whatever metaphysics one likes, including hypostatic Platonism.[32] I think a more significant difference between Platonism A and Platonism B is their different responses to the question

28 Cf. Frege (1893), xvii.

29 This comes out most clearly in Ricketts (1986) and Reck (2007).

30 This point is emphasized in Ricketts (1986), section 1.

31 See Frege (1918–19), 345; (1969), 271–2.

32 As Reck acknowledges in (2007), 6. However, Ricketts and Reck are probably correct in one direction of entailment: the "judgement-centric" metaphysics they attribute to Frege requires a minimalist view of truth on which the truth predicate adds nothing to the content of a judgement.

of our epistemic access to numbers. For the A-Platonist, there is a metaphysical gap between our judgements and the numbers we judge about, and this generates the well-known epistemic problem of how we overcome this gap and obtain knowledge of metaphysically independent and causally inert abstract objects like numbers.[33] My point is not that this is a devastating objection to Platonism A, but that, given Platonism A, it is a substantive question how we can have epistemic access to numbers. For the B-Platonist, however, the explanation of our epistemic access to numbers is very different: the laws of logic are the internal laws of judging itself, and being able to judge involves at least an implicit grasp of them, so once we appropriately systematize this logic and define all arithmetic notions in purely logical terms, we can prove the existence of numbers using logic alone (assuming, once again, that Frege's logicist programme is successful). Because the B-Platonist takes judgement, rather than a metaphysical notion of objectivity as mind-independence, to be basic, the question of how we know that numbers exist is a very different question for him than for the A-Platonist. The B-Platonist does not need to overcome a metaphysical gap between our arithmetical judgements and the numerical objects they are about, nor does he have to concern himself with the sceptical objection that we would continue to reason as we do even if the causally isolated "external" world of numbers did not exist. The only thing the B-Platonist needs to concern himself with are issues internal to judgement itself, in particular, the internal consistency of the system of logic in which he reconstructs arithmetic.[34, 35]

Lotze's Platonism: Ontological Commitment without Hypostasis?

It is not my intention to enter the lists in favour of reading Frege as an A-Platonist or a B-Platonist, but simply to argue that Lotze is a B-Platonist. Reck originally formulated his distinction between two kinds

33 See Benacerraf (1973) for an influential modern formulation of the access problem.

34 Historically, of course, this was what proved fatal to Frege's logicist programme in arithmetic. Ricketts and Reck read Frege's deep concern with the consistency of his logical system, and comparative lack of concern with our epistemic access to logic, as evidence that he endorses Platonism B rather than Platonism A.

35 Some readers might object that Reck's characterization of Platonism A and Platonism B in fact marks a different distinction: that between non-logicism (Platonism A) and logicism (Platonism B) about arithmetic. This is not quite accurate, however, for Reck's difference emerges even if we assume that logicism is correct. Assuming that arithmetic does reduce to logic, the A-Platonist has an explanatory burden to discharge that the B-Platonist does not: how can we, on the concrete side of the concrete–abstract divide, come to know the abstract logical objects that (assuming logicism) are the numbers?

of Platonism in terms of their views about numbers, but Lotze is not a logicist about arithmetic,[36] so I will argue that Lotze believes about propositions (contents of judgements) what Reck's B-Platonist believes about numbers.

First of all, Lotze, like Reck's B-Platonist, rejects the view that truth involves the correspondence of our thoughts to something external: "Nothing other than the connection of [the contents of] our representations among themselves can be the object of our investigation" in logic (*L* §304, 491); the measure of truth is not the "external world," to which our thoughts are to be compared, but whether a thought agrees with the "necessity of our thinking about all relations of the manifold whatsoever" (*L* §306, 493–4); the truth of logical laws is "independent of the relation of our cognition to an object beyond it" (*L* §311, 502). Even Plato is described as realizing "the truth which our world of [contents of] representations has within itself and independently of its agreement with the presupposed essence of things beyond it" (*L* §313, 506–7). As my bracketed insertions indicate, Lotze is not always as careful as he should be in distinguishing our representations (*Vorstellungen*) from their contents; read charitably, however, and in light of his antipsychologist doctrines, all of these claims are about contents (propositions), not about mental events or processes. Lotze holds that logic is not about the correspondence of our representations with something external to them, but with the agreement of the content of those representations with the logical laws governing all judgemental content as such. Recall Lotze's claim that validity "cannot be constructed out of any constituent elements which do not already contain it" (*L* §316, 513). In particular, therefore, validity/truth cannot be defined as correspondence.

What is more telling in favour of reading Lotze as a B-Platonist is his response to scepticism about logic. For the A-Platonist, logical laws concern inferential relations among abstract propositions, where propositions are taken to be metaphysically independent of our judgements about them. This generates a problem about our epistemic access to these laws. Reck's B-Platonist explains our epistemic access to logic in very different terms: logical laws articulate the relations of inference among (contents of) objective judgements, and the objectivity of judgement is explanatorily basic. The B-Platonist explains our grasp of logical laws as the articulation of a primitive capacity to recognize relations of inconsistency and entailment among judgements. Possession of this capacity is internal to the faculty of reason (the faculty of being able to make judgements in the first place); no subject can be said to be judging who does not understand that p and $\sim p$ cannot both be true. The B-Platonist has no need to explain how our judgements succeed in "matching" or

36 Lotze's views about mathematics are hard to determine, but I take him to be a non-standard kind of Kantian about arithmetic: propositions like $7 + 5 = 12$ are analytic, but we require *a priori* intuition to be given their objects. See *L* §353, 586–7.

"corresponding" to a metaphysically independent structure holding among metaphysically objective entities like propositions.[37]

This provides further evidence that Lotze is a B-Platonist. Lotze's famous discussion of validity in Book III, Chapter 2 of *Logik* continues his discussion in Chapter 1 of scepticism about logic.[38] The point of his distinction between validity and existence is to secure logic from scepticism by showing that the validity of our judgements does not require, and is thus not vulnerable to sceptical doubts concerning, a world of things existing "external" to judgement. Judgement does need to match some external standard in order to be valid but must merely obey its own internal laws of validity. Plato is again invoked as someone who precisely denied that judgement, including judgements in logic, must correspond to a world of entities beyond thought, and the sceptical consequences that this (according to Lotze) would entail. We once again see Lotze making exactly the argumentative moves described by Reck as Platonism B: there is no problem of epistemic access across a metaphysical divide, for logic merely articulates the internal laws of valid judgement itself.

Further evidence for reading Lotze as a B-Platonist comes from the difference between his responses to scepticism about logic and to scepticism about the principles of "pure mechanics," such as the legitimacy of inductive inference, or the principle that every event has a cause. Lotze's response to global scepticism about truth is that such scepticism is self-undermining. To articulate the sceptical position—indeed to articulate any position whatsoever—one must judge, and in making any judgement, one is judging that its content is *true*. Thus, articulating any position commits one to accepting that there are true judgeable contents, that is, propositions.[39] By contrast, Lotze admits that the denial of the causal principle or the systematic falsity of induction is consistently thinkable; there is nothing self-undermining or inconsistent about denying that all events have causes or that the future will resemble the past.[40] Instead, Lotze pursues a broadly Kantian response to scepticism about these principles, which, again, following Kant, he accords the status of synthetic *a priori* principles of experience: experience of existing things in space is impossible unless we assume these principles.[41] The reason for this distinction is that logical principles concern the domain of *validity*, contents of judgements, while these *a priori* principles of experience

37 Ricketts has more to say about Frege's epistemology of logic than Reck; see Ricketts (1986), 73, 83. Aside from the question of whether they have interpreted Frege correctly, I worry that this is a key point on which Platonism B is not more philosophically attractive than Platonism A. In particular, more would need to be said about how this epistemology of logic could be articulated without devolving into psychologism or idealism.

38 *L* §313, 506–7.

39 *L* §303, 489; §304, 491; §309, 498–99; §311, 502; §315, 508.

40 *L* §349, 578.

41 *L* §349, 579; §350, 581; §351, 583; §356, 591.

concern the domain of *existence*, things that exist "external" to judgement. Since there is no valid path from principles of validity to principles of existence, Lotze's response to logical scepticism does not answer the sceptic about the principles governing the existence of objects.[42] Lotze's adoption of two different anti-sceptical strategies for logic and for judgements about existent things in space and time shows that he accords propositions and ordinary objects very different metaphysical statuses. In particular, it provides further evidence that his metaphysics of propositions is not that of Platonism A—on which answering the sceptical objection is a matter of securing epistemic access across a metaphysical divide—but that of Platonism B, on which answering the sceptical question is simply a matter of articulating in a consistent fashion the internal laws that govern the validity of judgement.

Where does this leave us with our original question about the difference between hypostatic Platonism and Lotze's Platonism? I think they correspond quite closely to Reck's Platonism A and Platonism B, respectively. The A-Platonist understands the objectivity of propositions in metaphysical terms as their existing mind-independently. Our beliefs must then, somehow, be brought into conformity with them. From the point of view of the B-Platonist, the A-Platonist "hypostasizes" propositions and their constituents. The B-Platonist does not accept that judgemental contents, that is, propositions, are sequences of representations—much less that they are physical events in our brains and bodies—but does not hypostasize them.

Explaining why the B-Platonist does not hypostasize propositions will also explain how Lotze can be a non-hypostatic Platonist while accepting that there are propositions (OP from §3). Recall that the B-Platonist holds that the judgements *p* and "It is valid that *p*" have the same content. Thus, the following biconditional is trivially true (both sides have the same content):

1 *p* iff the proposition that *p* is valid.

From this, it follows by logic alone that:

2 (*p* or ~*p*) iff the proposition that *p* is valid or invalid.

Assuming that it is a basic law of logic that propositions are either valid or invalid, it follows that:

3 *p* or ~*p* iff there is a proposition that *p*.

Since the left-hand side is a law of logic, this shows that the B-Platonist can derive that there are propositions (OP) from purely logical principles plus

42 *L* §348, 577.

a trivial principle, (1), that asserts a biconditional between one and the same content under two guises (*p* and "the proposition that *p* is valid"). This means the fact that there are propositions is not a fact "over and above" the laws of logic; rather, it is a trivial consequence of the laws of logic themselves. Since the B-Platonist takes the laws of logic to be the internal laws of judging itself rather than laws that describe some external world of abstract objects, this means that it is internal to judging itself that there are propositions. In the terms introduced in the section "Lotze's Platonism," this means that the B-Platonist is also an *ontological* Platonist.

If we read Lotze's non-hypostatic Platonism as Platonism B, as I have argued we should, then he is an ontological Platonist about propositions. What then becomes of his foundational distinction between the validity of propositions and the existence of things, if he is admitting that *there are propositions*? The category of existence should not, I think, be identified with the category of what there is (existence in Frege's quantificational sense). Recall that Lotze consistently identifies existence (*Dasein*), the first category of actuality, as the existence of *things* (*Dinge*). Hypostatic Platonism is consistently characterized as the view that takes propositions and concepts to be existent *things*, thus transferring them into the category of existence. I propose that we read Lotze's category of the existent, of things, not as "what there is" but as that which is external to judgement in the specific sense that there being such things does not follow from the internal logical rules of judging itself. Lotze denies that there is a purely logical proof that there is a mind-independent world in space and time, or that there are causes for every effect; consequently, these objects are "existent things" in Lotze's technical sense, and skepticism about them must receive a different answer than logical skepticism. The "externality" of things is not their distinctness from our representations (for propositions are distinct from our representations, but are not things); rather, it is their being "external" to logic, that is, their not following from logic alone. This allows Lotze to consistently maintain that *there are* propositions while denying that they *exist*, that they are *external*, and that they are *things*. The hypostatic Platonist, on this reading, is not the theorist who holds merely that there are propositions, but the theorist who thinks that there being such propositions is a fact over and above the laws of logic. The hypostatic Platonist is the theorist who thinks that there is no purely logical proof (like (1)–(3) earlier) that there are propositions, and thus that there is a substantive question of our epistemic access to their existence.[43]

43 I would like to thank Fred Beiser, Emily Carson, Michael Forster, Consuelo Preti, Graham Priest, Erich Reck, Jamie Tappenden, and Clinton Tolley, as well as audiences at the 2016 "Logic in Kant's Wake" workshop at McMaster University and at the 2017 meeting of the Canadian Philosophical Association in Toronto, for their responses to earlier versions of this chapter. Special thanks are due to Sandra Lapointe

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for detailed comments on the penultimate draft, as well as for constant encouragement, nearly infinite patience, and world-famous hospitality. Thanks also to Ian Drummond for some very acute copy-editing of the penultimate version.

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7 Demystifying Cohen's *Logik*

Frederick Beiser

Cohen's Conception of Logic

Any student of contemporary logic who happens to look into the table of contents of Hermann Cohen's *Logik der reinen Erkenntniss* will immediately see that this is not a logic in our contemporary sense. There are no truth tables, no quantifiers and no formulae. We might expect a contemporary to shut the book and to dismiss Cohen for not knowing what logic is really about. But, before we judge Cohen, we should understand *his* conception of logic and what he was trying to do. History shows us that there are as many conceptions of logic as there are of art and morality; and it would beg the question to judge one by the goals and standards of another.

Cohen explains his conception of logic in the introduction to *Logik der reinen Erkenntniss*. Although he does not count or neatly separate them, there are four salient features to Cohen's exposition, three of which make his logic very unlike logic in our contemporary sense. I would like to explain briefly each of these features and his rationale for them.

The first basic feature of Cohen's conception of logic is that it is *epistemological*, i.e. it investigates the conditions for knowledge. Since Cohen thinks that we acquire such knowledge through science, he conceives logic first and foremost as the study of scientific method. Hence, in the introduction to his book, he calls logic "the thinking of science". More specifically, logic is concerned with the thinking involved in one kind of science: mathematical natural science (17).[1] Cohen gives a central place to this science because it is for him the model of certainty, exactitude and rigor.

The second fundamental feature of Cohen's conception of logic, which follows from the first, is that logic is non-formal. Since logic deals with knowledge, and since knowledge requires not only having a form but also a content, Cohen thinks that logic cannot simply be formal, i.e. it cannot deal only with the forms of judgement and inference. Hence,

1 All references are to the first edition of *Logik der reinen Erkenntniss* (Berlin: Cassirer, 1902).

in his introduction, Cohen dismisses "the ghost of a purely logic" (*das Gespenst einer formalen Logik*) (13). The chief problem with a purely formal conception of logic, Cohen thinks, is that it would reduce scientific method down to the forms of judgement and inference. But how we *know* the world, he insists, is not the same issue as how we *talk* about the world. The ideal of a formal logic arose from confusing the forms in which we *know* about the world with the forms in which we *talk* about it; it makes knowledge hinge upon a mere matter of grammar. The target of Cohen's critique of formal logic is not Frege, whom he does not seem to know, but "the latest English logic" (460), by which he seems to mean Boole. Unfortunately, Cohen is not explicit about his target except through this one phrase.

The third characteristic feature of Cohen's conception of logic, which is independent of the first two, is that logic is, in a broad sense of the term, *metaphysical*. Logic is metaphysical in the basic sense that the forms of logic are also forms of being. Logic, Cohen declares, "must not be removed from the sphere of interest of the old metaphysics" (14). Insofar as logic concerns our knowledge, and more specifically our *knowledge of the world*, it *ipso facto* deals with being. Cohen goes so far as to say it is necessary to rehabilitate Parmenides' old dictum about the identity of thinking and being (14; cf. 27). The whole sense and meaning of logic, Cohen writes in the conclusion of his book, is the identity of thought and being (501). This insistence on the metaphysical dimension of logic explains his resistance to psychologism. It is a grave error of psychologism, Cohen thinks, that it attempts to make a distinction between our thinking about the world and the world itself, as if logic were only concerned with human thinking, with the "laws of thought" or how we humans happen to think about things (39).

Although Cohen is eager to stress the metaphysical dimension of logic, he does not approve of all or every kind of metaphysics. There are various forms of metaphysics, he notes, and the form of metaphysics involved in logic concerns the principles or presuppositions of mathematical natural science. Kant's central contribution to philosophy, Cohen maintains, lay in his locating the precise form of metaphysics involved in natural science and in distinguishing this form from the other forms of metaphysics (9). The great error of the romantics and the post-Kantian idealists is that they did not observe the borders Kant drew between the metaphysics of natural sciences and the more speculative forms of metaphysics connected with religion. So, logic, in Cohen's sense, has nothing to do with Schelling's or Hegel's science of the absolute.

The fourth and final feature of Cohen's logic, which follows from the third, is that logic is anti-psychologistic. Cohen resists any reduction of the forms of logic down to how human beings think about the world or to the mechanisms of the mind. The problem with psychologism, in Cohen's view, is that it gives the forms of logic a purely subjective status,

as if we cannot know from them anything about the world itself. That said, Cohen admits that it is difficult to avoid all talk about psychology in logic; as soon as we talk about the activity of thinking, he concedes, we are referring to psychological events (21). Nevertheless, he insists that we must not allow an interest in such events to distract us from what logic is about, namely, how through such events our thinking acquires a content which is about the world itself.

Although it is far from our contemporary conception of logic, there was nothing new or eccentric about Cohen's conception of logic, which had its historical precedents. The chief precedent for Cohen's conception, which has gone largely unnoticed, was Adolf Trendelenburg's *Logische Untersuchungen*.[2] Trendelenburg was Cohen's teacher when he was a student at the University of Berlin from the Autumn of 1864 until the Autumn of 1865. Although Cohen would take issue with his teacher during his famous dispute with Kuno Fischer about the purely subjective status of Kant's forms of space, he still learned much from Trendelenburg, and not the least of these lessons concerned logic.

Three of the salient features of Cohen's logic—its epistemological, non-formal and metaphysical aspects—have their source in Trendelenburg's *Untersuchungen*. More than a half century before Cohen, Trendelenburg taught that the primary task of logic is to determine the method of the sciences (10). Using a phrase later made famous by Cohen, Trendelenburg said that logic had to begin with the "fact of science", i.e. the fact that sciences like physics had become successful and had become the model for our knowledge of the world (130). Trendelenburg also anticipated Cohen's views about the shortcomings of formal logic. Formal logic is bankrupt, Trendelenburg argued, because it does not address the method of thinking in the sciences (19–20). Finally, Trendelenburg also stressed the metaphysical dimension of logic. All the particular sciences have their metaphysical principles and presuppositions, he insisted, and the task of logic is to bring them to self-consciousness. Metaphysics concerns the foundations, the ultimate grounds, of each of the particular sciences (7).

Pure Thinking and Kant's New Method of Thought

The problems a contemporary reader will have with Cohen's *Logik der reinen Erkenntniss* do not have to deal simply with his conception of logic. The more basic problem deals with the cardinal doctrines of Cohen's logic, which seem to be a metaphysics on a grand scale. It is Cohen's central teaching in the *Logik der reinen Erkenntniss* that the

2 All references in parentheses are to the first volume of the second edition, *Logische Untersuchungen* (Leipzig: Hirzel, 1862).

method of science consists in what he calls "pure thinking" (*reines Denken*). But for this pure thinking Cohen makes extraordinary claims. Pure thinking, we are told, generates its own content, which is not given to it but produced by it. Furthermore, pure thinking produces the reality of its content, so that existence is not something added to its content but already lies within it. There is no distinction, in other words, between the universal and the particular, between a concept and the instances to which it applies; furthermore, there is no distinction between possibility and reality, between essence and existence. In making such bold claims, Cohen explicitly repudiates Kant's distinctions between understanding and sensibility, between concept and existence, which he regards as damaging to pure thinking (10–11). He is inviting, indeed urging, us to transcend the Kantian limits upon knowledge.

But as if all this were not bad enough, the worst is that Cohen does not explain or justify these steps. He just lays down his conclusions with no attempt to account for them. It seems, then, as if we are dealing with the worst kind of dogmatic metaphysics. In denying Kant's dualisms, Cohen seems to rehabilitate Hegel's doctrine of the concept. It is surely no wonder, then, that some of Cohen's neo-Kantian reviewers saw his logic as a relapse into Hegelian metaphysics.[3] This apparent return to Hegel is deeply ironic, given that Cohen was a neo-Kantian philosopher, and given that neo-Kantianism received its original inspiration in reaffirming Kant's dualisms and in rejecting Hegel's doctrine. Thus, in late Cohen, neo-Kantianism seems to have come full circle, reaffirming the very doctrines that it once repudiated.

So, Cohen's *Logik der reinen Erkenntniss* is an enigma. Whence this doctrine of pure thinking? Why does Cohen repudiate Kant's dualisms? Why does he go back to the rationalist tradition and rehabilitate doctrines that Kant went to such pains to refute? The crucial question to ask here is why Cohen attributes what he calls "autonomy" or "independence" to pure thinking. Why does he have confidence in the power of pure thinking to generate its own content and reality? Though these questions are very basic, the answers to them cannot be readily found in Cohen's book, which simply states his results without explaining how he arrived at them. The answers, if they lie anywhere, are in Cohen's philosophical development, in the history of his thought some 30 years before the publication of the first edition of his book in 1903.

We can best understand what Cohen means by pure thinking if we go back to that philosopher who had the greatest influence on him. That philosopher is, of course, Kant. In his first book on Kant, *Kants Theorie der Erfahrung*, Cohen stressed the central role in Kant's philosophy of

3 See Leonard Nelson, review of Cohen's *Logik der reinen Erkenntniss*, in *Göttingishe gelehrte Anzeigen* 8 (1905), 610–630.

one guiding idea. This idea is what Kant called the principle behind his "new method of thought".[4] According to this principle, we know a priori of things only what we create in them, or "nothing can be ascribed to objects except what the thinking subject takes out of itself". This principle presupposes that our rational activity is perfectly transparent to ourselves, and that it is so because we know entirely and perfectly what we produce. Kant stressed, just as Cohen later will, that this principle is behind the scientific method of Galileo and Newton; it means that we can understand nature only if we come to her with our own questions and force her to answer them.

It is this Kantian principle that inspired Cohen's concept of pure thinking.[5] His pure thinking is nothing more nor less than Kant's concept of a priori thinking, according to which we know of objects only what we produce of them. No less than Kant, Cohen makes this principle the foundation behind modern science. Remarkably enough, this debt to Kant is never made explicit anywhere in the *Logik der reinen Erkenntniss*; but it is implicit everywhere and presupposed all the time. Seeing this Kantian principle behind Cohen's pure thinking helps to make it seem less mystical and metaphysical.

But identifying Cohen's pure thinking with Kant's principle only takes us so far. The problem, as we have already indicated, is that there is a fundamental difference between Kant and Cohen regarding the limits of a priori knowledge. Kant thinks that a priori knowledge is an ideal limited to pure mathematics, and that we depart from this ideal the more we attempt to have knowledge of the world itself. Cohen, however, extends the ideal of a priori knowledge to mathematical physics, and indeed to the entire sphere of knowledge. For him, even knowledge of the world itself should be an instance of a priori knowledge. If Kant held that *all* knowledge were a priori—if he made his ideal of knowledge hold for all knowledge—then there would be, of course, no difference between Kant and Cohen. But Kant, deliberately, never goes as far as Cohen, because he thinks that our knowledge of the world cannot be entirely a priori, that if we are to know what exists we have to rely on experience, which receives the material of knowledge from sensibility. The concepts of our discursive understanding, Kant taught, just do not have the power to generate their own content, which must be given to it from some source outside thinking itself. The knowledge of physics presupposes a source of knowledge outside pure thinking, Kant maintains, and that source lies in our experience, in our sensibility, which receives the matter of sensation. It is, of course, just this Kantian insistence on the given element of

4 KrV, Bxviii. Cf. B xii, xiii.

5 It is more explicit in a passage from Cohen's *Einleitung mit kritischen Nachtrag zur neunten Auflage des Geschichte des Materialismus*, which was first published in 1896. See *Herman Cohen Werke*, ed. Helmut Holzhey (Hildesheim: Olms, 2005), V, 27.

knowledge that Cohen repudiates, and that he regards as a betrayal of the autonomy of pure thinking.

With his doctrine of pure thinking, then, Cohen was both radicalizing and breaking with Kant: radicalizing him, because he wanted to make his paradigm of a priori knowledge the paradigm of all knowledge; and breaking with him, because he insisted on dropping the restrictions involved in making sensibility an independent element of knowledge in addition to pure understanding.

We are now back to our original question, though we can formulate it in more precise Kantian terms. Why radicalize Kant's revolution? Why deny the given element in knowledge, and why affirm the autonomy of pure thinking?

Cohen's concern with Kant's concept of the given goes back to 1871 and the first edition of *Kant's Theorie der Erfahrung*.[6] There Cohen admits that Kant has failed to give a satisfactory answer to Herbart's famous question: "Whence the determinate qualities of determinate things?" (142). Although Cohen himself does not yet have an answer to this question, it is significant that he raises it, and that he thinks Kant's answer to it is inadequate. To explain the origin of the manifold, Kant had postulated his notorious concept of the thing-in-itself. Cohen counters that the thing-in-itself—understood as an entity or thing existing independently of our faculties of knowledge—is a hypostasis, an illusion, even though it is a necessary illusion because we need to explain the origin of our experience (252). More properly understood, the thing-in-itself is not a thing beyond experience, Cohen contends, but a noumenon, i.e. the ideal of a complete explanation of things in experience. For such a noumenon, all knowledge would be a priori, generated by the activity of the understanding itself. Cohen concludes his tract with the declaration that "the secret of idealism" would lie in "resolving the variety of things in the differences of ideas" (270). It is only in the *Logik der reinen Erkenntniss*, of course, that the key to the secret of idealism is finally revealed: namely, pure thinking.

The Infinitesimal

Although Cohen already had, if only inchoately and schematically, the ideal of pure thinking in 1871, he still had no conception of how to get there. He still had to face a very difficult problem: how could idealism explain the manifold? How could it convincingly show that the apparently given and contingent material of experience is in fact produced and necessary, the product of pure a priori thinking? A significant step toward the solution of this problem came in 1883 with the publication

6 All references in parentheses are to the first edition, *Kants Theorie der Erfahrung* (Berlin: Dümmler, 1871).

of Cohen's work *Das Princip der Infinitesimal-Methode und seine Geschichte*.[7] Here Cohen considers the implications of Leibniz's concept of the infinitesimal for philosophy. This concept plays a crucial role in natural science, Cohen assures us, so that it is important that philosophy, as the logic of the sciences, should consider it.

It is noteworthy, however, that, in this work, Cohen still does not have his mature conception of logic. The foundations of the concept of the infinitesimal, Cohen maintains, cannot be found in logic, at least not in any traditional sense. Logic investigates only relations of thought; but the infinitesimal concerns the relationship between thought and sensation, and so falls outside logic proper (2, 3). But because logic in the traditional sense cannot explain or justify this concept, Cohen argues, it is necessary to complement logic with another discipline which considers the relations between thought and sensation and the content of knowledge (1). This discipline is what Cohen first calls *Erkenntnistheorie*; but because he does not like the psychologistic associations of that word, he proposes instead the term *Erkenntniskritik*, which considers not the faculty of knowledge but the principles and presuppositions of scientific knowledge (6, 7).

In *Das Princip der Infinitesimal-Methode*, Cohen takes his starting point to treat the problem of the given from one section of Kant's *Kritik der reinen Vernunft*: the 'Anticipations of Perception'. The Anticipations state in the B edition: "In all appearances sensation, and the reality which corresponds to it in the object, has an intensive magnitude" (B 207). What is so striking about this statement for Cohen is that Kant analyses sensation—the apparently given and unanalysable component of experience—into degrees of intensive magnitude, which we can increase from zero to any given magnitude. This statement was for him a new application of the infinitesimal calculus to experience. Hitherto, that calculus had been applied to the *quantitative* component of experience—to its extensive magnitude—but not to its *qualitative* component—to its intensive magnitude. The Anticipations claims that the most basic *qualitative* dimension of experience—the degree of intensity of a sensation—can be treated, just as much as the quantitative, as the product of intellectual analysis. The Anticipations assume that (1) we can analyse the intensity of a sensation into basic infinitesimal components, which are degrees of intensive magnitude, and that (2) we can reconstruct its given intensity by gradually adding these degrees on a continuous scale. For Cohen, this demonstrated something fundamental about the concept of reality itself: "Reality lies not in the crudity of sensation, nor in the purity of sensible intuition, but it can be made valid through a *special* presupposition of *thought*" (14).

7 Hermann Cohen, *Das Princip der Infinitesimal-Methode und seine Geschichte* (Berlin: Dümmler, 1883). All references here are to page numbers of this edition.

Although the Anticipations were promising, they still went only so far. They held only for the intensive magnitude of a sensation—the degree of its intensity—but not its specific quality, which still seemed primitive and given. The Anticipations analyse the redness of red, the blueness of blue, the greenness of green, but they do not determine how red differs from blue or green. In other words, Kant had still not explained the manifold itself, i.e. the variety of different qualities as they present themselves to the senses. It is necessary, therefore, to take an additional step: to apply the infinitesimal to the variety of sense qualities themselves. But Cohen seems to think that this added step is not a large one, and that it can be achieved on the same basis as the analysis of the intensity of sensations.[8] If each sense quality can be analysed into its specific infinitesimal components, whatever they are, and if it can be generated by some law holding for their interaction, then it too would have its intellectual foundation. In his *Nouveaux Essais*, Leibniz had already proposed an analysis of Locke's secondary qualities—colours, tastes, sounds—along these lines.[9] These qualities would then be nothing more than *phaenomena bene fundata*, i.e. phenomena based on a good foundation, where their foundation consists in their infinitesimal components and laws of interaction.

We can now see what Cohen saw as the significance of Leibniz's concept of the infinitesimal. Leibniz developed this concept to analyse and explain some apparently unanalysable and inexplicable phenomena in physics, viz. tangents to curves, arc lengths, curvature of motions and so on. These phenomena were analysed in terms of precise laws between their elements, and on the basis of these laws the phenomena could be reconstructed or generated. The concept of an infinitesimal seems to have clear applications to other phenomena, not least among them sensation and the primitive qualia of experience. It showed that the apparently unanalysable *qualitative* element of experience could be analysed and reconstructed just as its *quantitative* or *extended* element. Just as one could analyse a curve by continually dividing it into ever smaller extended units, such as squares, so one could analyse a sense quality by analysing it into smaller non-extended units. Leibniz's great step forward in formulating his concept of the infinitesimal, Cohen maintains,[10] was generalizing it, so that it held not only for the realm of the quantitative or extended, but also for the realm of the qualitative or non-extended. It was thus a law of pure thinking, and not only a law for the realm of extension alone. The differential showed that it is possible to

8 See §102 of *Das Princip der Infinitesimal-Methode*, pp. 146–8.

9 Leibniz, *Nouveaux essais sur l'entendement*, in *Die philosophischen Schriften von Gottfried Wilhelm Leibniz*, ed. C.J. Gerhardt (Berlin: Weidmann, 1882), V, 383–5, §7.

10 See §58 of *Das Princip der Infinitesimal-Methode*, pp. 69–73.

conceptualize aspects of experience not formulable in terms of extensive quantity alone, i.e. in terms of length, width or height. Analysis in quantitative terms always left a remainder, namely the element of quality. But the differential entered into that element, analysing it into its elements and the laws of their interaction.

Cohen's affirmation of Leibniz's concept of the infinitesimal meant, of course, the rejection of Kant's dualism between understanding and sensibility. Kant's sensibility with its given and unanalysable qualia meant for Cohen an artificial and arbitrary restriction upon the limits of scientific explanation. Cohen was in effect returning to the rationalist tradition by adopting Leibniz's view that the realm of sensibility consists in confused concepts of the understanding. It is really the spirit of Leibniz who hovers over Cohen's *Logik der reinen Erkenntniss*. Of course, Kant is always in the background, but he is a Kant corrected by Leibniz.

The Ontological Status of the Differential

Whatever the prospects of Leibniz's programme for analysing sense qualities, we are still far here, it seems, from the kind of idealism that Cohen wants. Granted that we can analyse sense qualities into their infinitesimal components, someone might ask, why should this prove idealism? It seems as if these components could be material, so that we could have just as well reached a kind of materialism. This raises the hoary question of the ontological status of the infinitesimal, a question that Cohen considers with great care in both his *Das Princip der Infinitesimal-Methode* and his *Logik der reinen Erkentnis*.[11]

In both works, Cohen is clear and adamant that the infinitesimal cannot be an atom, something material, because nothing material can be infinitely small. The infinitesimal is not a unit of extension, he insists, but a unit of thought (*Denkeinheit*).[12] The law of continuity, on which the concept of the infinitesimal is based, he maintains, is really an *ideal*, a *rule* that nature never violates, and as such it is a natural *law* (55). True to his critical guidelines, Cohen insists that we must not hypostasize the infinitesimal. It is a mistake to ask whether the infinitely small exists, or to assume that there should be some sense experience to confirm it. The infinitesimal, which is the infinitely small, cannot be given in sense perception, simply because anything that we perceive is finite; it also cannot be an entity or thing, because any entity or thing is also finite. Rightly seen, then, the infinitesimal is simply an ideal, a

11 *Das Princip der Infinitesimal-Methode*, pp. 55, 57, 129, 142; *Logik der reinen Erkenntniss*, pp. 108–113.

12 *Das Princip der Infinitesimal-Methode*, p. 142.

methodological rule, which demands that, for anything finite, we never cease to analyse it.

But now, it seems, we have escaped one difficulty only to run into another. Granted that the infinitesimal is only an idea, a unit of thought, how does that explain the *existence* of phenomena, the *reality* of sense qualities in space and time? In other words, how does the existence of real phenomena depend on something that does not even exist? But here we only need to heed the special ontological status of ideas. To say that the infinitesimal is an idea does not mean that it is only a fiction, or that it is only convenient conceptualization, where such a fiction or conceptualization exists only in the mind—for the interrelations of infinitesimals consist in laws which govern phenomena. As a law, the infinitesimal governs and determines the qualities and real relations between things. To be sure, nothing exactly instantiates these laws, which are idealizations, but they are more than fictions and conceptual conventions for the simple reason that through them we can generate or reconstruct the phenomena that we investigate. This shows that these laws have a kind of objective validity, that they govern the phenomena and make them what they are.

In discussing the ontological status of the infinitesimal, Cohen often says that it has a validity or worth (*Geltungswerth*), a concept which he insists we must not confuse with existence. I read "*Geltungswerth*" as an allusion to what had become a common doctrine in late 19th-century logic: namely, Lotze's distinction between existence and truth. Famously, Lotze held that we must distinguish two questions: *Was gibt?* and *Was gilt?* We could not reduce the realm of truth down to existence for the simple reason that many propositions are true even though there is nothing corresponding to them in the realm of existence. This is true of mathematical propositions, of course, but it is also true, almost by definition, of hypothetical and conditional ones. More to the point, it is also true of scientific laws, which can be formulated in a hypothetical form, and which claim to be valid even though nothing in experience ever precisely conforms to them.

In the final section of *Das Princip der Infinitesimal-Methode*, Cohen reaffirms the idealism which he had stated some 12 years earlier at the close of *Kants Theorie der Erfahrung*. The governing idea of idealism, Cohen now states, is "no things other than in thoughts" (126). This was the same thesis he had stated more than a decade ago, only now Cohen had discovered his method for establishing its truth. That method consisted in the infinitesimal, in analysing things into their basic nonmaterial elements and the laws of their interaction. The *sui generis* Kantian realm of sensibility now disappears, Cohen is confident, because we can show that it too is the product of the infinitesimal and its laws (125). This is, of course, a very different kind of idealism from what Kant had intended.

The reality of nature no longer depends on my consciousness—not even consciousness in general—but upon thoughts or laws, which hold even if no one ever thinks of them. This we might call a *nomological* idealism rather than *subjective* idealism.[13] We are still left with the paradox of how the realm of existing phenomena can depend upon pure thoughts and laws, but that, it is fair to say, is the fundamental problem of idealism since Plato, and we should not, at least not now, expect Cohen alone to provide an answer.

13 Cohen thinks that one of the main advantages of his idealism is that it avoids the charge of subjectivism levelled against Kant. See §88 of *Das Prinzip der infinitesimal-Methode*, pp. 125–7.

8 The Logic in Dedekind's Logicism

Erich H. Reck

Any history of logic from Kant to the 20th century needs to take into account the emergence of logicism, since it is largely in this context that logic as we know it became prominent. Usually Gottlob Frege and Bertrand Russell are seen as its two main representatives, especially early on, and the more recent rise of neo-logicism, in writings by Crispin Wright, Bob Hale and others, proceeds along Fregean lines as well. In this chapter, I will focus on Richard Dedekind instead. Today Dedekind is sometimes mentioned in connection with logicism; yet at the end of the 19th century, he was the most prominent logicist, ahead of Frege. This alone invites further reflection. Reconsidering Dedekind also leads naturally to questions about what was, or could be, understood by "logic" in this context. Addressing them will involve developments in 19th-century mathematics that are relevant in two respects: by forming core parts of the background for the emergence of modern logic; and by pointing towards a distinction between two conceptions of logic that deserve more attention, both historically and philosophically.[1]

The chapter proceeds as follows. First, I will document Dedekind's characterization of his project as showing that arithmetic, understood in an inclusive sense, is "part of logic", and I will put that project into the context of broader developments in 19th-century mathematics. In the second section, I will look in more detail at Dedekind's procedure, including the fact that it involves set-theoretic "constructions" and a certain kind of "abstraction". Third, a brief summary of later appropriations and developments of Dedekind's contributions will be provided, especially in axiomatic set theory and category theory, although these are usually not seen as forms of "logicism". This will,

1 An early version of this chapter was presented at McMaster University, Hamilton, Canada, in May 2016. A later version formed the basis for a talk at the CSHPM/SCHPM meeting in Toronto, Canada, May 2017. I would like to thank the audiences at both events for their comments. I would also like to thank Sandra Lapointe for inviting me to them, as well as for comments on the later written version. This chapter builds on Reck (2013a, 2013b); there is also partial overlap with Reck (forthcoming), Reck and Keller (forthcoming), and Ferreirós and Reck (forthcoming).

in the fourth section, provide the background for the question of how Dedekind—and parallel to him, Frege—must have understood "logic" for their logicist projects to make sense, namely in a wider sense than the one dominant in the 20th century. The essay will conclude with some general observations, both about the contested status of the notion of "logic", i.e. a lack of consensus about its nature that is still often underestimated, and about its historical and philosophical relationship to modern mathematics.

Dedekind's Logicism: Programmatic Remarks and Historical Background

The text in which Dedekind's logicism is most explicitly stated is his well-known booklet *Was sind und was sollen die Zahlen?* (Dedekind 1888), which builds on his earlier *Stetigkeit und irrationale Zahlen* (1872). In both texts, he addresses issues concerning the foundations of "arithmetic" understood in a broad sense, from the theory of the natural numbers to traditional algebra and higher analysis, including the real numbers (and even the complex numbers in the end, although this is less explicit). In the Preface to his 1888 text, Dedekind adds programmatically that he will develop "that part of logic which deals with the theory of numbers" (Dedekind 1963, p. 31); this is elaborated further as follows:

> In speaking of arithmetic (algebra, analysis) as part of logic I mean to imply that I consider the number concept entirely independent of the notions of intuition of space and time, that I consider it an immediate result of the laws of thought
>
> (ibid.).

Dedekind never uses the term "logicism" himself (nor does Frege). As such remarks show, his goal is nonetheless to establish that arithmetic is "part of logic", and that involves relying solely on "the laws of thought" while rejecting any dependence on our "intuition of space and time".

There are clear echoes of Kant's philosophy in these remarks. This is so already by how the choice is framed, namely as one between using only logical laws or appealing to intuition as well, and even more, by characterizing the latter in terms of Kantian spatio-temporal intuition. One of the main reasons for Dedekind's refusal to appeal to space and time in this context is more mathematical, however. As he writes,

> It is only through the purely logical process of building up the science of number and by thus acquiring the continuous number domain that we are prepared to accurately investigate our notions of space and time [...]
>
> (*ibid.*, pp. 31–2).

In other words, we need a precise, prior account of the real numbers to understand space and time accurately, not *vice versa*. But what does Dedekind mean by "the purely logical process of building up the science of number"? Also, what are the "laws of thought" on which his approach is to be based, including the philosophical framework in the background, Kantian or otherwise? I will address the former questions first, including putting Dedekind's corresponding contributions in historical context. We will return to the latter question in later sections.

The crucial and arguably most innovative step in Dedekind's "purely logical process of building up the science of number" had already been presented in his 1872 booklet. It consists in the construction, by means of his notion of cut (Dedekind cut), of the real numbers out of the rational numbers. This step was crucial since it provided the missing link in a series of "domain extensions", from the natural numbers through the integers and rationals to the complex numbers. It was also the most innovative step, since all the others can be done in terms of pairs of numbers (or more precisely, equivalence classes of such pairs), e.g. rational numbers can be understood basically as pairs of integers etc. The main model in this connection was W.R. Hamilton's account of the complex numbers in terms of pairs of reals, which built on Gauss' earlier introduction of the complex number plane. Clearly aware of Hamilton's and Gauss' works (Gauss was his dissertation advisor), Dedekind also knew of parallel introduction of the rationals and integers based on the natural numbers.[2] The step from the rationals to the reals is different, because it involves the infinite in a more substantive way (see later).

This process of building up the familiar number domains, or the step-by-step reduction from the complex numbers all the way down to the natural numbers, is often seen as part of the "arithmetization of analysis". Dedekind takes on this perspective as well, e.g. when he talks about showing that "every theorem of algebra and higher analysis, no matter how remote, can be expressed as a theorem about the natural numbers", a project he associated with his mentor Dirichlet (Dedekind 1963, p. 35). In addition, the process is usually seen as driven by the elimination of infinitesimals in the Calculus, i.e. by rethinking the notion of limit in terms of the ε-δ-method, which ultimately relies on a precise, unified account of the rational and real numbers, precisely as provided by Dedekind. However, one can understand the arithmetization project also as part of demonstrating—and this is what Dedekind did, as we saw—that higher analysis is independent of the notions of space and time. Central to this demonstration is replacing the notion of magnitude, or

2 Cf. Sieg and Schlimm (2005), Reck (2016), and Ferreirós and Reck (forthcoming) for more details. Dedekind discussed this series of domain extensions already in Dedekind (1854).

of "measurable quantity", by the concept of real number (which allows for a related account of complex numbers too). And closely related with that replacement is the shift from relying on "intuitive" geometric evidence about magnitudes towards more precise and explicit "conceptual" reasoning (as illustrated, e.g. by Dedekind's proof of the Mean Value Theorem in *Stetigkeit und irrationale Zahlen*).

From a broader perspective, the following three developments in 19th-century mathematics should be seen as crucial elements of the background and motivation for Dedekind's foundational contributions. First, there is the shift away from taking geometry to be the ultimate basis for all of mathematics (a position one might call "geometricism", grounded in traditional geometric constructions and intuitive evidence) towards seeing arithmetic, in the aforementioned inclusive sense, as being independent and more basic (thus "arithmetizing" algebra and analysis).[3] Second, there is the push towards founding reasoning in mathematics, and in arithmetic especially, on explicitly defined concepts and logical derivations from them, as opposed to relying either on intuitive geometric considerations or on "blind calculation" (the adoption of a more "conceptual" methodology, also opposed to certain forms of formalism).[4] Third, there is the move towards reconstructing crucial, and often novel, mathematical entities—not only the real numbers, but also "ideals" in algebraic number theory (another of Dedekind's main contributions), "transfinite numbers" in Georg Cantor's work, and "points at infinity" in projective geometry—by using (what we would call) set-theoretic constructions, often involving infinite sets essentially.[5] All three of these developments were picked up and continued in Dedekind's works, in several cases by providing capstone contributions to the relevant fields. Finally and crucially for our purposes, with the set-theoretic treatment of the natural numbers in his 1888 essay, he pushed them a significant step further, thus in effect basing all "pure mathematics" of his time on "laws of thought" alone.[6]

Seen from this perspective, Dedekind's "logicism" thrives on a unification and systematic extension of broader developments in 19th-century mathematics. Since many mathematicians at the end of the century valued these developments highly, his relevant contributions and

3 In this connection, one can speak of the "birth of pure mathematics, as arithmetic" in the 19th century; cf. the title of, and the further discussion in, Ferreirós (2007).

4 Howard Stein and others have talked about the 19th-century birth of a kind of "conceptual mathematics" in this connection; cf. Stein (1988), also Reck (2013a, 2016).

5 With respect to this third point, seen as originating in the case of geometry, Mark Wilson has talked about the rise of "relative logicism" in the 19th century; cf. Wilson (2010).

6 For more illustrations and a further defense of this perspective, cf. Reck (2013a, 2016).

programmatic remarks did not go unnoticed. Thus, Ernst Schröder, the foremost German member of the Boolean school of algebraic logic, wrote of being tempted to join "those who, like Dedekind, consider arithmetic a branch of logic". Similar remarks can be found in David Hilbert's early works.[7] Within philosophy, the neo-Kantian Ernst Cassirer adopted Dedekind's logicism in the early 20th century.[8] And C.S. Peirce, who did not see himself as part of the logicist camp, acknowledged Dedekind as someone who "holds mathematics to be a branch of logic" (Peirce 1902, p. 32). By contrast, Frege's works were much less widely known at the time, partly because his contributions to mainstream mathematics were more minor. It took Russell's later appropriation of Fregean logicism to bring it to broader attention.

Two Sides of Dedekind's Logicism: Construction and Abstraction

A core ingredient of Frege's and Russell's versions of logicism is their introduction of a logical language and a corresponding formal calculus, i.e. a deductive system for higher-order logic. Each of them presented these aspects, in their respective versions, explicitly and in detail. They also advertised them as crucial advances over the logic of their predecessors, especially traditional Aristotelian logic. Nothing comparable can be found in Dedekind's works. This is one reason why he is sometimes not put on the same level, with respect to either logicism or the rise of mathematical logic. But a second, equally important ingredient of logicism, present also in Frege's and Russell's versions, consists in the introduction of a theory of sets, extensions, or classes. In fact, the use of such a theory lies at the core of the logicist reconstruction of the natural and the real numbers. In light of that fact, it is worth comparing Dedekind's approach to theirs in some detail. In addition, for Dedekind, logicism involves not only certain set-theoretic constructions, but a form of "abstraction" that distinguishes his version sharply from Frege's and Russell's. To get a better sense of the resulting, distinctive logicist position, including what is meant by "logic" in it, both processes have to be taken into account.

What are the main set-theoretic constructions used in Dedekind's logicist project? I mentioned earlier the one central to *Stetigkeit und irrationale Zahlen*, namely the introduction of cuts in the system of rational numbers. A closer look reveals that there are three main steps involved in it. First, Dedekind starts by considering all the rational numbers together,

7 For Hilbert, see Ferreirós (2009); for Schröder and more generally, compare Reck (2013a, 2013b).

8 Cf. Cassirer (1910), especially Ch. II; for further discussion, see Reck and Keller (forthcoming).

seen as an infinite system (an ordered field that contains all the natural numbers). In a second step, he considers cuts on that system in the usual mathematical sense, where each cut consists of two infinite sets. Third, he introduces the system of all such cuts, endows it with an ordering relation and arithmetic operations (induced by those on the rational numbers), and shows that the result is continuous (a line-complete ordered field). What makes the third step especially noteworthy is that it implicitly involves the full power set of the set of the rational numbers (an application of the Power Set Axiom), which leads from a countable to an uncountable set (as Cantor would soon establish). In that respect, the reconstructions of the integers and rational numbers in terms of pairs are less substantive. On the other hand, the latter involve the notion of pair as a basic "logical" ingredient. Such details are worth noting in our context, since they reveal what the notion of "logic" at play involves.[9]

Turning to the treatment of the natural numbers in *Was sind und was sollen die Zahlen?*, which set-theoretic constructions and basic notions play a role in it? Before answering that question, let me make a more general observation that highlights Dedekind's originality. A striking aspect of his 1888 essay is that it starts with a general framework of sets ("Systeme") and functions ("Abbildungen"), both understood as allowing for arbitrary cases (not only sets and functions involving elements, arguments, and values of all kinds, but also non-decidable sets and functions). While Dedekind builds on the generalized notion of function introduced by his mentor Dirichlet a few years earlier, together with the novel use of sets in Cantor's work, this was a radical innovation. In fact, Dedekind seems to have been the first person to propose using such a framework for systematically rethinking the foundations of arithmetic in the inclusive sense, and thus, the foundations for all "pure mathematics" at the time. He was also one of the first to treat sets and functions extensionally (assuming an Axiom of Extensionality for both). Finally, he considered his general notions of set and function, together with the framework to which they belonged, a part of "logic".[10]

Within such a "logical" framework, there are then several important steps in Dedekind's reconstruction of the natural numbers in his 1888 essay. First, he defines what it means for a set to be infinite (Dedekind-infinite, i.e. 1-1 mappable onto a proper subset). Then he defines the notion of a "simple infinity" (basically, the minimal closure of

9 Today we are used—from axiomatic set theory—to reducing the notion of pair to that of set (following Wiener or Kuratowski); but Dedekind does not suggest such a reduction. Nor does he reduce functions to sets (of tuples). More on both aspects later.

10 While not explicit in Dedekind (1888), this is clear from an 1887 draft of it. There he notes that the theory of sets, or of "systems of elements", is "logic"; cf. Ferreirós (1999), p. 225. Here and at various other points, I am strongly indebted to Ferreirós work, including Ferreirós (forthcoming).

a singleton set under a 1-1 function, thereby using a minimality clause equivalent to Peano's induction axiom). Next, the latter notion is shown to provide the basis for a "logical" treatment of mathematical induction (just as Frege had done by using his notion of the "ancestral relation"), and even for an explicit, systematic justification of recursive definitions and inductive proofs much more generally. After that come two core theorems in Dedekind's procedure, concerning (i) the existence of an infinite set, thus also of a simply infinite set, and (ii) the fact that any two simple infinities are isomorphic (so that the notion of simple infinity is categorical). As Dedekind adds, together these justify taking any simply infinite set to "play the role" of the natural numbers, in the sense that "translations" of all theorems concerning the natural numbers will hold for each of them (Dedekind 1963, pp. 95–6).

With respect to his overall procedure, but especially the last few steps just noted, several features call for further comment. The first concerns Dedekind's proof, or attempted proof, of the existence of an infinite set (his Theorem 66). What he appeals to in this connection are the following ingredients: "the totality *S* of all things which can be objects of my thought" (a kind of universal set); a 1-1 function *f* on *S* which maps any element *s* onto "the thought *s*', that *s* can be object of my thought" (serving as a successor function on *S*); and Dedekind's "ego" or "self" (a base element of *S* different from all values of the function *f*). The suggestion is, basically, to start with a distinctive element *a* of *S*, such as Dedekind's "self", and to construct a simple infinity by closing {a} under *f* in *S*. [11] As should be added, this procedure for the case of the natural numbers is parallel to Dedekind's 1872 construction of the system of all cuts in the rational numbers for the introduction of the reals. In the latter case, what we get is the construction of a complete ordered field; here we get the construction of a simply infinite set.

Dedekind's procedure in his 1888 essay, and especially the aspects just highlighted, were seen as problematic from early on. The main reason is that his use of a universal set leads directly to paradoxes (such as Russell's, Burali-Forti's, etc.), as Cantor informed him in the late 1890s. Beyond that, his construction of a simple infinity appears to depend on blatantly "non-mathematical" entities, namely "thoughts" and Dedekind's "self", as various critics complained, starting with Russell in 1903.[12] Dedekind acknowledged the former as a serious problem as soon as he found out about it. But is the latter really as damning as often assumed? Note that it is not hard to substitute Dedekind's original base element by the empty set, Ø, and his original successor function by the function that maps *x*

11 See Dedekind (1963), p. 64. Dedekind's particular choices of base object and successor function were obviously meant to assure that the outcome would be a simple infinity.

12 Cf. Reck (2013b) for further discussion of Dedekind's reception.

onto $\{x\}$, as suggested by Zermelo. Or we can start with Ø and use von Neumann's successor function, $x \rightarrow x \cup \{x\}$, as is standard in axiomatic set theory today. Dedekind himself did not suggest such substitutions. But both would seem to be consistent with his approach, even within what he considered "logic". Then again, even if we allow for them, there is still a question. Namely, what guarantees the existence, not only of the set Ø and each of its successors, but also of the set containing all of them? Dedekind's simple infinity is constructed as a subset of the universal set *S* (implicitly using a Separation Axiom), and the latter is problematic. Moreover, parallel questions arise concerning the existence of various functions and relations used by Dedekind along the way.

Another feature of Dedekind's 1888 procedure that calls for further clarification involves not set-theoretic "construction" but structuralist "abstraction". As noted earlier already, on his account, any simple infinity can "play the role" of the set of natural numbers. However, Dedekind does not leave it at that; he suggests the following further step (Remark 73). Starting with any simple infinity, e.g. one of those mentioned earlier (it doesn't matter which one, since they are all isomorphic), we "entirely neglect the special character of the elements, simply retaining their distinguishability and taking into account only the relations to one another"; that is, we perform an abstraction that "frees the elements of every other content". The result will be "the natural numbers", now understood as a separate, distinguished simple infinity (whose elements are characterized "purely structurally", as one might add). Finally, Dedekind calls the resulting numbers "a free creation of the human mind" (Dedekind 1963, p. 68).

From the standpoint of 20th-century axiomatic set theories (e.g. ZFC, the Zermelo-Fraenkel axioms with the Axiom of Choice), it is tempting to downplay or ignore Dedekind's appeal to "abstraction" and "free creation". It is also true that they are not fully clarified in his 1888 essay. At the same time, a parallel appeal to "creation" (although not yet to "abstraction") occurs in Dedekind's 1872 essay. There too, he does not simple want to have the cuts on the rational numbers to "play the role" of real numbers, as is standard procedure today. Rather, he insists on introducing novel, separate, and "pure" objects determined by them; and again, it is the latter that deserve to be called "the real numbers". Finally, in correspondence from the 1880s, Dedekind insists that his introductions of the real numbers and the natural numbers are meant to involve "abstraction" and "creation" in the same sense.[13]

The present essay is not the place to fully explore what is, or could be, going on in Dedekind's appeal to "abstraction". Nonetheless, this question needs to be addressed at least to some degree, since it concerns

13 Insisting on this point, i.e. on this reading of "Dedekind abstraction" for both his 1872 and 1888 essays, is not uncontroversial. Cf. Reck (2003) for a further defense. In Sieg and Morris (forthcoming), an alternative is presented (one in which Dedekind's 1888 essay is read as closer to set-theoretic practice).

one side of his logicism. In particular, the question needs to be raised in which sense, if any, not just the set-theoretic constructions used by him but also the kind of "Dedekind abstraction" just described could possibly be seen as part of "logic". The latter is even more urgent because Dedekind abstraction was quite unpopular for much of the 20th century. Indeed, it was either dismissed as incoherent (from Russell to Michael Dummett and beyond) or simply ignored (in set-theoretic reconstruction of Dedekind's foundational contributions).[14] Consequently, a discussion of its "logicality" is long overdue.

From Dedekind's Logicism to Axiomatic Set Theory and Category Theory

The most positive and detailed reception of Dedekind's foundational contributions occurred in axiomatic set theory, starting with Ernst Zermelo's work in the early 20th century (Zermelo 1908, etc.). Rather than dismissing Dedekind's approach as based on an inconsistent theory, what Zermelo did was to carefully reconstruct which set-theoretic constructions are involved so as then to reformulate the background requirements as "axioms". I already mentioned three of them parenthetically: the Axiom of Extensionality, the Power Set Axiom, and the Axiom of Separation. Zermelo also turned Dedekind's often vilified argument for the existence of an infinite set into the now standard Axiom of Infinity (even calling it "Dedekind's Axiom"), by using the sequence Ø, {Ø}, {{Ø}}, … instead of Dedekind's original simple infinity. A few further basic steps involved in Dedekind's works, corresponding to Boolean operations on sets (unions, intersections, and set-theoretic differences), can be covered by the Axiom of Separation as well, while for others the Axiom of Unions and the Axiom of Pairing were introduced.

In subsequent works, it was realized that certain steps taken by Dedekind implicitly involve the Axiom of Choice (aspects of his treatment of the notions of "finite" and "infinite") and the Axiom of Replacement (his general treatment of recursion and induction etc.), so that these had to be added as well. On the other hand, the notion of pair does not need to be assumed as basic, as Dedekind had done, since we can reconstruct it set-theoretically (along familiar Kuratowskian lines). And that reconstruction does not just affect the construction of the integers and rational numbers (as well as the complex numbers); it allows for a general reduction of the notion of function to that of set (by considering sets of tuples). Hence, we can work with one basic notion alone, namely that of set. Finally, an Axiom of Foundations can be added, since non-well-founded sets do not play an essential role in classical mathematics. And with this list of axioms in place, Dedekind's "logicist" constructions can

14 For a further discussion of such negative reactions, cf. Reck (2013b).

simply be repeated in axiomatic set theory, while avoiding antinomies such as Russell's, Burali-Forti's, etc.

But does this rehabilitate Dedekind's logicism? It clearly saves (almost all of) his technical results, and fairly directly so. Hence it allows for a "reduction" of all the pure mathematics with which he dealt to axiomatic set theory. Note also the following remark by Zermelo, which most likely reflects Dedekind's influence as well:

> Set theory is that branch of mathematics whose task is to investigate mathematically the fundamental notions of 'number', 'order', and 'function', taking them in their pristine form, and to develop thereby the logical foundations of all of arithmetic and analysis
>
> (Zermelo 1908, p. 200).

While Zermelo speaks of the "logical foundations" of mathematics in this passage, usually axiomatic set theory is not considered a form of logicism; similarly, Zermelo is not counted as a logicist. This is for several related reasons. First, the notion of set codified in its axioms tends not to be seen as purely "logical", e.g. because it involves the intuition behind the standard "cumulative" conception of set.[15] Second, set theory is usually taken to be another mathematical theory (as Zermelo does in the previous quote, too) as opposed to a separate "logical" basis, even though it is fundamental insofar as we can use it to interpret other mathematical theories within it. Third and perhaps most importantly, the existence claims crucial for set theory are no longer seen as substantive "logical truths" today, but as captured in terms of "axioms" understood in a sophisticated "formalist" way (along Hilbertian lines). And as a consequence, Zermelo's reconstruction of Dedekind's approach is taken not as rehabilitating logicism, but as reconceiving them in terms of formalism. Having said that, it is not clear that ZFC has to be understood formalistically.[16]

A second 20th-century approach that allows for a rehabilitation of Dedekind's technical results is provided by category theory. In certain respects, this is actually closer to Dedekind's original procedure, e.g. by taking the notion of pair as basic (in terms of the notion of Cartesian product) and by not reducing the notion of function to that of set (indeed, by treating functions as more basic than sets). Moreover, Dedekind's

15 In addition, the "unconstrained" and "non-conceptual" nature of the Power Set Axiom may be seen as rendering it "non-logical". Then again, it would be interesting to explore if adopting Gödel's "V=L" could be taken to restore the "logicality" of the theory. Gödel did, after all, arrive at his constructive universe by starting from Russell's logicist type theory.

16 Note also that axiomatic set theory is often grouped together with proof theory, model theory, etc., under the label "mathematical logic", thereby using some general sense of "logic".

approach to induction and recursion, thus his treatment of the natural numbers, finds an elegant and fruitful home in this context (via the notion of universal mapping properties, etc.).[17] Yet like in the case of axiomatic set theory, category-theoretic approaches tend not to be seen as forms of "logicism". Typically they are understood along Hilbertian formalist lines as well, e.g. by assuming different category-theoretic axioms as basic for different purposes and treating them formalistically.[18]

There is a certain view of "logic" that typically goes together with the adoption of formalism along such lines, a view that became widely accepted in the 20th century (but arguably has roots in Kant too). According to this view, what characterizes "logical truths" is that they are true "in all domains"—they are something like "tautologies" in Wittgenstein's sense, "analytic truths" in Carnap's sense, or "true in all models" in Tarski's sense. An immediate consequence of this view is that logic, by itself, cannot prove or otherwise justify any existence claims, including mathematical existence claims.[19] Instead, it is precisely "axioms" understood formalistically that are used for this purpose (axioms for whose consistency we can, within limits, argue in meta-logical ways, i.e. model-theoretically, proof-theoretically, etc.).

While this view is widespread, the following observation should be added. Along such lines, logicism turns out to be simply a nonstarter—not just the logicism of Dedekind, but also Frege's logicism and, say, the neo-logicism of Wright, Hale, etc.[20] Has logicism then been undermined decisively? Only if the notion of "logic" just described, together with some version of formalism, is inevitable. More basically, what this line of thought indicates is that Dedekind, Frege, and related thinkers worked with a different, less restrictive notion of "logic". And that notion remains in need of clarification, both historically and systematically.

Dedekind and Frege on Basic Logical Notions and Logical Laws

Let me restrict our discussion to the logicists Dedekind and Frege at this point, so as not to make things overly complicated. As has often been acknowledged, Frege was neither fully explicit nor entirely clear in his

17 Cf. McLarty (1993), or more generally the literature referred to in it.

18 Then again, various versions of category theory can be reconstructed in "mathematical logic" too, e.g., by framing them type-theoretically.

19 Or in Kant's earlier terms, "logic" (in the sense of "general logic", as opposed to "transcendental logic") does not involve any reference to specific objects, or indeed, to objects at all.

20 Insofar as Russell's mature logicism involves a "no-classes" theory of classes, and hence, an elimination of them as logical objects, his case is more complicated (also by involving the status of the Axiom of Reducibility and the Axiom of Infinity).

writings about what makes a notion or truth "logical". But roughly, it is the "generality" of logic that seems crucial for him. Logical notions are those used in all reasoning (or perhaps all "exact" reasoning); and logical truths are those that "govern" all objects, concepts, and functions. The contrast is with the notions and truths of the special sciences. For example, the truths of geometry "govern" geometric entities, such as points, lines, etc., and only those; similarly, the truths of mechanics "govern" physical entities and processes. The truths of logic, on the other hand, are not restricted that way. Again, logic provides a framework for all (exact) reasoning, in terms of its basic notions and its basic laws.

Dedekind is as sparse as Frege, or more so, in his discussion of what makes a notion or a truth "logical". But he seems to conceive of them in a way not too different from Frege, in several respects. First, for both the basic notions of logic include those of function and set/class (even if the two thinkers do not spell them out in exactly the same way). Second, for Dedekind, like for Frege, logic "governs" all reasoning, in particular all reasoning in mathematics. In fact, without logic, such reasoning would be impossible for him. Put differently, logical notions and truths are seen as indispensable by him. Interestingly, Dedekind's most explicit statement along such lines concerns the notion of function; as he writes:

> If we scrutinize closely what is done in counting a set or a number of things, we are led to consider the ability of the mind to relate things to things, to let a thing correspond to a thing, or to represent a thing by a thing, an ability *without which no thinking is possible*. Upon this unique and therefore absolutely indispensable foundation the whole science of number must, in my opinion, be established
>
> (Dedekind 1963, p. 32, my emphasis).[21]

With respect to sets, or "systems", Dedekind is less emphatic; he just notes:

> It *very frequently happens* that different things a, b, c, … can be considered from a common point of view […] and we say that they form a system S […]
>
> (*ibid*., my emphasis).

21 A similar remark occurs in Dedekind's well-known letter to Keferstein, where he characterizes the core of his project in the 1888 essays as follows:

> What are the mutually independent properties of the sequence of numbers *N*, that is, those properties that are not derivable from one another but from which all others follow? And how should we divest these properties of their specific arithmetic character so that they are subsumed under more general concepts and under activities of the understanding, without which no thinking is possible at all […].
>
> (Dedekind 1890, pp. 99–100)

Then again, the notion of set/class is so closely intertwined with the notion of function for Dedekind (e.g. both the domain and the range of functions are sets for him) that the latter is indispensable by implication as well, as one might assume.

Frege would, of course, not want to talk about "the mind" in connection with logic, since this invites psychologistic confusions. Yet one does not have to understand Dedekind's appeal to the "mind" in a subjectivist, individualistic sense, the sense Frege would find especially objectionable. To bring the two even closer together, one might also replace Dedekind's appeal to "thinking" by a more objective conception of "thought" (as Frege himself suggested). Furthermore, both thinkers work within a Kantian epistemological framework, even if they disagree with Kant about how arithmetic fits into it.[22] In fact, in Dedekind's case, the appeal to "abilities of the mind" might be seen as pointing towards Kant's "categories of the understanding", in the sense that the notion of function should count as such a category. "Logic" is then the discipline that deals with presuppositions for all thinking; and the ability to "think functionally" is an important, so far underemphasized example.[23]

In any case, for Dedekind, and in a related way for Frege, "logic" includes a general framework of functions and sets/classes. Within such a framework, we can reconstruct the basic notions and principles of arithmetic in the broad sense, thus all the "pure mathematics" of the late 19th century. And we can do so without appealing to intuitive considerations in the traditional geometric sense, as Dedekind insisted in his two foundational essays and as was crucial for Frege too. Put in more traditional terminology, what "logic" provides in our context is the framework in which we can reconstruct the *logos* of pure mathematics—its basic notions and laws—in an explicit, systematic way.[24] So much seems clear about Dedekind. Less clear is what exactly the relevant logical principles are supposed to be. One would expect him to have made explicit his basic "laws of thought", not just his basic "logical notions". But that is not the case, i.e. he never formulated such laws explicitly. In that sense, both the constructions and the abstraction in Dedekind's writings remain without precise backing.

22 In my account, the parallels between Frege's and Dedekind's logicisms are emphasized, i.e. their two perspectives are assimilated, like on this point. For an interesting approach that highlights the differences much more, cf. Benise-Sinaceur et al. (2015). I plan to respond to the latter in a future publication.

23 Cf. Klev (2017) for a recent interpretation of Dedekind's conception of logic along Kantian lines. For a related reading that is more neo-Kantian, cf. Reck and Keller (forthcoming). Note that along both lines "logic" includes Kantian "transcendental logic", not just his "general logic".

24 With respect to this appeal to "logos", I have in mind Marburg Neo-Kantianism as exemplified by Cassirer and his two teachers, Hermann Cohen and Paul Natorp; cf. again Reck and Keller (forthcoming).

This lack of explicit basic laws was Frege's main complaint about Dedekind's logicist project, especially in Frege's *Grundgesetze der Arithmetik, Vol. I* (1893).[25] And as already acknowledged, Dedekind did not specify a logical language and a related deductive calculus either. Concerning the former, in hindsight it seems natural to reconstruct his approach either in set-theoretic language or, more directly (since functions are basic for him), in type-theoretic language, and particularly, in the language of a simple theory of types (because of his extensional conception of sets and functions). Now, earlier we considered Zermelo's reconstruction of the basic construction principles needed for Dedekind's purposes in terms of set-theoretic axioms. In contrast, all one can find in Dedekind's own writings is the use of a general comprehension principle for sets; or better, he seems to assume a universal set together with a general principle for forming subsets (a general separation principle).[26] But even that much has to be reconstructed from his general procedures, i.e. he does not make such laws explicit himself.

Why did Dedekind not formulate basic "logical laws" explicitly? It is hard to be sure. Perhaps he was simply the first to work with a general framework of sets and functions for foundational purposes, so there was no precedent for it. (Boole and his followers had formulated some laws for classes, but in a more restricted way and not for foundational purposes.[27] Frege's relevant work was slightly later.) Or Dedekind assumed an older conception of "logic", one according to which only the special sciences have "basic laws" or "axioms" while logic does not.[28] Beyond that, why did he consider his project to be "logical" even though it involved existential claims? Here a comparison to Frege is helpful again. Why did Frege considered his "Basic Law V" for classes (or value ranges) to be logical? He saw it as a "conceptual" and, therefore, "logical" truth (as opposed to an intuition- or perception-based geometric or scientific truth), it seems. Dedekind might well have assumed the same. Or again, we could take his talk of "abilities of the mind" very seriously and try to spell it out along Kantian lines, as indicated earlier.

So much for the "logical" constructions involved in Dedekind's approach. Earlier, I pointed out that a form of "abstraction" plays a core role for him as well. Here too one might, especially after Frege's criticisms, have expected the formulation of basic principles by him. But once again, Dedekind did not make explicit such principles; he only gave

25 Cf. Reck (forthcoming) for a further discussion of Frege's relation to Dedekind.

26 In Ferreirós (1999), a closely related "dichotomy principle" for sets is attributed to Dedekind.

27 Once he became aware of it, Dedekind was very interested in Schröder's work on logic, which remained in the Boolean tradition but also incorporated Dedekindian techniques.

28 Cf. Ferreirós (forthcoming) for this second suggestion.

some related hints (about "neglecting the special character of objects", "only retaining their distinguishability", etc.). To reconstruct Dedekind's logicism fully, we would thus have to add "abstraction principles" for him as well. Here again, Frege's approach, or a Fregean neo-logicist approach, might be compared profitably, especially with respect to the form of such principles.[29] Yet doing so still leaves us with the question of why we should consider those principles to be "logical".[30] Perhaps they too have to be seen as general "conceptual" truths (as opposed to intuition- or perception-based truths). But all of this is clearly in need of further clarification. In other words, many questions remain about the notion of "logic" involved.[31]

Conclusion: The Contested Notion of "Logic" and Its Mathematical Background

I assume contemporary readers will find it most natural to reconstruct Dedekind's foundational contributions along set-theoretic lines, or perhaps along category-theoretic lines, and in either case, ultimately in "formalist" terms. This is tempting especially if one rejects that "logic" can underwrite any existence assumptions, which have to be supplied by formalistically understood "axioms" instead. Yet Dedekind presented his project as one of showing that arithmetic is "part of logic". Moreover, several of his contemporaries, both on the mathematical side (Schröder, Hilbert) and on the philosophical side (Cassirer, Peirce), followed him in that characterization, at least initially. Similarly, Frege and Dedekind took themselves to be involved in parallel projects, even if Dedekind was not explicit enough about his basic laws, as Frege complained. For both Dedekind and Frege, the needed construction and abstraction principles were meant to be "logical" in the end; and for both, they involved general notions of function and set/class.

A broad, inclusive notion of "logic" is at play in both Dedekind's and Frege's forms of logicism, as I have argued. But this notion is not only hard to reconstruct in its details, it is also contested. And it is clearly in conflict with a notion according to which only statements "true in all domains" count as "logical", so that existence claims are ruled out from the start. Partly because of the antinomies of set theory, partly

29 Cf. Linnebo and Pettigrew (2014) for the kind of principles I have in mind. Note that they are modeled on neo-logicist abstraction principles (but with a "structuralist" twist). This may suggest ways of thinking about them as "logical", similar to Frege's attitude about his Basic Law V.

30 Cf. the discussion of "Hume's Principle" as "quasi-definitional" in neo-logicism.

31 A different argument might be that, since Dedekind abstraction is "structuralist", it involves a form of "permutation invariance" that qualifies it as "logical" (along Tarskian lines).

also because of the rise of formalism (as tied to Hilbertian axiomatics), that notion of logic became dominant in the 20th century. Yet from a historical point of view such a shift constitutes a narrowing of the sense of "logic". If we accept this narrowing, only the truths of first-order logic, or perhaps those of simple type theory, qualify in the end. All "logic" itself can provide, then, is a deductive framework for the mathematical sciences, while any existential assumptions have to be added as non-logical "axioms". One thing reconsidering Dedekind's logicism can show is that this view was not always taken for granted, i.e. it can remind us of an older, broader sense of "logic".

"Logic" in this broader sense grew out of certain mathematical developments in the 19th century, especially the switch from taking geometry as the basis for all of mathematics (Euclidean "geometricism"), to rethinking all "pure mathematics" first in "arithmetic" terms (the "arithmetization" of analysis and algebra) and then in purely "logical" terms. Both Frege and Dedekind were part of that movement. Frege's contributions have impacted philosophy more, especially in the analytic tradition, while Dedekind's are more relevant for mainstream mathematics, as his influence on axiomatic set theory and category theory illustrates.[32] In any case, it would be an impoverishment, both historically and philosophically, to forget the "logical" origins of such 20th-century theories, even if they are understood "formalistically" today. Finally, the question of how exactly either of them goes beyond "logic" seems worth reconsidering. For all we know, the described narrowing of our understanding of "logic", hidden as they are from the usual ahistorical perspectives, might become subject to change again.

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32 See also the recent rise of Homotopy Type Theory and the project of Univalent Foundations, which might be seen other successors of Dedekind's (and partly Frege's) "logicist" project.

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9 What Russell Meant When He Called Moore a Logician

Consuelo Preti

On September 11, 1898, G.E. Moore wrote to Bertrand Russell to give him a synopsis of the views he had developed in his Trinity Fellowship dissertation, submitted the month before.[1] Moore's 1898 dissertation was an analysis of Kant's arguments on freedom and on reason, a revision of the version Moore had submitted in 1897, which had failed to win him a fellowship. Chapter II of the 1898 dissertation included a formulation of the nature of judgment described by Moore to his friend Desmond MacCarthy as "a perfectly staggering doctrine."[2] As Moore explained to Russell,

> I carefully state that a proposition is not to be understood as any thought or words, but the concepts + their relation of which we think. It is only propositions in this sense, which can be true, and from which inference can be made. Truth therefore does not depend on any relation between ideas and reality, nor even between concepts and reality, but is an inherent property of the whole formed by certain concepts <that> stand in a specific relation to the concept of existence…There would need, I think, to be several kinds of ultimate relation between concepts—each, of course, necessary.

Upon receipt of this letter, Russell replied, "I agree most emphatically with what you say about the several kinds of necessary relations among concepts, and I think their discovery is the true business of Logic (or Meta[physics] if you like)."[3] Once he'd digested Moore's complete dissertation (he didn't read it in full until November 1898),[4] Russell went on to press a series of debate points with Moore, writing to him that "when I see you, I should like to discuss some difficulties which occur in working out your theory of Logic." A few years later, in correspondence with Couturat (May 5, 1900), in advance of the International Congress

1 The Trinity College Fellowship was a six-year post which carried no teaching responsibilities or residence requirements.
2 Moore to MacCarthy, August 14, 1898 (Add. 8330 2/5/6).
3 September 13, 1898 (McMaster Russell Archives (710.052981).
4 Russell to Moore, December 1, 1898 (Add. 8330 8R/33/10).

in Paris that had such a radical effect on the development of his ideas, Russell reported that "I see that I have chosen a topic that cannot be treated briefly, largely because my arguments depend in part upon a new logic (*vide* Moore, 'The Nature of Judgment,' Mind April 1899)."[5] By July 1900, in correspondence again with Couturat, Russell claimed "My friend G.E. Moore is, in my opinion, the most subtle in pure logic."

Now what could Russell have meant by all this? How exactly could Russell have called Moore a *logician*? The main features of the story I want to tell are as follows: (1) that an amalgam of logic, psychology, and metaphysics made up the study of philosophy during this period, and that the way these notions were understood at this period needs to be untangled; (2) that the way to understand how Moore can be construed as a logician is to understand how Bradley could be construed as a logician; but (3) that how Bradley can be construed as a logician has its roots in how Kant can be construed as a logician. Kantian conceptions of logic, 19th-century reactions to it, and, in particular, how Kant was read at Cambridge are key elements of the intellectual reconstruction of Moore's thought at this period, and what I will mainly focus on below.

Philosophy at Cambridge in the 19th Century

We need to set the historical/conceptual scene in order to understand the conception of logic that was dominant in late 19th-century Cambridge. Philosophy at Cambridge was long known as moral science. The inauguration of the Moral Sciences Tripos (first held in 1850) had its basis in 19th-century university reform controversies. One important dispute centered on the mathematics to be taught at Cambridge. Continental developments in the calculus, partial to the analytical, Leibnizian notation, were ignored and derided at Cambridge for decades, in favor of local loyalty to Newtonian fluxions. This led to a serious decline in Cambridge mathematical study, later bemoaned by Russell in his complaint about the "definitely bad" quality of the mathematics which comprised his Mathematical Tripos Part I. (Russell 1959a, p. 38). Where Cambridge mathematics went wrong, however, was not echoed in philosophy at Cambridge. Though continental headway in mathematics was ignored in Cambridge for some time, not every continental development was entirely disdained there. In particular, new directions in the science of psychology played a major role in the study of philosophy.

The Moral Sciences Tripos had at first consisted of moral philosophy, logic, history, political economy, general jurisprudence, and the laws of England. According to Sidgwick's summary of the state of philosophy at Cambridge (1876), this was because it was formed on the basis of subjects "in which the University happened to possess Professors," and not the more natural divisions by which philosophy then was studied. Moral

5 Griffin (1992, 191).

science, Sidgwick claimed, should more naturally consist of logic, metaphysics, and psychology (Sidgwick 1876, 242). Indeed, by the late 1890s (when Russell and Moore sat their exams), the Moral Sciences Tripos had been divided into two parts. Part I consisted of (I) Psychology, (II) Logic and Methodology, and (III) Political Economy.[6] The Part II Tripos consisted of a number of complex options. The regulations required that "every candidate shall be examined *either* in Ethical and Metaphysical Philosophy *or* in Ethical and Political Philosophy, also in one or two but not more than two of the four special subjects." The compulsory subject Ethics and Metaphysics was divided into two sections. In Metaphysics (I(a)), there were six sections: (I) Knowledge, analysis, and general characteristics; (II) Fundamental forms of the object of knowledge; (III) Certainty; (IV) Criteria applicable to special kinds of knowledge; (V) Sources and limits of knowledge; and (VI) Coordination of knowledge. For the Ethics part, there were four sections: (I) Analysis of the moral consciousness;[7] (II) The end or ends of rational action; (III) Exposition and classification of particular duties and transgressions; and—notably—(IV) Relation of Ethics to Metaphysics, Psychology, Sociology, and Politics.[8]

There is a welter of archival evidence that shows that Moore's undergraduate Tripos preparation included exposure to the psychology prevailing in Cambridge (and elsewhere) at this period.[9] Thus Moore's intellectual environment was characterized by the attempts of thinkers of the day to developing a modern conception of the relation between thought and reality—one that importantly included a deepening awareness of a gap between the logical and metaphysical properties of mental states and their purely psychological/subjective ones.[10] Highly visible during this period were Moore's and Russell's teachers G.F. Stout, James Ward, Henry Sidgwick and John Ellis McTaggart, all of whom participated in spirited debates (often spanning multiple issues of *Mind*) with each other, with F.H. Bradley at Oxford, and with the American and continental psychologists, on concerns that crosscut psychology and

6 Among the readings recommended for the Part I.1 (Psychology) part of the examination were Ward's 1886 article "Psychology," a *locus classicus* for decades; Hermann Lotze's *Microcosmus*, vol. I (1897).; and Bradley's *Ethics* (1876), *Principles of Logic* (1883), and *Appearance and Reality* (1893). Herbart was included, too.

7 Analysis of the moral consciousness included, among other things, moral perception, moral judgment, object of the moral faculty, and freedom of the will.

8 The special subjects in 1894 and 1896 were History of Philosophy, Advanced Psychology and Psychophysics, Advanced Logic and Methodology, or Advanced Political Economy. Russell and Moore both chose History of Philosophy for their respective special subjects. In 1894, the examination in History of Philosophy was Bacon to Descartes; in 1896, it was The Philosophy of Hegel. Russell did the former; Moore, the latter (CUC, 1896).

9 And direct evidence from Russell: 'What Shall I Read?' (*CPBR*, Vol. 1; p. 345–365).

10 Thus Dummett (1993, 1) was flat-out wrong when he claimed, with respect to the origins of analytic philosophy in Austria and Germany, that "Russell and Moore sprang from a different *milieu*."

logic, logic and metaphysics, and metaphysics and psychology.[11] But to us, Moore's early philosophical background looks like a set of quite incongruent views: Bradley's Absolute Idealism, McTaggart's idiosyncratic Hegelianism, Sidgwick's hedonism, Kant's ethics, and the mental science of Stout and Ward. It is anything but obvious how these coalesced into a position so influential that it sent Russell into somersaults of acclaim—and changed the history of philosophy in the 20th century.[12]

There are two significant historical developments that I would argue are an important part of an account of this period. The first is the so-called *Psychologismusstreit*—a quite virulent dispute between (so-called) psychologist and anti-psychologist views that swept through nearly every academic discipline throughout Germany and Austria, migrating to Great Britain and beyond. The other was the inauguration of physiological or laboratory psychology.[13] Both of these developments lie behind the kind of psychology—the mental science—that had developed on the continent and then prevailed in the study of philosophy (moral science) at Cambridge in the late 19th century.[14] It is probably clear by now that disciplinary turbulence was the order of the day during this period. In particular, the expressions "logic," "psychology," and "metaphysics" could mean just about anything depending on who employed them; what country they were in; what interests they had; how little they took account of other thinkers' work; and how confused they were accused of being. No contemporary understanding of these subjects will serve to explain how Moore, his contemporaries, or his teachers and examiners will have understood them. So we need to sort out the mess of terminological ambiguity.

11 Stout and Ward were not only deeply familiar with the all of continental turmoil in their field but were leading and influential players in its discussion and dissemination in Britain. Stout was the leading conduit for Austrian mental science via his own work (Stout 1896) and also through his editorship of *Mind*. Ward's seminal (1886) article was a *locus classicus* for decades as a then contemporary formulation of the nature of psychology.

12 Russell (1903, xviii); (1975, 61; 70; 146); 1959a; 1959b.

13 I will say less about that here, except to say that the philosophers at Cambridge were not inclined to consider this development as sufficiently philosophical for the claims they wanted to defend about the mind and its objects.

14 An important source of evidence for the reconstruction of the intellectual environment at this period are the journals. *Mind* was among the first of these for the Anglophone thinker: it was founded in 1876 explicitly as "[T]he first English-language journal devoted to Philosophy and Psychology" and was a vital hub for contemporary thinking in philosophy and psychology on the continent, in Britain, and elsewhere. Digital search functions now possible reveal how much of *Mind* was (unsurprisingly) peppered thickly with debates and inquiries into (among other things) the nature and processes of mind; validity and truth; subject of thought and object of thought; act of mind and object of thought; and the customary divisions of traditional logic (concepts, inference, and judgment). These debates display the attempts on the part of the thinkers of the day to grapple with a more modern, developing conception of the relation between thought and reality.

Nineteenth-Century Psychology, Logic and Metaphysics

We should start briefly with the so-called *Psychologismusstreit*.[15] This was a dispute between psychologism and anti-psychologism that cut across nearly every intellectual discipline of the day. It has received less attention that it could have in a complete account of the origins of analytic philosophy at Cambridge, most likely because even a rough look must contend with the fact that "psychologism" has a spate of definitions, not all of them cohesive.[16] Kusch (1995, 119–121) comprehensively catalogues a set of definitions and criteria in the literature of the day, among which are the following: any philosophical view that regarded philosophy as applied psychology; advocated subjectivism and relativism; conflated genesis and validity; or that combined psychology and empiricism. There were metaphysical, ontological, epistemological, logical, ethical, aesthetic, and mathematical varieties of psychologism (1995, 108). I want to argue here that the intellectual context in which Moore formulated his views can be best understood by fitting it under the theoretical canopy of the *Psychologismusstreit*, with a particular emphasis on the disputes characteristic of this period on the formulations of subjectivity and objectivity in the study of cognition and judgment. This, I also believe, will help to understand how Kant was read in Cambridge, will explain the evolution of Moore's views via his postgraduate work on Kant's ethics, and will explain how Russell could have called Moore a logician.[17]

15 See Kusch (1995) for the definitive detail.

16 Kusch (1995) provides a set of tables documenting the number of accusations across schools of thought or individual thinkers.

17 I think this line of reconstruction helps to put a bit of an end to the usual story of Frege's influence on analytic philosophy and the (common) attribution to Frege of a groundbreaking anti-psychologism in logic. In fact, Frege's own views were part of the *Psychologismusstreit*; the intellectual context in which the *Psychologismusstreit* played out in the 19th century incorporated Frege's work but was broader than his mathematical logic. It must be said that the role of Frege's work in the specifically Cantabrigian origins of analytic philosophy is, in effect, nil. It was not discussed in the mental science literature most likely to have been a direct influence on Moore. References to Frege in the philosophy journals amount to 14 citations between 1879 and 1900, for instance. None include extended discussion. "On Concept and Object" and "On Sense and Reference" are given a one-sentence summary in *Mind* in 1892 and noted as having appeared in two German journals (literature reviews of the contents of other journals was then a common practice). The mathematical training that Russell received at Cambridge, as we know, did not include any notice of Frege's work. Frege was quite isolated at Jena, and his contributions were unheeded or downright derided by his peers (Schroeder (1898) is particularly scathing, for instance). Frege was aware of the difficulties: in the preface to his *Basic Laws of Arithmetic* (Ebert and Rossberg (2013, xiii), he wrote morosely, "the prospects for my book are dim. In any case we must give up on those mathematicians who, encountering logical expressions like 'concept,' 'relation,' 'judgement,' think: *metaphysica sunt, non leguntur!* and also those philosophers who, sighting a formula, cry out: *mathematica sunt, non leguntur*!"

The mental science—that amalgam of logic, psychology, and metaphysics—that shaped the British intellectual scene came from two different sources: (i) the Germanophone tradition, which, given the mid-century discoveries in physiology and allied sciences, had begun to get out from under the legacy of Kantian and Hegelian metaphysics; and (ii) the British tradition, steeped in but officially rejecting the associationist views of the classical empiricists (as well as those of 19th-century empiricists like Mill). That is, a traditional element of philosophical enquiry was, of course, the nature of thought: mind, reason, knowledge, etc. Logic—understood as what characterizes thought—was itself divided into three traditional categories: concepts, inferences, and judgment. Components of thought included ideas and presentations. For the mental scientists at Cambridge, the conceptual crisis inaugurated by the *Psychologismusstreit* was deeply felt, amplified by the threat coming from the laboratory "to which the name Psycho-Physical Materialism is given" (Ward 1893, 54). What Stout and Ward did, I would argue, was to carve out an understanding of "psychology" that would operate as a transition between 19th-century mental science and 20th-century philosophy of mind: an anti-subjective, systematic, scientific taxonomy of the nature of subjectivity.

Psychology at Cambridge was to be understood thus as an "empirical" psychology, whose inspiration was Brentano,[18] in that it proceeded by scientific method: analysis of the contents of mind via a set of systematic necessary and sufficient conditions.[19] It was not "modern" psychology, which to Stout and Ward meant "physiological" psychology.[20] Nor was it the "psychology" of the classical or then contemporary empiricists, which Stout and Ward understood as (and further dismissed as) suspect *epistemology*. For Stout and Ward, a scientific psychology was one that took the realm of subjectivity as a proper concern of scientific enquiry but was employed to give an account of the features of subjectivity *common* to all subjects. "Psychological" analysis was what the psychological observer could objectively attribute to the "psychical," a term that described the immediate experience of the subject observed,[21] but *their* psychology was *science*, in that it tended toward an objective examination of subjectivity *itself*. Stout and Ward conceived of the nature of subjectivity as essentially directed onto its objects: subjectivity as inextricably *relational*. In short, psychology at Cambridge was characterized

18 Who himself called it 'descriptive' in contrast to 'genetic' (Brentano, 1874).

19 So a "scientific" psychology did not necessarily mean empirical in the sense of physiological: Brentano's was empirical, and it was scientific, but it was not physiological.

20 Ward (1893).

21 Moreover, the psychologist can make a distinction between, for instance, subject and object, which is not necessarily something the psychological subject can apprehend for himself (Stout, 1926, 28).

by disciplinary attention to the distinction between the nature of subjectivity treated scientifically (and as a source of objective knowledge *about the mind*), in contrast to both the associative and the synthesizing role of the thinking subject with respect to reality and truth in pre-19th-century epistemology and metaphysics.

Stout and Ward also took their notion of psychology, as a matter of disciplinary principle, not to imply commitment to any particular metaphysical view, and indeed they assumed that they could maintain its independence from any such view.[22] The priority for both of them was: (I) to defend a conception of philosophical psychology against the encroaching physiological methods that were transforming psychology into a lab science on the continent; and (ii) to (at the same time) formulate a properly systematic, scientific psychology, entirely independent of metaphysical and epistemological considerations they associated with traditional philosophy.[23] And this makes sense in the context of the study of mind that they had inherited. One issue was: how to account for knowledge, when, inevitably, its origin is in subjective processes of mind. Another issue was connected with the traditional opposition of the sensible faculties and the reasoning faculties, which entailed a number of attendant epistemological and metaphysical conclusions that were to be rejected at Cambridge. One of those was that reason or thought had special powers of transcendence, synthesis, and unification, all of which permitted its penetration of and grasp of the ultimate nature of reality, which was itself logical (in the sense of necessary, universal, unified, unconditioned one).[24] Thus, for the 19th-century psychologist (mental scientist) at Cambridge, the pressure was on to give a legitimately scientific formulation of mind and thought—and one that was independent of any particular philosophical entailments that could be drawn from it.[25] The one Stout and Ward defended, as we alluded to above, was a relational one that sidestepped (i) any particular metaphysical commitment to the object side of the relation but which (ii) implicitly defended an objectivist account of the relationality.

Now, a central element here is that Stout and Ward both systematically criticized Kant and Bradley as confused on just these conceptual issues. Stout and Ward, when they were in a critical mood, called Kant a "psychologist," by which they meant a bad epistemologist, precisely on grounds that the Copernican insight did nothing much more than elevate the mind's own (alleged) principles of organization into an (alleged)

22 Broad (1945).

23 Stout (1926, 28–9).

24 Bradley, Green, Stirling, and others.

25 It didn't help that the conception of 'reality' was subject to ambiguity: Bradley, for instance, took it as logical; Ward took it as that of objectively scientific common sense. This affected their accounts of psychology, of course.

justification of knowledge that ran subjectively—aground. But they also called him a *really bad* psychologist ("psychologist" in their sense, that is), in that he wasn't even right on the mind's own principles of organization.[26] Kant's views on "logic" as part of his Critique of cognition—and more generally his examination of reason and attendant notions—was read either metaphysically (when it was being embraced) or psychologically (when it wasn't, and mostly it wasn't, in Cambridge). Bradley fared little better in Cambridge: the Cambridge mental scientists of the 1890s were very concerned about the associationism of the empiricists (both classical and contemporary) and likewise called them "psychologists," meaning "bad epistemologists"[27] (a criticism in fact that Bradley shared). But, as Cantabrigians would see it, "psychology" for Bradley meant "logic" because "logic" meant "thought"; both "logic" and "thought" meant "reality," and "reality" meant the Absolute, which was a universal, necessary, perhaps logical (in *our* sense) entity, all of which was derided by Ward as the worst mental science ever and waved away with less than disdain by Stout.[28] We need to set Bradley aside for the time being, however. In order to fully understand Bradley's role in the influences on Moore, we need to clarify how "logic" in Kant's wake was understood at Cambridge.

Reading Kant in Cambridge in the 1890s

I noted earlier that the *Psychologismusstreit* was a useful canopy under which to investigate the philosophical influences on Moore. Here is why I think so. It turns out that for Moore's teachers, Kant was public enemy number one with respect to a suspect subjective psychologism concerning notions proprietary to the new mental science (or even, as Sidgwick argued, a half-decent ethics[29])—and Moore's approach to Kant was deeply influenced by his teachers' reading of Kantian views.[30] Most crucially:

26 The Kantian offenses against a proper scientific psychology, as far as they were concerned, included the outdated notion of mental faculties; the lack of a causal account of how the *ding an sich* was connected to phenomena; and no realistic account of how the Categories actually applied in thought, let alone what justified them.

27 Ward (1886).

28 "A view so unclear I suspect a clerical error" (Ward, 1887) is one of the milder criticisms Ward made of Bradley in print. Bradley took himself to be discussing what he calls 'psychology' in his articles in the 1880s and in the PL, but he would not take his 'psychology' to be that of the classical empiricists he criticized. And as far as the Cambridge mental scientists were concerned, he was no psychologist in *their* sense.

29 See Sidgwick (1907, chapter V).

30 By Moore's teachers, I mean Stout, Ward, Sidgwick, and McTaggart. Moore attended lectures by all of them in preparation for his Tripos Part II. McTaggart examined the Moral Sciences Tripos in 1896 (CUC 1896, 339) and prepared Moore for the History of Philosophy special subject (The Philosophy of Hegel). Sidgwick and Ward examined Moore's fellowship dissertations in 1897 and 1898, respectively (Baldwin

Moore's 1898 dissertation was a critical account of reason and freedom in Kant, and Moore would have been expected to explain the Kantian conception of these notions in his own analysis of it. Kant's conception of reason was squarely within the conception of "logic" of the day. It had to account for the nature of the understanding: that is, of judgment, of the logical form of judgment, of the components of judgment (concepts), of intuitions, and of truth and falsity (among other things). Kant's account, as we know, included the formulation of a *transcendental* logic, to provide an account and a justification, of reason by reason, of our acquisition of knowledge, including an account both of judgment, and of the nature of the reality we can genuinely know. The upshot: reason itself contains *a priori* elements that make knowledge possible. As for Freedom, Kant made a distinction between rational and empirical psychology via the notion of a *transcendental freedom,* wherein our will is determined to be free to be rational. Kant's account of freedom included an account of the self (which according to both Ward and Stout was misformulated against the standards of their contemporary mental science), and an account of moral agency, which (Kant believed) the psychology (epistemology, in our sense) of *his* day could not provide. Kant also bequeathed some of the terminology that the late 19th-century mental scientists at Cambridge took to be highly suspect: (i) a so-called *faculty psychology*, whereby the 'soul' is characterized by a variety of innate faculties, which are its capacity to carry out its activities; and (ii) a division of these activities into thinking, cognizing, feeling, and willing.[31] Last but not least, at the foundation of all of the activities of the mind (reason, understanding) in this tradition was the notion of *unity*: the mind was the active, integrating sensory experiences according to universals or universal laws of integration (or a transcendental self).

I would argue that the way Kant was read at Cambridge was of a piece with what Kitcher (1990) has called the "psychological" readings of the first *Critique*. (Kitcher 1990, 3–7).[32] All of Moore's teachers

and Preti, 2011). Stout (1931; 1952, Gifford Lectures), Sidgwick (in *Mind*), and Ward (posthum, 1922) all published their views on Kant, most taken from their lectures, which Moore attended. Sidgwick was also entirely *au courant* with the streams of mental-science influenced philosophy at Cambridge and, moreover, explicitly distanced himself from the Bradleyan metaphysics of day (Baldwin and Preti, 2011, xxxvi). This needs greater attention than I can give it here, however.

31 Kant got this from Wolff (Murphy and Kovach 1972).

32 Noting here that ambiguity reigns when it comes to parsing out what terms like "psychological," "anthropological," and "subjective" will have meant. The key is that Kant himself made the attempt to understand the nature of subjectivity (and, in particular, cognition) by way of a formulation of its a priori conditions. His German critics tended to think he failed on his own terms, and his English critics tended to think he failed to formulate a convincing non-subjectivist account, his transcendental arguments notwithstanding.

were entirely familiar with both Germanophone and Anglophone criticism of Kant's views, which informed their own contributions to the literature. That "psychological" readings dominated Kant scholarship during this period, as Kitcher (and others[33]) note, was in part due to Kant himself. Kant took the project in the *Critique of Pure Reason* to examine the faculties of sensation, understanding, imagination, and reason—*prima facie* psychological ('anthropological') if anything is—in order to determine the nature of the knowing mind. In order to avoid a collapse into subjectivism, Kant sought to tie some aspects of knowledge to a priori conditions of the nature of our cognitive faculties: specifically, what they had to be like in order for knowledge to be possible. In this respect Kant did address psychological elements directly, criticizing the rational psychology of his predecessors who tried to derive a substantive claim about the soul from "I think"; criticizing empirical psychology for lacking the pure or non-empirical grounding that a genuine science must have; and (arguably) all but explicitly defending a transcendental psychology.[34] The project of establishing the a priori conditions of our cognitive faculties just is, as Kitcher notes, Kant's transcendentalism. What she emphasizes is how much it is centered on what could be understood to be, implicitly, a "transcendental psychology" (1990, 16–9).[35]

Classic criticisms in the wake of the publication of the *Critique* from this angle included that the *Critique* was an attempt to derive normative conclusions from factual premises; another was that Kant tried to ground necessary conclusions from psychological premises. Kitcher argues that such criticisms dogged its interpretation for 100 years after its publication (thus, we should note, featuring squarely in the literature of the *Psychologismusstreit*).[36] Early continental readers, according to Kitcher, took it that Kant failed to distinguish between questions concerning the logical conditions of knowledge and questions about the subject of knowledge.[37] Kitcher cites Fries, for example, as taking it that the *Critique* was a failed exercise in presenting arguments that attempted to

33 Cf. Guyer (1998).

34 Cf. the paralogisms concerning the self.

35 Kant, it should be said, never explicitly defends a transcendental psychology. But his critics did not hesitate to attribute it to him from the start. I am not here taking a position on this reading of Kant, only trying to elucidate how much of it was characteristic of the way Kant was read in Moore's *milieu*.

36 See Kusch (1995) and Kohnke (1991), among others.

37 Leary (1978) argues that there is a connection to be made between the criticisms of Kant on the part of Fries, Herbart and Beneke and the onset of what he calls "the philosophical justification of psychology as a natural science." Stout was thoroughly familiar with the work of these thinkers (Stout 1896). Ironically enough, many of the early criticisms of Kant centered on what was perceived to be Kant's *failure* of the strongest defense possible of a theory of subjectivity.

derive a priori formal conditions of the mind from empirical or a posteriori mental facts instead of presenting a thorough account of the latter (Kitcher 1990, 22–3).

Reinhold[38] tried to insert a Cartesian ego as a first principle on Kant's behalf from which to derive the nature of psychological concepts like the pervasive but ambiguous *Vorstellung*. Herbart, she claims, attributed to Kant the failure of trying to ground philosophy on psychology (1990, 6, 9); in the language of the day, this will have meant grounding metaphysics in an illegitimate subjectivism. Kuno Fischer put it even more strongly: "The question of whether the critique of reason is supposed to be metaphysical or anthropological is a real problem, unavoidable in the history of the development of German philosophy since Kant."[39]

Commentators in English to this scrum of Germanophone Kant criticism in their turn did not hesitate to add their own criticisms of Kant in terms of a suspect subjectivism that they took to infect his account. Green, Hamilton, Stirling, Caird, and Sidgwick (cf. Walsh 1981, 723–29) also took exception to Kant's psychologism, and all feature in Moore's exposure to and writings on Kant.[40] These critics imputed to Kant a variety of "psychologisms": from (i) accounts of his views refashioned according to the kind of Hegelian he ought to have been; to (ii) more sober criticisms of his account of the mind and its objects. Hamilton stressed the apparent divide between phenomena (knowable) and noumena (unknowable) and the tension between these and Kant's conception of the ego, which is (somehow) both source and ground both for knowability of objects and for objects themselves and also (somehow) a moral agent. Green took the view that everyone misunderstood Kant (except presumably himself), arguing that Kant ought to have realized that without "the unifying principle the manifold world would be nothing at all" (PE 75). Caird,[41] an eminent Kant scholar of the day (and who examined Moore's 1897 dissertation), tried to rehabilitate Kant in the face of what he took to be a variety of incoherent and incompatible claims, such as the claim that perception and thought were to be strictly distinguished, but were nevertheless inseparable elements of phenomenal knowledge. Caird, however, rendered Kant as defending the dialectical nature of human thought, proceeding in stages of self-transcendence to a purer grasp of reality, which was ultimately indistinguishable from the nature of Thought itself. That is, Caird read into Kant what idealist views he

38 Karl Leonhard Reinhold (October 26, 1757–April 10, 1823).

39 Kitcher (1990, 6–7). All of these thinkers were entirely familiar to Stout (and to Ward), and Moore himself refers repeatedly to Fischer in his dissertations. 'Anthropological,' at this time, referred to any account of human processes, like that of mind.

40 See, for instance, his undergraduate lecture notebooks (Add. 8875 10/1/1; 10/2/1; 10/3/1; 10/3/2; 10/3/3).

41 See Lindsay (1877).

took seriously himself (Walsh 1981, 724–5).[42] Stirling, who for his part was opposed to the idealist/Hegelian reading of Kant, read Kant charitably as having desired to make good the claim that the subjective elements of mind could nonetheless supply necessity and objectivity, but argued that Kant failed to succeed (Walsh 1981, 727).

Sidgwick, for his part, published a number of critical commentaries on Kant, an important one in 1883 on the critical philosophy.[43] Sidgwick, for instance, makes a reproach that also turns up in Stout's undergraduate Kant lectures:[44] that "if we are unable to penetrate to things *beyond* experience, why would we be any more able to discover the conditions which lie—if I may say so—behind it?" (1883, 320). Sidgwick also raises the objection, among other things, that Kant fails entirely to coherently account for what he calls the "objectivity of our empirical cognitions." This takes Sidgwick to a crucial criticism of Kant's transcendentalism. Sidgwick contends that Kant's view imports a tension into his conception of 'object':

> that I can know objects to be merely modifications of my sensibility, combined in certain ways by my understanding; while at the same time I also conceive them as different from the modifications of my sensibility and as perduring when the latter cease.
>
> (1883, 318)[45]

This persistent conflation, Sidgwick argues, is due to Kant's apparent insouciance with respect to his own distinction between *phenomenal* and *noumenal*:

> Kant always regards the one object as phenomenal of the other, but often identifies the two so completely that he speaks of both indifferently by the same name in the same passage, even in the very

42 Walsh claims (1981, 724) that for Caird, "the important question to ask about Kant was not what he believed but what he got right." In the light of Caird's nonplussed reaction to Moore's 1897 Kant commentary, it is all the more ironic that, in what is a slightly mysterious notation on the 1898 dissertation, Moore exhorts himself (or is reporting the advice of another) that "you should not merely find your own views in Kant, but unless you carefully compare him with what you yourself can really understand and think to be true you are in great danger of never finding what he meant at all…" (Baldwin and Preti 2001, li).

43 Walsh in fact characterizes Sidgwick's reading of Kant as containing the essentials of an attack on "transcendental psychology," and even of an attack on the possibility of a critical philosophy itself.

44 Moore's notes are preserved (Add. 8875 10/1/).

45 Lindsay (1877, 483) explains that the "Hegelian Contributions to English Philosophy," especially those of Caird and Green, emphasize the failure of psychological formulations as atomistic or "isolated" to one mind, which nevertheless try to give an account of *mind*.

> transcendental discussions in which the distinction between the two is of fundamental importance.
>
> (1883, 318)

I have tried to underscore a few key elements that stand out as regards the main tendencies in Kant interpretation in the 19th century: (i) that a variety of readings of Kant that saw him as confused about the study of subjectivity then held sway, consistent with the disputes of the *Psychologismusstreit* across epistemology, psychology, metaphysics, and logic; (ii) that a clue to Moore's developing metaphysical realism had its source in the anti-transcendentalism of his teachers' attacks on what they took to be Kant's subjectivism; and (iii) that these elements were a notable component of the Kant criticism in Moore's *milieu*. Kant was criticized for failing to commit to a full-dress idealism (by his Idealist critics), for bad psychology (by the Cambridge mental scientists), and for bad epistemology (by everybody else). I would argue that, in all of these lines of attack, there is a similar perspective: that Kant failed to show how the a priori conditions that (allegedly) ground our cognitive faculties result in anything but some kind of mentalism: in either a metaphysical idealism, or an epistemological subjectivism. For the Idealists, it was obvious that they could not but do so, in spite of Kant's protestations; for the mental scientists, such a claim was just bad psychology with respect to their own contemporary standards.

And That Is How Russell Could Have Called Moore a Logician

I began this paper with the question as to how Russell could have ever called Moore—let alone repeatedly and admiringly—a logician. The answer to this, I have been arguing, is to situate Moore's influences in the context of the disciplinary turbulence that marked out the late 1890s, at Cambridge and on the continent. On the terminological order of the day, Kant and Bradley were logicians; both Kant's formulation of judgment in the Critique and Bradley's in the *Principles of Logic* (PL) featured foundationally in Moore's preparation for the Moral Sciences Tripos. Moore's innovative account of the nature of judgment precisely broke new ground—new for late 19th-century analyses of mind and its objects—in that it both rejected the Kantian/Bradleyan formulation of judgment as unifying (conflating, his teachers will have said) thought and its objects and offered a metaphysics of propositional content that underscored the relational, non-subjectivist formulations of the Cambridge mental scientists. That Moore's teachers found none of the psychological, epistemological, or metaphysical implications of Kant's or Bradley's respective theories of the nature of judgment plausible, is, I would claim, the most significant part of the story of the influences on

Moore's own views on the nature of judgment. I will conclude here with a gesture toward how I believe Bradley fits into this story.

Bradley's historical standing as the British Absolute Idealists' British Absolute Idealist is what gets the most attention in investigations of his role in Moore's early developing thought. That Bradley's PL was part of Moore's pre-dissertation reading is not at issue, but I would argue that a proper contextual understanding of the role of Bradley's work in Moore's developing thought would involve situating Bradley's work within the framework not just of the metaphysics but of the mental science of the day. Recall that the late 19th-century mental sciences included *logic*, understood of course to mean the workings of the mind in *judgment* or (more generally) cognition; in itself, of course, a legacy of Kant. Certainly, the mental science influencing Moore when he began his Fellowship work reflected the more scientific and up-to-date analyses of his teachers rather than Bradley's more traditional idealist considerations concerning knowledge, justification, and reality. But Bradley's anti-psychologism, no less than that of Stout and Ward, was itself influenced by the continental mental scientists who had begun to examine distinctions as to psychological and logical questions (Lotze, in particular, whose work was well known and influential during this period to all the British philosophers). PL was Bradley's foundational attempt at grappling with that distinction. It is thus possible to make the case that Stout, Ward and Bradley can all be seen as integrating the logic, psychology, and metaphysics characteristic of the continental mental scientists into their own views, which, in their turn, were fundamental in the progression of British mental science (and Moore's developing thought) at this period.[46]

Epilogue

I want to end here with a very brief sketch of an explanation of the effect that Moore's views on the nature of judgment had on Russell. Russell's fulsome acknowledgments to Moore—over his lifetime—have for the most part been greeted with attitudes ranging from bemusement to dismissal. The most thorough examination of Moore's influence on Russell is Griffin's (1991). But even Griffin falls short, I think, in being able to explain just what it was about Moore's ideas that could have had the seismic effect on him that Russell emphasized again and again. Here then is rough take of what I think that is. As I have argued, Moore managed

46 See, for instance, Dyde's review of PL (1884; 85); also Adamson (1884). Dyde puts it this way (289): "First of all it must be made particularly prominent that no treatment of the principles of Logic is worthy of consideration that does not attempt to explain the connection between consciousness on the one hand and the world on the other. The failure to recognize the urgency of this need has led to the tremendous amount of almost useless writing commonly known as formal logic."

to articulate dramatically (even, Russell thought, too dramatically[47]) a realist metaphysics of judgment built up from the views that he had absorbed via Ward and Stout on a relational view of consciousness and its objects during his Tripos preparation. Russell, of course, had been exposed to the same material. But I would argue that Moore had the advantage, because the Moral Sciences Tripos drew a much straighter line between metaphysics, logic, psychology, and *ethics* than it did between metaphysics, logic, psychology, and *mathematics*. Schooled in the bad Cambridge mathematics at his Tripos Part I, Russell found it difficult to shake the Bradleyan conflation of metaphysics, logic, and psychology while trying to work out his "Tiergaarten program" of this period, conceived as a *dialectic* of the sciences (Griffin 1991).[48] Since part of this included a commitment to mathematics being a stage in that dialectic, and the formulation of a conception of its ground, Russell's thinking just at this period did not as effortlessly include a formulation of mathematical *judgment* as being relational. For Moore the distinction between the mind and objects of thought emerged more effortlessly from his focus on ethics and on the nature of *moral* judgment, that is, in Moore's developing thought, the objectivism of the mental scientist concerning judgment blended with the objectivism that Moore was looking for to ground ethics and ethical judgment. Moore had, to all appearances, temporarily sought refuge in the Bradleyan Absolute, which he seems to have initially thought would offer an adequate objectivism for the object of moral judgment. But as I have shown, the Bradleyan Absolute was ultimately no match for the anti-psychologistic formulations of the nature of judgment that Moore absorbed from his teachers. Russell's characterizations of "Moore's logic"—Moore's theory of judgment—are what show the light beginning to dawn on Russell. And once Russell digested it, the rest was history. Here the hope is that this history has been additionally clarified with this look at the role of logic in Kant's wake in Moore's *milieu*.

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47 Russell to Moore, September 18, 1898 (Add. 8330 8R/3/8).

48 It could even be suggested that Russell gave such fulsome credit to Moore because he realized that he *should* have seen for himself the significance of "the non-existential nature of propositions…and their independence of any knowing mind" (1903, xviii).

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10 Sigwart, Russell and the Emergence of Scientific Philosophy

Sean Morris

When considering 19th-century logic in relation to Bertrand Russell (1872–1970), the relationship is most often thought of as a negative one. On the usual story, logic, that is, formal logic, had long been considered stagnant and waited for the great innovations of Frege, Peano, and Russell himself to move the subject forward. The result was the modern mathematical logic that would dominate so much of 20th-century analytic philosophy. On this account, Russell largely rejected the old Aristotelian logic in favor of the new logic and the undreamed-of progress that it brought to the discipline and to philosophy more generally. While in part true, this story neglects a central feature of post-Kantian logic. Logic in the 19th century certainly did include what we typically describe as formal logic, but it also included what is now usually described as methodology.[1] Under the latter fall such topics as the connections between the concepts of the various sciences, the role of observation in scientific theories, and epistemological matters more generally, such as how science should proceed so as to establish systems of reliable beliefs. Formal logic in this context was perhaps rightly described as being empty or mere form, whereas in methodology, logic found content.

In this chapter, I will argue that the early Russell was very much connected to this post-Kantian tradition of logic that included methodology. Specifically, I will argue that Russell's work in the 1910s on the conceptual foundations of the empirical sciences—specifically the projects surrounding and including his 1914 *Our Knowledge of the External World*—emerged out of methodological concerns that he had developed already in his idealist period of the 1890s. This interpretation of Russell's philosophical concerns will allow me to argue for an important continuity between Russell's idealist period and his analytic turn that followed, specifically that underlying Russell's philosophy was always a commitment to some version of naturalism—the view that there is no higher form of knowledge than what the natural and mathematical sciences themselves provide.

1 On this point, see Heis, ARL.

Over the last 30 years or so, one of the major developments in Russell scholarship has been to emphasize the importance of Russell's beginnings in the idealist tradition. As a result, we have gained a much better understanding of the motivations for Russell's analytic philosophy by viewing it as a reaction to his idealist beginnings. Much less has been said, however, about any positive lasting influence that his idealist period may have had on him as his philosophy continued to develop. I aim to fill this gap. In doing so, I will place Russell within the broader movement of scientific philosophy, which includes philosophers of a variety of traditions including phenomenology, neo-Kantianism, and early analytic philosophy. Approaching Russell as a scientific philosopher will help to bring out the continuity of his thought in a way that seeing him first as an idealist and then as an analytic philosopher does not. My approach will show Russell's urging of scientific method in philosophy in the 1910s as part of the long-standing naturalist strand in his thinking. While the naturalistic turn of Russell's philosophy of the 1920s forward has long been recognized—notably by Quine himself[2]—my chapter will show that it was not so much a turn as the culmination of a view present already in the 1890s.[3] In light of my interpretation, we should see the central aim of his program for scientific method in philosophy not as one of empiricist foundationalism, that is, not of one aiming at the reconstruction of science upon an indubitable foundation of sense data, but as a reorganization of knowledge so as to make apparent which scientific claims are more and less dubitable. As we will see, Russell takes this to be a naturalistic endeavor and not as any kind of foundationalist reconstruction of scientific knowledge aimed at refuting skepticism.

2 For example, by Quine himself in his 1966 "Russell's Ontological Development," p. 85. More recently, Andrew Lugg has made the case for Russell as a naturalist from at least his work beginning in the 1910s that runs through the rest of his philosophical career; see Lugg's "Russell as a Precursor to Quine," *The Bertrand Russell Society Quarterly*, November 2005/February 2006 (www.lehman.edu/deanhum/philosophy/BRSQ/05nov/lugg.htm). That Russell turned increasingly towards naturalism in the 1920s is uncontroversial. Lugg's contribution, which is very convincing, is that this line of thought was already present in the 1910s. I hope to further Lugg's interpretation by showing that Russell's naturalist commitments were present already in his earliest philosophical work of the 1890s. It is no objection to my view, or to Lugg's, that Russell perhaps did not fully embrace his naturalism until the 1920s. Rather, we can understand his naturalism of the 1920s not so much as him turning away from a previous philosophical view but instead as him fully embracing a view he had always held.

3 This is not to argue that Russell held this view with complete consistency from the 1890s to the 1910s and beyond or that he always recognized the full consequences of such a view. I will only argue here that the naturalistic strand of Russell's thought is a constant, early, middle, and late. That it would evolve in various ways over the course of the roughly 50–60 years of his philosophizing should hardly be surprising. Indeed, it would be more surprising, and probably less interesting, if this aspect emerged in the 1890s and then never changed.

More specifically, I will proceed by looking to Christoph Sigwart (1830–1904)[4] as an important historical precedent for many of the views that would emerge in Russell's philosophy during the 1910s. During the 1890s, Russell expressed unusually high praise for Sigwart. For example, in the preface to his first major philosophical work, the 1897 *Essay on the Foundations of Geometry*, Russell remarks "In Logic, I have learnt most from Mr Bradley, and next to him, from Sigwart and Dr Bosanquet" (EFG, p. 9). And in an earlier 1894 letter to his soon to be first wife, Alys Pearsall Smith, he writes, "I have found that my German professor is an intimate friend of Sigwart, who is my favorite among modern German philosophers, so we were mutually interested in what each other had to say of him." Russell, however, gives very little indication of the reasons for such praise. I will here conjecture that Sigwart served as a key influence on the development of the kind of philosophical project that Russell aimed to carry out during the 1910s, that is, his program for scientific method in philosophy, particularly in regard to its more naturalistic elements. I am not claiming that Sigwart was the unique influence on Russell in this respect, but rather that Sigwart represents an approach to philosophy that was widespread during Russell's formative philosophical years and one that continued to influence him throughout his philosophical career. In what follows, I will argue that this early period had an important positive lasting effect on Russell and that Sigwart represents a central kind of early influence on Russell, specifically in bringing philosophy into line with scientific methods and developments.

The structure of my chapter is as follows. In Section I, I begin by briefly characterizing the tradition of scientific philosophy. Next, in Section II, I examine Sigwart's philosophy. I first present some of the general features of scientific philosophy that he holds to, especially a renewal of cooperation between philosophy and the sciences. I then discuss some of the more particular features of his philosophical project, key among them his focus on the methodology of the sciences. Finally, in Section III, I turn to Russell, arguing that many of the features of his program for scientific method in philosophy have precedents in Sigwart. Among these are a commitment to a version of naturalism; that philosophy should proceed piecemeal like the natural sciences; that the method for discovering axioms for the mathematical and natural sciences is inductive;[5] and most importantly, that philosophy done scientifically has as its central epistemological task an ordering of knowledge so as to distinguish those claims that are more dubitable from those that are less so. Before I begin, I should note that my treatment of Sigwart is selective.

4 He was the son of German philosopher Heinrich Christoph Wilhelm Sigwart (1789–1844).

5 Another potential influence on Russell on this point is Stanley Jevons; see Lydia Patton (2017).

I have focused on those aspects of his philosophy that might have had an influence on Russell and have ignored those that clearly would not have.[6]

I

The term 'scientific philosophy' began its rise to prominence in the mid-19th century in reaction to the perceived subjectivism and speculative nature of post-Kantian idealism, particularly as found in the philosophy of Hegel and his followers.[7] Characteristic of this Hegelian approach was the striving for "a grand synoptic view of the world founded, built, and polished by one thinker in splendid isolation" (Richardson, 1997, p. 427). There was also a more general movement in this period towards distinguishing philosophy from the sciences, taking religious or artistic approaches as the model for philosophy.[8] The scientific philosophers rejected both of these things. On the first, they instead championed the methods of modern science, which emphasized a community of researchers engaged in collaborative research to solve various problems by relying on the methods and results of their fellows. Philosophy done in this way would itself adopt the idea of philosophical questions and problems being solved by specialists working on a narrow range of problems so as to contribute to the progress of the whole. And on the second, such approaches emphasized the subjectivist aspects of post-Kantian idealism. In contrast, the scientific philosophers urged that philosophy work in cooperation with modern science rather than try to distinguish itself from science. By doing so, philosophy itself could share in the kind of progress achieved by the sciences.

So what, then, does philosophy look like under this new scientific conception of it? Methodology emerged as the new subject matter of philosophy. For example, the positivist Richard Avenarius saw a properly scientific philosophy as itself falling among the empirical sciences. On his conception of scientific philosophy, philosophy became "a general science of the sciences," which took as its subject matter the given special sciences and the role of the scientist in relation to them. His philosophical concerns focused on establishing unity among the concepts of

6 For this reason, I have largely skipped over any discussion of Sigwart's grounding of logic in psychology. For some discussion of this, see Picardi, SHF. Russell would not have been impressed by this aspect of Sigwart's thought since Russell had announced his rejection of this sort of psychologism by the mid-1890s. Similarly, Sigwart appeals to God in certain aspects of his philosophy. This, too, would not have been something that Russell would have found any sympathy with. Russell is clear in his praise for Sigwart, and I am trying to provide an account of what so impressed Russell.

7 In characterizing the movement of scientific philosophy, I follow the accounts found in Richardson (1997) and Friedman (2012). Friedman mentions Schelling in this context as well. Friedman cites Helmholtz's 1855 "Über das Sehen des Menschen" as a sort of founding document of the movement.

8 Richardson cites Schopenhauer, Nietzsche, and Kierkegaard as examples.

the historically given sciences. Methodology was to be pursued through psychological means and so, in this way, placed itself among the empirical sciences. A similar concern with methodology emerges also in the work of some of the better known neo-Kantians.[9] Since they, however, were less concerned with science as a purely empirical discipline and had no qualms about the a priori, their focus on methodology had more to do with the form of scientific knowledge, that is, with what has become known more as the logic of science (Richardson, 1997, pp. 428–9). In general, these concerns with methodology, where again, methodology was taken as a part of logic more generally, moved the scientific philosophers away from metaphysics and towards epistemology. As we will see in discussing Russell, all of these characteristics appear prominently in his scientific philosophy of the 1910s, but he leaves the origins of his views here undisclosed. So let me turn first to Sigwart to try to discern which specific aspects his view might have influenced the young Russell.

II

While Christoph Sigwart is now a largely forgotten figure of 19th-century philosophy, he was one of the more important German logicians in the latter half of that century, producing, along with Lotze, what was probably one of the two most widely read logic texts in Germany at the time.[10] Sigwart is now most often remembered, and dismissed, as the paradigm of a psychologistic logician—that is, for his commitment to the view that logical laws are grounded in human psychology[11]—no doubt as a result of Husserl's criticisms of him in the *Logical Investigations*.[12] I do not want to take up this criticism of his philosophy here. As we will see, psychology as the science of human thought[13] is certainly important to his view, but

9 Richardson provides some detailed discussion of Alois Riehl and also cites Bruno Bauch, Paul Natorp, and Ernst Cassirer as engaging in similar projects.

10 On this point, and on logic in general during this period, see Jeremy Heis, ARL.

11 Sigwart characterizes the importance of psychology for logic as follows: "That is, logic is grounded …, not upon an effete tradition, but upon a new investigation of thought as it actually is in its psychological foundations, in its significance for knowledge and its actual operation in scientific methods" (*Logic*, vol. I, p. x).

12 Exactly what these criticisms come to is a complicated issue, as is the charge of psychologism generally. For a careful study of psychologism, see Kusch (1995). Kusch observes that psychologism is a term of criticism, not used to describe one's own philosophical position. Sigwart is among those who deny it of themselves but apply it to others.

13 Psychology, according to Sigwart, aims

> [t]o determine what Thinking in general is, how it differs from other psychical activities, in what relations it stands to these, and what are its different varieties…. It is true there is no generally accepted Psychology to which we can refer, but for our present investigation it will be enough to refer to our ordinary use of language.
> (*Logic*, vol. I, sec. 1, p. 1)

I want to put aside whether or not this leaves him with any defensible account of the laws of logic. Instead, I will emphasize the importance he attributed to this budding science in bringing philosophy more strictly into line with the sciences.[14] Sigwart was one of the many German philosophers of this period who appealed to psychology in reaction to the speculative metaphysics of post-Kantian idealism, particularly the *Naturphilosophie* of Hegel and Schelling. Here, there is much overlap between the aims of the German psychologistic movement of which Sigwart was a part and the aims of the scientific philosophy movement more generally.[15] Furthermore, I will argue that Sigwart's attempts to bring philosophy back into line with the science of his day gives us an indication of what the early Russell appreciated so much in a figure like Sigwart.

Sigwart's aim in the two volumes of his *Logic*, published originally in 1873 and 1878 respectively, is to provide an account of the methods by which we reliably obtain knowledge, with the most developed sciences providing the paradigm for such claims to knowledge.[16] As he describes the work, it is an "attempt to reconstruct logic from the point of view of methodology, thus bringing it into active relations with the scientific problems of the present day…" (vol. I, p. ix). Immediately, we see Sigwart here urging that philosophy be brought back into cooperation with current scientific developments. Sigwart's use here of the term 'logic' is

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Later in the work he acknowledges that his conception of psychology may seem antiquated in light of the then developing fields of experimental and cognitive psychology, though he still thinks there is a place for his own particular approach (vol. II, p. vi).

14 Sigwart's psychology is not that of physiological and experimental psychology, both of which were on the rise by the time that the second volume of his *Logic* was published. He is clear that he does not see his own work as being in opposition to these developments and welcomes what they can contribute to his own undertaking (vol. II, p. vi).

15 Psychologism can be characterized most simply as the view that logic is part of or grounded in psychology. The height of the movement in Germany took place roughly between 1890 and 1914, though its origins can be traced back to Mill's *Logic* of 1843. Much of the inspiration for the movement was to bring philosophy back into contact with the sciences by locating logic itself within psychology. The aim was to avoid grounding logic in more mysterious extra-scientific powers of intuition or direct rational insight. For this reason, the movement has also been characterized as a form of naturalism; see Kusch (1995) for a thorough discussion of the movement. I would like to remain neutral here on whether psychologism as a whole should be counted as part of scientific philosophy. After all, the varieties of psychologism are many; again see Kusch. Certainly we see in Sigwart what seem, at least retrospectively, to be obviously non-scientific elements. But again I am focused on those elements that I think did appeal to Russell (e.g. appeals to God obviously wouldn't have impressed Russell but philosophy guided by science would have).

16 Eva Picardi describes Sigwart's aims as being an early version of the naturalized epistemology later made famous by Quine; see her SHF, p. 164. This is another interesting connection to Lugg's naturalistic reading of Russell, which has as its ultimate aim the claim that Russell is a precursor to the naturalism that would later take center stage in Quine's philosophy.

that common in the 19th century, inclusive of methodology, though he distinguishes the two in contexts where it matters. Much of his work then includes topics that go well beyond what we might now think of as logic, that is, formal logic. Indeed, Sigwart claims that it is specifically in methodology that logic finds its significance for science:

> Much rather do we hold it necessary that Methodology, which is generally made to take a subordinate place, should be regarded as the special, final and chief aim of our science [that is, logic]. And since this Methodology must have for its principal object the growth of science from the natural data of knowledge, we hope to satisfy also to some extent those who endeavor to avoid the barrenness and abstract character of the formal scholastic Logic by making it include a Theory of Knowledge….
>
> (vol. I, sec. 4.4, p. 21)

As part of this general aim of restoring the cooperation between philosophy and the sciences by way of his methodological investigation, Sigwart denies any special philosophical source of knowledge and, in attempting to identify the methods that reliably yield knowledge, takes the sciences themselves as his starting point. "[L]ogic," he says, "is grounded … not upon an effete tradition, but upon a new investigation of thought as it actually is in its psychological foundations, in its significance for knowledge and its actual operation in scientific methods" (vol. I, p. x).[17] Here, Sigwart does appeal to psychology, but he sees this only as a means of reconnecting philosophy with the sciences. He appeals to psychology as the science that studies thought as we actually find it in scientific practice, rather than as existing, for example, in some mysterious Platonic realm. This, he believes, is the best we can do for philosophy unless we are willing to tolerate a philosophy that stands on an extra-scientific basis, such as one that relies on some power of direct rational insight or intuition into an indubitable foundation for science. Sigwart appeals instead to a study of identifiable human faculties that can be studied from the perspective of natural science.[18]

In line with his thinking here, Sigwart then explicitly denies the possibility of any kind of Cartesian epistemological foundation for the empirical sciences. He describes Descartes' aim as bringing about a new beginning for knowledge, where a complete skepticism first leads us to reject any previously held belief that could possibly be false and

17 My citations to Sigwart's *Logic* include the volume, section (when given), and page numbers. His remarks here make all the more sense when remembered that his logic includes methodology.

18 I hesitate to describe Sigwart's method as wholly empirical because he is certainly not doing psychology in the sense of experimental psychology.

then second, to reconstruct all knowledge upon indubitable premises.[19] Against this view, Sigwart urges instead that we can never make such a thorough and complete break with our past beliefs:

> There can … be no method whatever by which we may begin Thought entirely anew. All that we can look for is a method of carrying it on from already existing data, which must always form the starting point for future Thought, even when acknowledged to be uncertain.
>
> (vol. I, sec. 2.1, p. 12)

Sigwart then begins with science in its current state. He does not think we must first tear down all of our apparent knowledge so as to then begin anew with an indubitable foundation for the empirical sciences to stand upon. This latter method, he thinks, would leave "empirical science … altogether impossible" (vol. I, 2.2, p. 13).[20] Rather than trying to begin knowledge entirely anew, Sigwart explains that it is in examining the history of science that we can locate those methods that have reliably brought us to knowledge:

> In pointing out the paths which thought must take to the ends it has in view methodology must call to its aid the History of Science, it must investigate the methods through which those ends have actually been reached, or through which at least an approximation has been successfully made, and it must show the grounds upon which these methods have been based, and examine their justification.
>
> (vol. II, sec. 63, p. 19)

He restates this approach, with some elaboration, one page later:

> Our task now leads us to endeavour to point out the ways by which the ideal before us may be reached—or at least approached—under the universal conditions of human thought, and starting from such ideas and judgments as have been formed in the natural course of psychological development. Another element now appears in our investigation, the history of science, in which we find recorded

19 It seems here that Sigwart attributes to Descartes what we will see Russell identifying as universal skepticism. Russell does not necessarily identify Descartes skepticism with universal skepticism. For my point, it is not so important to determine which of the two is a better reader of Descartes. As we will see, it matters more that they both think universal skepticism a non-starter for rebuilding science on a new indubitable foundation.

20 We will later see Russell making a very similar claim in stating that any account of knowledge must begin with our instinctive beliefs, or common knowledge.

> progressive attempts to attain to concepts and judgments which are logically perfect and appropriate to the highest ends of thought, and which shows us, on the one hand under what conditions and through what means problems have been solved, and on the other hand what circumstances have until now hindered a complete solution and what expedients have at least facilitated an approximation to the end.
>
> (vol. II, sec. 63, p. 20)

Sigwart's basic idea here appears to be that science provides us with a paradigm for knowledge. Unfortunately, though, he is not especially clear then on what these logically perfect judgments are that science aims at and sometimes achieves. He does provide us with two central features of them: (1) that they must be composed of ideas that are completely determined and the same for everyone and (2) that we must be aware of their logical ground (vol. I, sec. 39). The various sciences then achieve such judgments to varying degrees. He cites mathematics as a science that has achieved the ideal of logical perfection, whereas psychology lies on the opposite end of this spectrum (vol. II, sec. 64.2, p. 22). The central point I want to draw out here, though, is that Sigwart clearly thinks that logical perfection can be found by looking at what the sciences have actually achieved rather than by approaching this problem by way of some philosophy prior to the sciences themselves. The history of science is intimately connected to his methodological investigations because it is in examining this history that we can locate the methods that either have succeeded in providing logically perfect judgments or, at the very least, have moved us towards this ideal.[21]

In Part III of *Logic*, Sigwart undertakes to discover the rules for making logically perfect judgments of this kind. He focuses here on three main topics: deduction and proof, judgments of perception, and induction. Actual logically perfect judgments, Sigwart says, can be found in the realm of deduction, that is, when we have deductions from premises, which we already recognize as self-evidently true. In other cases, our premises are only hypotheses, assumed to be true but not known to be so at the outset, at least not self-evidently so.[22] In either case, judgments based upon such deductions can be classified as being of the logically perfect sort, though either actually or hypothetically in accord with the degree to which we know the premises to be true (vol. II, p. 181). Sigwart employs the rules of Aristotelian syllogistic in licensing us to pass

21 The history of science is another point at which Sigwart would see psychological investigations as playing an important role since, as he sees it, history is a science dependent on psychology (vol. II, p. vi).

22 As mentioned in the previous paragraph, Sigwart considers deductions in mathematics to be of the first sort, whereas we find examples of the second sort in the empirical sciences.

from one judgment to the next as we work through a deduction. So here we have what we would ordinarily think of as a judgment based on logical deduction.

Sigwart recognizes, however, that we do not always proceed in this way and introduces a second method specifically aimed at discovering the premises of a logically perfect judgment, which he calls 'reduction.' This method, he describes as "the reverse of deduction; it finds premises for given propositions from which they might follow deductively, and it serves to bring into consciousness the highest starting points of deduction" (vol. II, p. 181). While we might apply such a method in finding the basic premises of any sort of judgment, reduction is especially relevant in discovering the logical axioms:

> This is especially true of logical axioms themselves. Locke is perfectly right in saying that many men are all their lives unconscious of the principle of contradiction, although in the concrete case they maintain the incompatibility of affirmation and negation with the fullest conviction. The principle of contradiction, like all other logical principles, is found only by reduction….
>
> (vol. II, sec. 82.5, p. 208)

He explains that we recognize in each particular case that a predicate cannot at the same time both be affirmed and denied of a subject and that this impossibility does not just depend on the particular case at hand. By way of reduction, we look for some general logical principle from which each particular case derives its absolute certainty. Furthermore, much the same can also be said of the discovery of the mathematical axioms, as well the fundamental laws found in the natural sciences since these laws are reached by induction which is just special case of reduction (vol. II, sec. 82.5, pp. 208–9; sec. 82, p. 203). In the next section, we will see that Russell also appeals to such a method. The method of reduction is essentially what he identifies as his inductive, or regressive, method for discovering the premises of mathematics and of the sciences more generally.

After laying out the rules for reaching perfect judgments by way of logic, Sigwart introduces a second role that logic has within methodology—the task of systematizing our knowledge so as to make apparent the logical and conceptual relations found among its various branches. Here, he says, we strive "to represent all the knowledge attained to at any given time as a whole of which the parts are all connected in logical relations" (vol. II, sec. 103, p. 508). This part of methodology he identifies as *system*:

> When the totality of knowledge thus obtained is regarded at any given time as relatively complete, and perception has covered the universe accessible as completely as present limitations will allow,

> then the need arises of surveying the whole, of arranging the results of knowledge in a comprehensive inventory, of representing them as parts of a comprehensive whole, and of expressing the relation of the parts to the whole by means of logical relations. Such an arrangement of our knowledge into a whole is called a SYSTEM.
>
> (Sigwart, Logic, vol. II, sec. 103.1, p. 509)

As throughout the work, it is that form of knowledge found in the mathematical and natural sciences that he has most clearly in mind here.[23] Sigwart's basic idea here is a pre-formal axiomatic approach to the sciences. Such systematization Sigwart describes as taking two forms. The one, *systematic deduction*, he says, organizes a body of knowledge by way of deductive relationships using syllogistic logic.[24] In this case, we try to locate a small body of first principles—either axioms or hypotheses—from which the rest of the science can be deduced. The other, *systematic classification*, or a division of concepts[25]—applied primarily to the empirical sciences (vol. II, sec. 64.4)—he says, organizes a body of knowledge by way of the links among its concepts, from higher to lower, that is, from general to specific, where the more specific concepts are those most directly connected to perception (vol. II, sec. 103.2–4, pp. 509–11).

While the systematization of knowledge in either form serves the purpose of gaining clarity generally, a more specific benefit that Sigwart highlights is that the deductive approach traces the degree of certainty through any body of knowledge. As we have seen already, Sigwart distinguishes axioms from hypotheses in that the former are self-evidently true, whereas the truth of the latter may still be questioned. Sigwart seems to assume that since the same degree of certainty is preserved from premises to conclusion, a body of knowledge that ultimately rests on axioms will be itself self-evident and certain. Mathematics, he claims, has this status. In contrast, hypotheses, such as the laws of gases in mechanics, are arrived at only through a process of induction and so conclusions drawn from these hypotheses will only be as certain as the induction upon which those hypotheses rest. The deductive arrangement of a theory then allows us to trace the degree of certainty obtained throughout the entire system (vol. II, 103.3, pp. 509–10; also vol. II, 103.25, pp. 527–8). Again, in discussing Russell in Section III, we will see that he, too, aims at such an arrangement of knowledge, taking it to be largely what remains as the central task of philosophy once we give up foundationalist approaches to knowledge.

23 For example, Sigwart appeals to the deductive development of a natural science from its basic laws to illustrate his notion of system.

24 He lays out his system of syllogistic logic primarily in secs. 79–82.

25 Sandra Lapointe has informed me that Sigwart's method here is very much like that of Herbart. While Sigwart does cite Herbart in a number of other places in *Logic*, Herbart is not cited in this discussion.

While this deductive arrangement of knowledge begins with axioms and hypotheses from which the more specific claims of a science can be deduced,[26] the arrangement of knowledge by way of classification typically follows the reverse order. Here, we begin with the empirically given concepts that make up the specific claims of a science and then, by way of a process of abstraction, obtain concepts of greater and greater generality. We begin with the empirical concepts since these concern the things given to us in perception, and these are what "our most certain knowledge refers to…." So, Sigwart explains, "[W]e must therefore start from these data as being the most firmly established" (vol. II, 103.4, p. 511; 103.12, pp. 515–16). Although these judgments of perception are the most certain, they cannot fulfill the role of self-evident axioms. Since perceptive judgments are unique to an individual, they do not satisfy the requirement that axioms be composed of ideas that are the same for all individuals. Given this situation, Sigwart concludes then that we are left to take the existence of an objective external world only as a postulate:

> It cannot be allowed, then, that the general propositions which guarantee the objective validity of our judgments of perception, are obvious as simple, self-evident truth; nor that we find them in a form which, by itself, makes the reference of our perceptions to an existing thing, and of certain perceptions to a certain thing, *a priori* certain. But it still remains open to us to acknowledge the existence of an external world which is the same for all, as a postulate of our search for science and knowledge which we cannot avoid believing, although we recognize that it is self-evident.[27]
>
> (vol. I, sec. 48.4, p. 322)

This postulate itself, he explains, can only be justified through his methodological investigation where

> [t]he history of science shows us a continual process which enters upon a new stage whenever our hypotheses lead to contradictions. There is no other confirmation of our belief that any given thing is, than the complete consistency of all our judgments with reference to the existent; the return of the circle unto itself.[28]
>
> (vol. I, sec. 48.4, p. 324)

26 Although, as we saw, these axioms and hypotheses might first be reached by the reverse process of reduction.

27 In a footnote, Sigwart adds that his view here seems to be in agreement with Baumann's realism.

28 Russell argues similarly for the hypothesis that there is a reality corresponding to our sense data in both POP, pp. 22–5 and OWEW, pp. 102–4. He observes, however, that there is nothing logically impossible in the counter hypothesis that reality is all just a dream. For this reason, Russell adds that the hypothesis of an external reality is the simpler of the two and also results in greater systematization.

Through the reaching of concepts of greater and greater generality, this abstractive process eventually leads us to the ultimate a priori constituents of matter, "which are of a purely mathematical nature…" (vol. II, 103.25, pp. 527–8). "Thus," he concludes, "it is that the highest and most general concepts finally relinquish all sensuous content; they are logico-mathematical schemata, in which we try to grasp the essence of the existent, and have their origin in the nature of our thought" (vol. II, 103.25, p. 528). Sigwart's discussions of exactly how this goes are fairly condensed, but the basic idea seems to be that as we abstract away from particular features of reality, we eventually arrive at matter as such. Matter, apart from any specific kind of matter, is undifferentiated by any particular attributes, falling then under the concept of the continuous, and involves relations much like those found in considering spatial magnitudes. In this way, matter in its most abstract form appears to consist of magnitude and geometrical concepts (vol. II, sec. 78.2, p. 169).

In addition to this aspect of classification, since judgments of perception are about the things that constitute the physical universe, such a

> classification of the totality contained in the universe would be, if we imagine it complete, the final and perfect result of all empirical investigation, the conclusion of all the processes we have been considering, the all-embracing completion and logical perfection of knowledge.
>
> (vol. II, 103.6, p. 512)

Given the current state of our actual knowledge, however, Sigwart remarks on the impossibility of completing such a perfect classification from a single highest concept, thus distinguishing his own project from that of speculative *Naturphilosophie*, which, he says, has "been rightly ridiculed" for making claims of just this sort (vol. II, 103.10, p. 514).[29] Instead, he proposes that, while keeping this ultimate aim in view, we aim at the most practical logical arrangement of knowledge given our current means for doing so. Instead of a single complete system of all concepts, he urges that we focus on specific systems, for example, just the physical concepts. Whether we will ever unify these into a single system, for example, with the mental concepts, will remain an open question perhaps to be resolved by the further progress of science and philosophy (vol. II, 103.11, p. 515). The unification of the sciences remains, then, largely an empirical question to be sorted out from within the sciences themselves rather than a prescientific philosophical commitment. Here is a final conceptual connection to Russell. In the next section, we will see him also taking up this criticism of speculative metaphysics,

29 Dendy translates this as "speculative natural philosophy."

distinguishing his piecemeal scientific approach to philosophy from that of the grand metaphysical systems of the past.

From the earlier discussion, we see that Sigwart clearly does not favor the speculative metaphysics of post-Kantian idealism. Still, he does not reject metaphysical pursuits altogether as some later scientific philosophers do. I take it that he understands what constitutes metaphysics broadly, so as to include not just theorizing over some deeper, hidden reality that goes beyond what science has to offer but also claims to knowledge of such a deeper reality. Instead, he takes a cautious and critical attitude towards the subject, attempting to carefully track where metaphysical assumptions set in and maintaining a careful boundary between the logical and metaphysical aspects of his project. For example, he tells us that his approach to logic as a theory of knowledge will avoid falling "into disputed regions of metaphysics…." (vol. I, sec. 1.6, p. 8). And just after laying out his view that methodology is the aim of logic, he notes that

> we exclude all questions relating to the metaphysical significance of the processes of Thought, and keep strictly to the prescribed limits within which we regard Thought as a subjective function. We do not extend our claims upon it so far as to demand a knowledge of Being, but limit them to the sphere of that necessity and universal validity which, even in ordinary language, are always and everywhere regarded as the distinguishing and essential characteristics of what is logical.
>
> (vol. I, sec. 4.4, p. 21)

Furthermore, just as with his urging that methodological investigations start by looking to science in its given state, he sees no essential distinction between science and the proper understanding of metaphysics:[30] "It is not by its method that metaphysics is distinguished from other science (this must ultimately be the same for all knowledge), but by the universality of its problem, a problem as necessary as that of knowledge in general." And so metaphysics should be answerable to the current state of our scientific knowledge:

> In that it brings to light the principles presupposed in all scientific effort, metaphysics stands at the beginning of all science; and in that its assumptions can only be verified by the result, by the complete occurrence of all knowledge, it stands at the end of science.
>
> (vol. II, sec. 105.7, pp. 555–6)

30 The attitude here is much like that found in Herbart, another author with whom Russell was very familiar. Sigwart, too, cites Herbart in a variety of places throughout *Logic*. I thank Sandra Lapointe for pointing this out to me.

As we saw already with methodology, metaphysics, too, does not rest upon some form of direct philosophical insight distinct from what we find in the sciences. Its methods must be the same as for any other branch of knowledge and is distinguished only by its generality. While in a sense, metaphysics gives the grounds for all science, a correct metaphysics can only be judged by the results it yields within science. If metaphysics is to be pursued at all, it must be done so only to the extent that science demands and is judged only by the success of its results from within science. Metaphysics, as Sigwart understands the subject, becomes the concern of logic, that is, logic inclusive of methodology, and is distinguished ontologically only in being the most general of all ontologies.

III

Let me now turn directly to Russell. Russell's most developed statements of scientific philosophy come in 1914, in a series of works that include "On Scientific Method in Philosophy," "Mysticism and Logic," "The Relation of Sense Data to Physics," and his book *Our Knowledge of the External World*.[31] In these works, we find all of the themes described in Section I: a renewal of the cooperation between philosophy and the sciences, a concern with the logical connections among the concepts of the special sciences, and a critical attitude towards philosophers who see philosophy as primarily concerned with developing a substantive metaphysical account of the world. And also like the earlier scientific philosophers, Russell was particularly concerned with the lack of progress found in recent philosophy in contrast to that found in the sciences. Russell gives little to no indication of who might have influenced him in this approach to philosophy. It is of course possible that it was completely original to Russell's own thinking. In this section, however, I will argue that Russell's program for scientific method in philosophy had important historical precedents in a variety of philosophers that he had read especially during his idealist period,[32] and so, did not emerge out of a philosophical vacuum. As stated at the outset of this chapter, I will focus specifically on some parallels found between Russell's views and those of Sigwart discussed in the previous section. While we will

31 I think the much earlier "Mathematics and the Metaphysicians," from 1901, also fits in with some of the key aspects of scientific philosophy. I discuss this further in a footnote later.

32 Among these are Helmholtz, Herbart, Lotze, Wundt, Kant (in at least a sense), and, of course, my focus here, Sigwart. None of these figures receives the high praise, at least with regard to logic, that Russell pays to Sigwart. This of course does not mean that he could not have been influenced by them still, whether knowingly or not. My aim here is to show that Russell spent his formative years surrounded by a number of thinkers urging the scientific approach to philosophy that I have been using Sigwart to illustrate.

see various aspects of Sigwart's methodological project reappearing in Russell, the point I wish to emphasize most is that Russell, too, emphasizes that there is no special, extrascientific, philosophical source of knowledge. In light of this, we should see the central aim of Russell's scientific method in philosophy as much the same as Sigwart's. Russell's goal is not some version of foundationalist empiricism, setting all of our scientific knowledge upon an indubitable foundation of sense data.[33] Rather, he, too, aimed at the reorganization of our scientific knowledge, so as to distinguish the more from the less dubitable claims. Certainty, then, was never his aim.[34] Rather, Russell sought to highlight where best to make changes within a scientific theory (where this included also the mathematical sciences) in light of some form of recalcitrant data.

Now, of course, Russell presents these 1914 views quite some time after his initial mention of Sigwart in his 1897 *Essay on the Foundations of Geometry*, but there is more continuity here with his later philosophy than the years alone might suggest. In his fourth year at Cambridge, Russell turned to philosophy, after studying mathematics in the previous three years, and fell under the influence of his teacher, James Ward.[35] Ward himself was particularly concerned with issues in the burgeoning field of psychology, and his 1886 entry in the *Encyclopedia Britannica* was for quite some time the standard reference on the then current state of the field. Unique to Ward among Russell's teachers and particularly important to my account here is that Ward thought that philosophy should begin with the sciences rather than seek to provide them with some epistemological or metaphysical foundation apart from them.[36] In addition, it was Ward specifically who introduced Russell to Sigwart's writings (Griffin 1991, p. 41).

Adopting this view for his own philosophy, Russell took on the rather ambitious philosophical project, which began with his 1897 book on the foundations of geometry, of reworking of Hegel's *Encyclopedia of*

33 A fairly standard reading of Russell's project is to see it as the culmination of British empiricism.

34 This view can be found throughout Russell's writings; see, for example, his *Problems of Philosophy*, p. 25.

35 Russell also attended lectures by Sidgwick and Stout during this period, but according to Griffin, their influence on Russell at this time, especially that of Sidgwick, was minor compared with the influence of Ward. Griffin bases this conclusion on an examination of Russell's student papers and notes (1991, pp. 31–45). Griffin notes that Stout, at the time, was a relatively junior philosopher—a former student of Ward, 17 years younger than Ward, and with no significant independent publications yet—and so, would have seemed the less impressive figure to the young Russell (p. 34), though in the years following the 1890s Stout became the more significant influence (pp. 33–4).

36 On this aspect of Ward's thought specifically, see Griffin (1991), p. 37. Russell takes this approach in his *Essay of the Foundations of Geometry*, following Kant (sec. 7–8). Ward was himself a Kantian of a sort; again, see Griffin (1991).

the Philosophical Sciences.[37] Russell aimed to do so by adhering to the actual findings of current scientific research. This contrasted with the typical Hegelian approach of starting from a particular metaphysical standpoint and then trying to develop the various sciences from it, as well as from the approach typically taken by the empiricists of starting from a purely empirical data or from common sense and then working back to the claims of the various science. Here, Russell was again most likely working under the influence of Ward, who also urged that philosophy begin with the various sciences and then work out the accompanying metaphysics from there (Griffin, 2003, pp. 88–9).[38] Following his work on geometry, Russell then moved to the philosophy of physics, particularly to issues in dynamics. This eventually brought him to the foundations of mathematics by way of the philosophy of continuity and infinity.[39] We saw already that Russell cites Sigwart as an important influence in the preface to his *Essay on the Foundations of Geometry*, but Russell continued to see Sigwart as a significant figure as Russell turned towards dynamics. Sigwart's *Logic* appears on one of Russell's 1897 reading lists on the philosophy of dynamics (Russell, CP, vol. 2, p. 493),[40] and the book stands out here in being only work on the list having no explicit connection to dynamics or the philosophy of physics more generally. Again, I conjecture that it is the general approach found in Sigwart—that of beginning with the sciences rather than with a prior philosophy—that remained important to Russell's thinking here.[41]

Russell's detour into the foundations of mathematics engaged him almost exclusively for nearly the next ten years, but upon completing *Principia Mathematica* around 1910, he took up where he had left off,

37 In "My Mental Development," Russell remarks of his early project, "The scheme was inspired by Hegel, and yet something of it survived the change in my philosophy" (p. 11). I take it that the turn Russell is referring to is his turn to analytic philosophy.

38 This account of Russell's work is developed in detail in Griffin (1991). Griffin also observes that Russell's notes from Ward's lectures show Ward to be extremely well informed about current psychology and physics. In addition, starting while Russell was still completing his Mathematical Tripos, Ward provided Russell with a constant flow of classic philosophical works as well as many books on the philosophy of mathematics (1991, pp. 41–2).

39 Russell recounts this development in the preface to his 1903 *Principles of Mathematics* (xvi–xvii). This was, of course, after he had rejected his idealist beginnings, but again, it also shows a certain kind of continuity in the kinds of problems he was concerned with and in his approach to them.

40 This is the first of two reading lists that Russell had on the subject of dynamics in 1897. It should be noted that the entries through Sigwart's *Logic* are in an unknown hand. The rest of the list is in Whitehead's hand (CP, vol. 2, p. 490).

41 There are, of course, topics generally relevant to the philosophy of physics in Sigwart's work, such as discussions of causality, but the other works on the list are much more clearly focused on issues in the philosophy of physics in and, in nearly all cases, specifically on issues in dynamics.

returning to issues in the philosophy of physics, or what he came to refer to as the problem of matter. As we will see, a particular concern of his in this period was relating sense data to the entities described by physics. Traces of this concern, however, can be found already in some of his 1897 manuscript material (CP, vol. II, pp. 86–7).[42] So, throughout this period, he was continually focused on some aspect of the philosophical analysis of the sciences.[43] This is not to claim that he already had his scientific method of philosophy laid out by the late 1890s, but only that this was clearly a trajectory that he had set himself on quite early.[44] Indeed, it was while focusing on the mathematical problems that dynamics led him to that he developed the more technical logical constructions found in his mature scientific philosophy. Let me turn now to his mature statements of scientific philosophy, so as to show how Sigwart might have served as a kind of historical precedent for such a view.

Russell opens his 1914 "On Scientific Method in Philosophy" by describing two motivations for engaging with philosophical questions: the one, coming from ethics and religion, as is found in the work of Hegel, Plato, and Spinoza; and the other, coming from science, as is found in the work of Leibniz, Locke, and Hume. In these ethical and religious motivations, Russell finds "on the whole a hindrance to the progress of philosophy" and urges instead that philosophers seek inspiration in the sciences (SMP, p. 75). In line with what motivated the scientific philosophy movement against religious and artistic models for philosophy, Russell describes this hindrance as resulting from the sort of subjective

42 Here, Russell specifically mentions trying to work out the relationship between "data of sense" and matter (CP, vol. 2, p. 86).

43 There are concerns from 1903 to 1910 that appear to depart from this view such as Russell's seeming engagement with issues in the philosophy of language, but many interpreters have shown how these concerns actually arise out of the philosophy of logic that was meant to support his logicism. For such a reading, see, for example, Peter Hylton's *Russell, Idealism, and the Emergence of Analytic Philosophy*, ch. 6, or his "Russell's Theory of Descriptions." Russell himself, for example, is careful to note that his concern with meaning is not with the more ordinary sense in which words have meaning but with a special logical sense (POM, p. 47). Russell did later turn more directly towards concerns in the philosophy of language. On this point, see Burton Dreben, "Quine and Wittgenstein: The Odd Couple," pp. 45–8 and 53–6.

44 Russell published "Recent Work on the Principles of Mathematics" in 1901. Although this paper is particularly focused on developments in the foundations of mathematics, it illustrates many of the general themes found in Russell's later scientific philosophy, especially in urging that philosophers study these developments so that modern mathematics can be incorporated into philosophical work. Also significant is that Russell included the paper in his 1918 collection of essays, *Mysticism and Logic*, changing the title to "Mathematics and the Metaphysicians." It is in the collection that all of Russell's essays are collected laying out his mature scientific philosophy. I take it that the change in title was meant to highlight his critical attitude towards more strictly metaphysical approaches to philosophy.

perspective encompassed in attempts to understand the world from an ethical perspective:

> To regard ethical notions as a key to the understanding of the world is essentially pre-Copernican. It is to make man, with the hopes and ideals which he happens to have at the present moment, the centre of the universe and the interpreter of its supposed aims and purposes. Ethical metaphysics is fundamentally an attempt, however disguised, to give legislative force to our own wishes.
>
> (SMP, p. 83)

Instead, Russell urges that philosophy be done in the spirit of modern science, that is, as a collaborative effort among experts, focusing on specific philosophical problems rather than on each trying to come up individually with an account of the universe as a whole.[45] By proceeding in the way that Russell proposes,

> it becomes possible at last for philosophy to deal with its problems piecemeal, and to obtain, as the sciences do, such partial and probably not wholly correct results as subsequent investigation can utilise even while it supplements and improves them.
>
> (SMP, p. 87)

It was by following the opposite tendency towards grand systems that philosophy had become stagnant:

> Most philosophies hitherto constructed all in one block, in such a way that, if they were not wholly correct, they were wholly incorrect, and could not be used as a basis for further investigations. It is chiefly owing to this fact that philosophy, unlike science, has hitherto been unprogressive, because each original philosopher has had to begin the work again from the beginning, without being able to accept anything definite from the work of his predecessors. A scientific philosophy such as I wish to recommend will be piecemeal and tentative like other sciences; above all, it will be able to invent hypotheses which, even if they are not wholly true, will yet remain fruitful after the necessary corrections have been made. This possibility of successive approximations to the truth is, more than anything else, the source of the triumphs of science….
>
> (SMP, p. 87)

45 Sandra Lapointe informs me that Stout also held such a view. This would fit with Griffin's view that Stout seems to have become more important to Russell after his student days (Griffin, 1991, pp. 33–4).

A key, then, to philosophy achieving the kind progress found in the sciences is to take up the methods of science. And in particular, philosophy should restrain itself from the grand metaphysical systems of the past, which led only to stagnation. The general attitude here shown towards the more speculative *Naturphilosophie*, as we saw, is one also urged by Sigwart, and specifically in his dismissal of this tradition's attempts to find a single system unifying all knowledge, a view which Sigwart identified as conflicting with the current state of science.

Russell does not think, though, that there is no task left for philosophy apart from what the sciences already accomplish. As he envisions it, scientific philosophy will still maintain its distinctness from the special sciences; philosophy will be a science but a science of its own rather than just a part of the already established special sciences. Russell puts forward two characteristics as marking out philosophy's specific domain. First, it will be completely general, that is, it will have no specific subject matter of its own in that it will deal only with the universe's most general features, found in any science whatsoever (in a sense, its subject matter is everything). And second, it will be a priori, that is, the empirical results of the special sciences will neither prove nor disprove the results of philosophy (SMP, pp. 85, 86). Philosophy, under this characterization, Russell concludes, "becomes indistinguishable from logic," which consists only of the most general statements concerning anything whatsoever and of the enumeration of logical forms, that is, the identifying of the various kinds of propositions and facts along with the classifying of the constituents of such facts (SMP, p. 86). Despite being distinguished from the special sciences, Russell does not think that philosophy yields any special brand of knowledge not obtainable by the sciences:

> While admitting that doubt is possible with regard to all our common knowledge, we must nevertheless accept that knowledge in the main if philosophy is to be possible at all. There is not any superfine brand of knowledge, obtainable by the philosopher, which can give us a standpoint from which to criticize the whole of the knowledge of daily life. The most that can be done is to examine and purify our common knowledge by an internal scrutiny, assuming the canons by which it has been obtained, and applying them with more care and with more precision. Philosophy cannot boast of having achieved such a degree of certainty that it can have authority to condemn the facts of experience and the laws of science.[46]
>
> (OKEW, pp. 73–4)

46 He expresses a similar view in his 1912 *Problems of Philosophy*, pp. 149–50.

Philosophy and the sciences are meant to be complimentary, to cooperate in their strivings for knowledge. A successful scientific philosophy will place philosophy among the other sciences rather than distinguish itself from them. Russell takes philosophy here to serve as a tool of criticism in that it scrutinizes the principles that science takes for granted, examining carefully what grounds we have for accepting or rejecting them. Only after such scrutiny does philosophy accept such principles.[47] Again, we saw this general attitude also in Sigwart in his championing of the importance of methodology for philosophical investigations and, more specifically, in his rejection of any kind of Cartesian foundationalism concerning the sciences.

What then remains as the central task for philosophy, according to Russell, is akin to something like Sigwart's project of systematization. While Russell maintains that we have no place outside of science from which to criticize the whole of our knowledge, he does think that we may order it by way of its varying degrees of certainty. As remarked in the previous quote, he begins here with what he describes as our common knowledge, which, though vague and inexact, on the whole, demands our assent as true. Most fundamental here is the knowledge gained from direct sensory experience, taking this to be of the most certain. Next, we find knowledge gained by testimony such as is found in newspapers or history books. Here, certainty can vary considerably. In some cases, it is not far off from that of direct sensory experience. For example, Russell observes that we might question the existence of Napoléon only as a sort of joke, whereas there is real historical debate over the existence of Agamemnon. Finally, we have "the systematization of all this knowledge of particulars by means of physical science, which derives immense persuasive force from its astonishing power of foretelling the future." Here again, we find some claims, such as the law of gravity, that command near certain ascent and others, such as claims about the actual constitution of matter, that remain in question (OKEW, pp. 72–5). Russell's scientific philosophy has then the task of carefully scrutinizing our common knowledge (continuing the previous long quote):

> The philosophical scrutiny, therefore, though sceptical in regard to every detail, is not sceptical as regards the whole.... The reason for this universal abstention from a universal criticism is not any dogmatic confidence, but its exact opposite; it is not that common knowledge *must* be true, but that we possess no radically different kind of knowledge derived from some other source. Universal scepticism, though logically irrefutable, is practically barren; it can

47 Russell describes his project this way in *Problems of Philosophy*. While he does not talk of scientific philosophy specifically, much of what he says here reflects the view that he would give a name to in the 1914 works.

> only, therefore, give a certain flavour of hesitancy to our beliefs, and cannot be used to substitute other beliefs for them.
>
> (OKEW, p. 74; Russell's italics)

In his slightly earlier *Problems of Philosophy*, he describes the result of this scrutiny as a sort of hierarchy, or ordering, of our common knowledge—what he here calls our 'instinctive beliefs'—from the most accepted claims to the most tenuous:

> Philosophy should show us the hierarchy of our instinctive beliefs [i.e., our common knowledge]....It should take care to show that ... our instinctive beliefs do not clash but form a harmonious system. There can never be any reason for rejecting one instinctive belief except that it clashes with others; thus, if they are found to harmonize, the whole system becomes worthy of acceptance.
>
> (POP, p. 25)

Although the chance of error can never be completely eliminated, Russell further explains that this hierarchy allows us to identify more easily where error is more and less likely to lie and, in this way, yields a systematic account of our knowledge:[48]

> [B]y organizing out instinctive beliefs and their consequences, by considering which among them is most possible, if necessary, to modify or abandon, we can arrive, on the basis of accepting as our sole data what we instinctively believe, at an orderly systematic organization of our knowledge, in which, though the *possibility* of error remains, its likelihood is diminished by the interrelation of the parts and by the critical scrutiny which has preceded acquiescence.
>
> (POP, pp. 25–6)

While this hierarchy of knowledge may bear some similarity to the foundationalist project of reforming scientific knowledge so that it rests on a basis of first principles obtained by a sort of immediate rational insight,

48 As we will see, the same sort of project is outlined in the 1912 *Problems of Philosophy*, pp. 149–52, which also takes place within the context of his claim that philosophy can offer no higher form of knowledge than what the sciences provide. An even earlier statement of this general approach occurs in his 1906 "The Regressive Method of Discovering the Premises of Mathematics," pp. 279–80. He does not discuss this project in "On Scientific Method in Philosophy," but his idea that philosophy should be piecemeal certainly motivates the need for an ordering of knowledge. He remarks here that philosophy will yield hypotheses that, while not absolutely true, will remain fruitful once the necessary corrections are made. It seems to me that Russell's ordering of knowledge has a central role to play here in determining where such adjustments need to be made (SMP, p. 87).

it is really a naturalizing of such a project, where, though absolute certainty is never attained, we strive from within science to get as close to this ideal as we can.[49] As we have seen, on Russell's view there is no firmer knowledge to be found. There is only the critical perspective on our knowledge offered from within science itself.[50] While it may be that universal skepticism can never be wholly ruled out, the acceptance of such general skepticism has no value for scientific thinking. It is, as Russell said, "practically barren." Again, here we have in Russell's hierarchy of beliefs a harkening back to Sigwart's systematization. As we saw in the previous section, by identifying the basic principles of a given science as either self-evident axioms or as hypotheses, Sigwart, too, looks to trace the degree of certainty through any given body of knowledge. And all the more importantly, as with Sigwart, Russell agrees that unconstrained skepticism leaves natural science unbegun.

Within this context of his scientific philosophy, Russell also takes up a version of the second task that Sigwart thought systematization could accomplish. That is, in observing that the concepts most directly connected to sensory particulars are the most certain, Sigwart then sought to relate them to the more theoretical concepts of science through a process of greater and greater abstraction, that is, through his method of classification. While the notion of abstraction has been notoriously difficult to make precise sense of,[51] Russell does take up the issue of relating sensory particulars to theoretical terms, appealing instead to his logical constructions from sense data.[52] As mentioned at the beginning of this section, a particular concern for Russell following his 1897 geometry book was the status of the matter described by modern physics.

49 In *Problems of Philosophy*, Russell explicitly situates his work within the context of Descartes' philosophy; see ch. 2 especially. Russell places his views within a similar context for his views in *Our Knowledge of the External World*, observing at one point that "Modern philosophy, from Descartes onwards … still believed … that *a priori* reasoning could reveal otherwise undiscoverable secrets about the universe, and could prove reality to be quite different from what, to direct observation, it appears to be" (p. 16).

50 He does think that Descartes' method of doubt is not the completely barren universal skepticism of other philosophers and that this method can be usefully adopted as providing a critical perspective on our knowledge claims (POP, pp. 150–1).

51 Berkeley is probably the classic critic of abstraction; see, for example, his *Treatise Concerning the Principles of Human Knowledge*.

52 In *Problems of Philosophy*, Russell does adopt something like Sigwart's method of classification. He describes here a process of subsumption in the empirical sciences leading us to wider and wider generalizations. He concludes here,

> The progress of science is constantly producing such subsumptions, and therefore giving a constantly wider inductive basis for scientific generalizations. But although this gives a greater *degree* of certainty, it does not give a different kind: the ultimate ground remains inductive….
>
> (pp. 106–7; Russell's italics)

Returning to this topic in 1914, he describes a worry arising from the supposed verifiability of physics; that is, based upon observations and experiments, we can calculate in advance results that are later confirmed or disconfirmed by observation. According to modern science, however, all we have access to by way of observation are the immediate data of sense—patches of color, sounds, tastes, smells, and so on. The matter described by physics, in contrast, is made up of entities such as molecules, atoms, and electrons, all lacking in such sensible qualities. It seems then that the reality described by physics is wholly inaccessible to us by way of the means described by science itself. How then could we ever establish a correlation between these immediate sensible objects and the non-sensible objects supposed by modern physics as their cause? As Russell describes the problem:

> But how is the correlation itself ascertained? A correlation can only be ascertained empirically by the correlated objects being constantly *found* together. But in our case, only one term of the correlation, namely, the sensible term, is ever *found*. Therefore, it would seem, the correlation with objects of sense, by which physics was to be verified, is itself utterly and forever unverifiable.
>
> (RSDP, p. 113; Russell's italics)

It seems that, in this way, science posits a reality unknowable by scientific means. We have in this way returned to an unknowable thing-in-itself (OKEW, p. 83).

One option Russell describes would be to just accept an a priori principle establishing the desired correlation and thus eliminating any empirical basis for it. While many philosophers have followed this path, Russell recommends against it since physics itself would then cease to be an empirical science. Instead he proposes that we define the objects of physics as functions of sense data. Towards this end, Russell attempts to construct the world of physics from logic and sense data.[53] Putting aside much detail, Russell says, for example, that we might take the objects of common sense, such as tables and chairs, to be collections of their various appearances, both sensed and unsensed (RSDP, p. 120). Through similar, though increasingly complicated constructions, he aims to eventually arrive back at the fundamental objects of physics, thus demonstrating that there need be no gap between the world of sense and that of physics. While his technique differs considerably from that employed by Sigwart, we see again a certain affinity with Sigwart in Russell's respective attempts to forge a link between observable elements

53 Here, we have Russell's version of the kind of conceptual unification found in the tradition of scientific philosophy generally.

of empirical science and the more theoretical ones. And to bring us back to our starting point, this is just the sort of conceptual unification that we find throughout the tradition of scientific philosophy more generally.

In addition, Russell's constructions also have the benefit of avoiding, what he describes as gratuitous metaphysical assumptions, among them precisely that of a hidden, inaccessible underlying reality, again suggesting Kant's thing-in-itself (OKEW, pp. 111–12). Taking his cue from his early logicist project of constructing mathematics from a purely logical basis, Russell describes his method as follows:

> Given a set of propositions nominally dealing with the supposed inferred entities, we observe the properties which are required of the supposed entities in order to make these propositions true. By dint of a little logical ingenuity, we then construct some logical function of less hypothetical entities which has the requisite properties. This constructed function we substitute for the supposed inferred entities, and thereby obtain a new and less doubtful interpretation of the body of propositions in question. This method, so fruitful in the philosophy of mathematics, will be found equally applicable in the philosophy of physics….
>
> (RSDP, p. 122)

Given his commitment to there being no higher form of knowledge than what science itself can offer, the lessening of doubt described here is not in the service of defeating skepticism. Rather, Russell aims merely to avoid more questionable assumptions, keeping his philosophy of physics as close as possible to the observable. He also describes his method as adhering to Occam's razor in avoiding unnecessary metaphysical assumptions (OKEW, p. 112), summing up this approach in his supreme maxim for scientific philosophy: "*Wherever possible*, *logical constructions are to be substituted for inferred entities*" (RSDP, p. 121; Russell's italics). While his method here does not rule out metaphysics altogether, it is now constrained, as in Sigwart, by the new scientific approach to philosophy:

> And on the theoretical side, ultimate metaphysical truth, though less all-embracing and harder of attainment than it appeared to some philosophers in the past, can, I believe, be discovered by those who are willing to combine the hopefulness, patience, and open-mindedness of science with something of the Greek feeling for beauty in the abstract world of logic and for the ultimate intrinsic value in the contemplation of truth.
>
> (OKEW, p. 24)

Russell sees his methods as ruling out versions of speculative metaphysics that would countenance such things as a hidden underlying reality,

inaccessible to the methods of science, or that would make claims to a special source of knowledge accessing just this sort of reality. Metaphysics survives in Russell, however, in making claims, for example, about what the fundamental structure of reality is like. But as we have seen, such investigations are now carried out from a philosophy that is inseparable from science itself.

Let me conclude by drawing one further link back to Sigwart. In all of this, a question remains as to how we discover the basic principles of any given science, as these will be crucial to Russell's program, especially in his attempts to provide the ordering of knowledge discussed previously. Again, we might propose some form of direct rational insight that gives us immediate insight into an axiom's truth. This is clearly out of place within a philosophy that takes current science as its starting point. Recall that on this issue, Sigwart proposed his method of reduction. We attempt to work backwards from a body of knowledge as given until we reach the axioms or hypotheses at its basis. Strikingly, this is just the approach that Russell describes as his regressive, or inductive, method for discovering the basic principles of both mathematics and the natural sciences more generally. Russell's most extended discussion of this method appears in his 1907 "The Regressive Method of Discovering the Premises of Mathematics."[54] Although he is primarily concerned here with the premises of mathematics, his discussion gives full consideration to the premises of the natural sciences as well. He observes that in both the foundations of mathematics and in the natural sciences, the premises for our conclusions tend to be much less obvious than the conclusions themselves. For example, we assent much more readily to a truth of arithmetic such as $2 + 2 = 4$ than we do to many of the axioms from which such a truth can be proved in a system such as that of Russell and Whitehead's *Principia Mathematica*. Similarly, Russell observes that the individual observations, or facts, in the natural sciences are much more obvious than the general laws from which they can be deduced. "Hence," Russell concludes,

> we tend to believe the premises because we can see that there consequences are true, instead of believing the consequences because we know the premises to be true. But the inferring of premises from consequences is the essence of induction; thus the method in investigating the principles of mathematics is really an inductive method, and is substantially the same as the method of discovering general laws in any other science.
>
> (RM, pp. 273–4)

54 This view also appears in a more succinct form in both the 1910 and 1925 editions of *Principia Mathematica*, p. v and p. 59. I take it that it is a view then that Russell remained committed to throughout the period under discussion. Here, he describes the method as inductive.

He sketches a similar approach in *Our Knowledge of the External World*, here specifically describing his method in terms of reduction:

We start from a body of common knowledge, which constitutes our data. On examination, the data are:

> found to be complex, rather vague, and largely interdependent logically. By analysis we reduce them to propositions which are as nearly as possible simple and precise, and we arrange them in deductive chains, in which a certain number of initial propositions form a logical guarantee for all the rest. These initial propositions are *premisses* for the body of knowledge in question … The discovery of these premisses belongs to philosophy; but the work of deducing the body of common knowledge from them belongs to mathematics, if "mathematics" is interpreted in a somewhat liberal sense.
>
> (OKEW, p. 214)

In both of these works he specifically connects this process of finding the premises for a given science to his epistemological project of an ordering of knowledge (RM, p. 275; OKEW, p. 215).

I have tried to show in this chapter by way of examining Sigwart's logical work that Russell's scientific philosophy—especially its naturalistic elements—has important precursors in the methodological aspects of 19th-century logic. Analytic philosophy has all too often viewed its logical tradition as beginning with the new logic of Frege and Russell, neglecting nearly all that came before. But this loses all sight of the valuable contributions of 19th-century logicians to the sorts of methodological projects that would become central to much of 20th-century analytic philosophy of science. More generally, we see in the logical work of Sigwart, and others like him, the emergence of a kind of philosophizing that looked for reunion with science, so that philosophy might partake in the kind of progress the sciences have achieved. The desire for such progress was to become a central theme in Russell and his followers, Carnap and Quine not least among them.[55]

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55 For comments on an earlier draft of this chapter, which greatly improved it, I thank Sandra Lapointe. A version of it was also presented at the 2016 meeting of the Society for the Study of the History of Analytical Philosophy. I thank the audience there for their helpful comments, especially Landon Elkind, Peter Sullivan, and Clinton Tolley. I also thank McMaster University for the award of a 2013 Bertrand Russell Archives Visiting Researcher Stipend as it was while visiting the archives that ideas for this chapter began to take shape.

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11 Kant and Formalism

Hilbert, Russell and Whitehead

Nicholas Griffin

The reputation of great books in philosophy precedes them – you often feel you know them long before you actually get around to reading them. This is even more frequently so with great books in mathematics. It was certainly true of myself in respect of David Hilbert's *Foundations of Geometry* (1899). I knew it was the first rigorous axiomatization of Euclidean geometry, and I knew of Hilbert as a formalist, and I thought of formalism as the diametrical opposite of anything Kantian in the philosophy of mathematics. A formalist axiomatization of geometry would use undefined terms, the meanings of which would be given entirely by the axioms in which they occurred. A Kantian account of geometry, on the other hand, would be couched in terms of spatial intuition – that 'lazy limbo of mystery', as Russell ([1899b], p. 106) called it. Surely, Hilbert would be the one to wrench geometry away from the long tradition of intuitive interpretation – physical, visual, imaginative, or other – that had dogged its development through the 19th century,[1] which, if it hadn't originated with Kant, had been given new life and new respectability by him. In this Kantian tradition, where proofs had not been found, geometry could be conjured from the obscure *a priori* recesses of the mind by hand-waving and a magical invocation of 'intuition'. Even worse, where proofs were available and their results offended philosophical preconceptions, the results could be dismissed by the same magical means, as inconsistent with our spatial intuition. From this Kantian nightmare, I thought Hilbert had rescued geometry.

Imagine, then, my amazement when I came to read *Foundations of Geometry* for the first time and found it had an epigraph from Kant: 'All human knowledge thus begins with intuitions, proceeds thence to concepts and ends with ideas'.[2] Not only that, but in his very short Introduction, Hilbert says that his task is to establish the axioms of geometry, and that this is 'equivalent to the logical analysis of our perception of

1 On this tradition in England, see Richards [1988].

2 *Critique of Pure Reason*, A702/B730, as quoted in Hilbert [1899], p. 2. (The standard Kemp-Smith translation is not significantly different.)

space' ([1899], p. 2). This, from the founder of formalism: finding the axioms of geometry is equivalent to the logical analysis of *our perception* of *physical space*, for it must be physical space if we perceive it. And finally, in introducing his five groups of axioms, Hilbert says that each group 'expresses certain related facts basic to our intuition' ([1899], p. 3). This is hardly formalism at the start of the 20th century; it's more like Kantianism at the middle of the 19th century. Now, admittedly, apart from this one occasion on the first page, Hilbert never uses the word 'intuition' again in the entire book. But what is going on? Is this to be taken seriously? Or is it the kind of ritual homage that led Soviet scholars in the 1930s to have epigraphs from Stalin, or North American English professors in the 1980s to acknowledge their debt to Derrida?

There is no denying the enormous influence that Kant had on 19th-century thought, both philosophical and scientific. There is no more striking testimony to that than Russell's astonishing remark in *My Philosophical Development* that initially he thought of *Principia Mathematica* as a 'parenthesis in the refutation of [Kant]' (Russell [1959], pp. 74–5). Indirectly, of course, the remark makes it clear that Russell thought that the rejection of Kant's philosophy was a necessary step toward creating a tenable philosophy of mathematics. Hilbert – perhaps because, unlike Russell, his primary concerns were mathematical, rather than philosophical – felt no such necessity. And this divergence with respect to Kant, between two mathematical thinkers both working on similar agendas, is worth noting. Moreover, Hilbert's remarks in *Foundations of Geometry* go beyond a simple neutrality about Kant's philosophy: they suggest that he saw the *Foundations* as the carrying out of a Kantian project.

It is, in fact, rather difficult to make out how indebted Hilbert was to Kant. Kant is mentioned by name only in the epigraph of the published book, but Hilbert's lectures on the foundations of geometry which preceded the book, and those which immediately followed it, have more to say about intuition than appears in the book.[3] In the earliest of these lectures (on projective geometry in 1891), he divides geometry into three parts: the geometry of intuition, axiomatic geometry, and analytic (or Cartesian) geometry, in which geometry is reduced to analysis (Hilbert [2004], pp. 21–2). The geometry of intuition is based on 'simple facts of intuition' (p. 21), its concerns are pedagogical, practical and 'ästhetisch',[4] and Hilbert divides it into school geometry (congruence, triangles, polygons, circles, etc.), projective geometry and *analysis situs*. Hilbert regarded analytic geometry as having detached geometry from its intuitive roots, a process that was to some degree reversed in the 19th

3 The relevant texts are printed in Hilbert [2004].

4 I keep the German word because it is not clear to me whether Hilbert intends it in the sense in which Kant used it or in the more usual sense, alluding perhaps to such things as the use of proportion and perspective in painting.

century by the development of projective geometry. In the 1891 lectures on projective geometry, Hilbert uses the 'simple facts of intuition' to identify eight 'fundamental laws of intuition' (p. 28). These are essentially the incidence axioms of projective geometry, though Hilbert does not call them 'axioms'. Nonetheless, in identifying axiomatic geometry as the second division of his subject, Hilbert characterizes it as the investigation of the axioms underlying the geometry of intuition (p. 22).

Hilbert's use of the axiomatic method in these early lectures was nowhere near so well developed as it became in subsequent lecture courses, and, of course, in his published book. It was in the next published lecture course (Lectures on the Foundations of Geometry, 1894) that he starts to deploy it seriously for the first time. In doing so, he makes it quite clear that he regards geometry as a natural science based on experience (Hilbert [2004], p. 72) and that the axiomatic treatment of it is intended to supply an analysis of the simple facts of intuition that experience reveals. One very straightforward way in which this reveals itself is in his tendency (familiar from the published book) to arrange his axioms in groups, each group (as he says in the book) expressing related basic facts of intuition. These features remained constant through the 1899 book and beyond. Geometry continues to be viewed as a natural science in the Lectures on Euclidean Geometry (1898–99), the immediate predecessor of the book (cf. Hilbert [2004], p. 221), and its axiomatization continued to be an analysis of 'our power of intuition' (*ibid*., p. 230).[5] It seems clear, therefore, that Hilbert's remarks in his book to the effect that the axiomatization of geometry involves an analysis of our spatial intuition were entirely serious.

Moreover, he uses the term 'intuition' elsewhere and quite frequently, especially later when describing the objects presupposed by his finitary arithmetic. These, he says, are 'extralogical concrete objects… given to our faculty of representation… that are intuitively [*anschaulich*] present as immediate experience prior to all thought' – and he even brings in Kant's good name once again to bless the whole proceeding.[6] This certainly shows that some Kantian influence was enduring. And, indeed, it may be that Kant's influence got stronger during the 1930s, possibly as a result of his collaboration with Paul Bernays, who seems to have been more Kantian than he.[7] But these Kantian elements of Hilbert's thought – either in 1899 or later – do not seem to be doing a great deal of work. In his mature, formalist system (from about 1922 on), their sole

5 Cf. also *ibid*., p. 303 and, for both points, the 1902 lectures on the Foundations of Geometry (Hilbert [2004], pp. 540, 541).

6 Hilbert [1926], p. 376 (rearranged). Hilbert makes the same claim in very similar terms in many writings in the 1920s.

7 Bernays apparently was close to the neo-Kantian philosopher Leonard Nelson, who worked at Göttingen until his death in 1927. See, for example, the opening pages of Hilbert [1931].

purpose, so far as I can tell, is to furnish a series of elements – viz. a sequence of numerals |, ||, |||, etc. – on which models can be constructed. At this point, one might have expected a transcendental argument, but (at least in the texts I've seen) Hilbert doesn't offer one. He says merely that such objects are 'requisite for mathematics and… all scientific thinking, understanding, and communication' ([1926], p. 376) – a transcendental claim, maybe, but hardly an argument. It seems that, at best, Hilbert's Kantianism covered the very first step in the development of finitary arithmetic and went no further. Its purpose in Hilbert's formalism seems to have been to provide a philosophically credible back story for the parts of the system that were really beyond the purview of mathematics. And even there, its purpose seems only terminology-deep – Kantian language is used but the Kantian concepts behind it are not really seriously exploited.[8] And this is as it should be – one doesn't need to be a Kantian in order to do model theory. On the other hand – and perhaps more surprisingly – being a formalist does not necessarily set one in opposition to Kant.

The axioms in Hilbert's *Foundations of Geometry* are stated in terms of points, lines, and planes; but, as Hilbert famously remarked, one could equally well talk of tables, chairs and beer mugs.[9] In this he was surely not looking back either to some kind of physicalism or to alternative Kantian intuitions, but forward to model theory. The physical (or other) objects on which points, lines, and planes could be modeled, and whatever intuitions one might have about them, played no role at all in explaining the nature of those concepts. 'Point', 'line', and 'plane' were expressions that had no meaning in the system beyond that supplied by the axioms which governed their use.[10] The result was a structure that could be modelled on many different kinds of things. The design of the structure was guided by our knowledge of Euclidean space, but the structure itself – 'a framework of concepts', as Hilbert was fond of describing it (Hilbert [2004], pp. 72, 104 (1894); p. 540 (1902) – was no more about Euclidean space than it was about any other object which satisfied its axioms. In all of this, it is hard to see Kantianism playing any role at all, and the extent of Hilbert's debts, real or imagined, to Kant remains to me a mystery.

8 A direct realist empiricist, for example, could well claim that physical objects would serve as the 'extralogical concrete objects' that Hilbert's finitary arithmetic requires.

9 This frequently cited remark was made in connection with a lecture in 1892 (cf. Blumenthal [1935], pp. 402–3), well before the logical analysis of our perception of space in 1899.

10 As Hilbert explained to Frege in a letter of 29 December 1899 (Frege [1980], p. 39). The point is not explicitly made in the book, which was why Frege had raised the matter.

Unlike Hilbert, Russell had taken his Kantianism seriously. Indeed, he had taken it much more seriously than perhaps any other philosopher whose primary concerns were in the philosophy of mathematics. In *An Essay on the Foundations of Geometry* (1897), a book which (despite the similarity of title) was as different from Hilbert's as chalk from cheese, he had attempted (among other things) a transcendental deduction of projective geometry from the Kantian concept of a form of externality. The results were quickly attacked from two different directions. G.E. Moore [1899] argued that Russell, his claims to the contrary notwithstanding, had failed to avoid the deadly sin of psychologism. On the other hand, Poincaré [1899] complained that the axioms for projective geometry that Russell claimed to derive from the form of externality were mathematically vacuous. Both failings were the result of taking Kant too seriously. In response to Moore, Russell came to the conclusion that the entire technique of transcendental arguments was inevitably psychologistic. If a transcendental argument was purely logical (as Russell in the *Essay* had claimed that his were), then it could yield no result not obtainable by purely deductive means. In response to Poincaré, Russell could do little but admit his sins, for it was true that his account of the axioms of projective geometry was light-years away from anything approaching the standards of precision appropriate to the subject at the beginning of the 20th century. And this was because he had had to dig the properties of projective space out of the hopelessly vague concept of a form of externality.[11]

In 1898, therefore, Russell reacted strongly against Kant about whom, to the best of my knowledge, he never said another good word.[12] This was an important part of the revolution in Russell's philosophy that took place at the end of the 19th century, but it was only part. It is sometimes thought that Russell's rejection of a Kantian philosophy of mathematics was immediately followed by his embrace of logicism. But this is not the case. He abandoned Kant approximately two years before he discovered Peano, whose logic would make logicism possible. During

11 The three so-called 'axioms' for projective geometry that he offered were the following ([1897], p. 132):

I. We can distinguish different parts of space, but all parts are qualitatively similar, and are distinguished only by the immediate fact that they lie outside each other.

II. Space is continuous and infinitely divisible; the result of infinite division ... is called a *point*.

III. Any two points determine a unique figure, called a straight-line, any three in general determine a unique figure, the plane....

12 Lest this seem anachronistic, Russell heard Moore's criticism of Kant from Moore himself in advance of the publication of Moore's review, at meetings in May and June 1898 and later the same year when he read Moore's second fellowship dissertation (Moore [1898]).

this time he was in the uncomfortable position of realizing that his old approach would not work without having found a new approach that would. Meanwhile, Poincaré had to be replied to.

On the axioms of projective geometry Russell did a complete makeover. In place of the three statements he misleadingly dignified with the title of 'axioms' in the *Essay*, he offered the following in his reply to Poincaré:

Axiom I. There is a class A of objects (A_1, A_2, A_3, ...) such that any two of these objects, e.g. A_1, A_2, uniquely determine another object (a_{12} say) belonging to a different class a. But the object a does not reciprocally determine uniquely the objects A by which it was determined. If the object a_{12} determined by A_1 and A_2 is not identical with the object a_{13} determined by A_1 and A_3, then the three objects A_1, A_2, A_3 uniquely determine an object (α_{123} say) belonging to a new class α, which again does not determine uniquely the objects from which it is determined. Moreover a_{12} and α_{123}, are independent of the order of the determining objects; and α_{123} is also determined by A_1 and a_{23}, or by A_2 and a_{31}, or by A_3 and a_{12}.

Axiom II. Two objects of class α (α_1 and α_2) determine uniquely an object ${}_{12}a$ of class a; and if ${}_{12}a$ is not identical with ${}_{13}a$, then α_1 and α_2 and α_3 determine uniquely an object ${}_{123}A$ of class A, which is also determined by α_1 and ${}_{23}a$. Two objects of class a, or four of class A or of class α, determine nothing. Thus all the objects determined by means of objects in the classes A, a, α belong in turn to these three classes.

Axiom III. When two objects α_{123}, α_{124} are respectively determined by A_1, A_2, A_3 and by A_1, A_2, A_4, the object of class a determined by α_{123} and α_{124} is the same as that determined by A_1 and A_2.

Axiom IV. Three objects α_{123}, α_{124}, α_{125}, determined respectively by A_1, A_2, A_3; A_1, A_2, A_4; A_1, A_2, A_5, collectively determine nothing. Three objects α_{123}, α_{145}, α_{167} (provided the first and second do not determine the same object a as the first and third) collectively determine the object A_1.

Axiom V. Let a_{23} be the object determined by A_2 and A_3, and A the object determined by a_{23} and α_{145}. Then the object a determined by α_{123} and α_{145} is the same as that determined by A_1 and A.

Axiom VI. When two objects ${}_{123}A$, ${}_{124}A$ are respectively determined by α_1, α_2, α_3, and α_1, α_2, α_4, they determine together the same object ${}_{12}a$ as is determined by α_1 and α_2.

Poincaré must have been astonished.

But probably not impressed. Because, when Russell's paper was published in the *Revue de la métaphysique et de morale*, the axioms were merely stated and all of Russell's actual derivation of projective geometry from them was omitted. Russell had declared himself 'not very satisfied

with it' (we shall see why shortly) and Couturat, the editor, thought that it was too technical for the journal.[13] (The full paper as Russell had originally written it appeared only in 1990 in his *Collected Papers*.) The omission of the development of the actual geometry from the axioms must have made the axioms look more like a leap of faith than they actually were. Poincaré, in his reply, ignored them entirely (Poincaré [1900]) – and so has almost everyone else.

Those who have commented (e.g. Torretti [1978] and Griffin [1991]) have hardly been more enthusiastic. Both dismissed the axiom set as incomplete because Russell gives only axioms of incidence and no axioms of order.[14] This was because we failed properly to understand what Russell was up to. In fact, as Sébastien Gandon ([2012], Chapter 1) has pointed out, Russell in 'The Axioms of Geometry' was attempting to develop projective geometry based entirely on the concept of incidence – which might be thought of as the paradigmatically projective concept. (Russell, as we shall see, certainly thought of it thus.) In 1899, he knew he could get close. The key was von Staudt's quadrilateral construction, by which, given three collinear points A, B, D, a unique point C, on the same line and projectively interchangeable with A, could be constructed, by means of incidence alone. The quadrilateral construction was Russell's key to proving a central thesis of projective geometry, that the cross ratio (or, as Russell called it, the anharmonic ratio) of four collinear points was projectively invariant.[15] In projective geometry, there are no invariants involving fewer than four points. In particular, in projective space, given three collinear points, it cannot be said of one of them that it is between the other two.

Now Russell could prove the uniqueness of the quadrilateral construction (AOG, pp. 407–8). But not quite everything was settled because, as Felix Klein [1873] had noted, in order to show by means of the quadrilateral construction that *every* four collinear points had a cross ratio, one needed a proof, not just that the construction was unique, but that every point on the line could be produced by the construction. But how could that be achieved? The construction could at most give denumerably many points, whereas the line was a continuum. Yet unless there was a construction for every point on the line, there was no guarantee that *any* four points on it have a cross ratio, and the fundamental theorem of projective

13 Russell to Couturat, 29 August 1899, and Couturat to Russell, 5 September 1899, respectively (quoted in Russell [1990], p. 392).

14 Coxeter, in a classic text ([1947], pp. 20–2), divides the axioms of projective geometry into two groups: seven axioms of incidence and six axioms of separation (or order) plus an axiom of continuity.

15 The cross ratio of four collinear points A, B, C, D is AC/BC: AD/BD. The ratio is invariant since, if collinear points A, B, C, D are projected to collinear points A', B', C', D', respectively, then $AC/BC : AD/BD = A'C'/B'C' : A'D'/B'D'$. (See Cremona [1893], §63, for a source we know Russell used.)

geometry, that given two sets of three points on two lines there was one and only one projective mapping from the points of the one set into the points of the other, could not be proved. It was a serious defect of Russell's approach, and Russell was aware of it. He acknowledged the problem in the part of the paper that was omitted in the *Revue* ([1899] p. 409) and it was this that led to his misgivings about his axiomatization.[16] Other axiomatizations (e.g. those of Pasch [1882] and Peano [1889]) rescued the fundamental theorem by employing the very axioms of order that were missing from Russell's account.

The problem might seem insuperable, for the concept of cross ratio is an inherently ordinal notion; how is such a thing to be extracted from axioms of incidence only? Ironically, the solution was already to hand, though Russell was not yet aware of it. In 1898, Mario Pieri showed that, given three new axioms (over and above axioms of incidence of the sort Russell had supplied), it was possible to derive those parts of projective geometry that depended on ordinal notions like cross ratio. Now the natural way to express these axioms is in terms of order,[17] making it look as if Torretti and I were correct and that Russell should have added axioms of order in 'The Axioms of Geometry'. In fact, however, Russell's instincts were good (much better than Torretti and I had supposed), for Pieri's achievement was to show that one did not need the concept of order to state these axioms; they could be stated in terms of incidence alone, though doing so would make them much more complicated. As Coxeter says, you have a choice: 'a number of simple axioms involving two undefined relations, or fewer but far more complicated axioms involving only one such relation' ([1949], p. 33). Not surprisingly, Coxeter, writing an introductory text, chose the first. But from a philosophical point of view, the second is what is noteworthy. The idea that you can derive a concept of order from the concept of incidence is a wonderful example of the contribution mathematical ingenuity can make to philosophical analysis.[18]

16 At this point, Russell did not know how serious the defect was. In a part of the paper that was published, he seems to ignore it: 'To show that these axioms suffice [for projective geometry], it is only necessary to prove (by starting from the axioms) the uniqueness of von Staudt's quadrilateral construction, since all projective Geometry ... follows from this construction' ([1899], p. 405). Russell at this time was still coming to grips with Cantor's transfinite set theory and it was another two years before he recognized the proof of the power-set theorem. In 1899, he must have supposed it would be possible to prove that denumerably many constructions would suffice to show that every point could be constructed. Had he, at that time, fully accepted Cantor's work he would surely have been less cavalier.

17 Cf. Russell's own account of them (Russell [1903], pp. 386–7).

18 The whole matter is lucidly described by Gandon [2012], pp. 33–6, to whose account I am indebted.

Russell did not come across Pieri's work until he discovered the Peano school in August 1900 at the mathematical congress in Paris. There, presumably, he heard Pieri speak on geometry as a purely logical system (Pieri [1901]). He read Pieri [1898] the same month (cf. Russell [1983], p. 363) and it informed all his subsequent work on projective geometry – it was, he said, 'the best work on projective geometry' ([1903], p. 382n). Pieri's work confirmed Russell's initial supposition that incidence was the projective concept *par excellence* and that projective geometry should be conceived as exclusively concerned with incidence – projective space as an incidence structure, as Gandon put it ([2012], p. 42). In a brief 1902 reply to the French geometer Georges Lechalas, Russell described Pieri's approach using incidence as the only undefined notion as 'the projective manner of introducing order' (Russell [1993], p. 464). The issue is finally worked out, as far as Russell is concerned, in *The Principles of Mathematics*, where he develops these ideas in two consecutive chapters. In Chapter 45, he treats projective geometry following Pieri and using incidence as the only primitive. This gives the ordinal notion of cross ratio as a projective invariant, though the method he admits is 'somewhat complicated' ([1903], p. 393). We might call this 'pure projective geometry'. In Chapter 46, he follows the earlier work of Pasch [1882], but this time, following Vailati [1892], he introduces a primitive three-place order relation. Unlike projective geometry, where two points determine a line, here two points determine a line segment: the set of points between them. The result he calls 'descriptive geometry'. This distinction between projective and descriptive geometry is not commonly observed – indeed Russell himself doesn't observe it in his *Encyclopedia Britannica* article on non-Euclidean geometry (Russell, [1902]).[19] The distinction is, however, preserved by Whitehead in two short, widely ignored monographs he produced a few years later: *The Axioms of Projective Geometry* [1906] and *The Axioms of Descriptive Geometry* [1907], where both are developed in more mathematical detail. Presumably, both would have found their definitive formulation in the missing geometry volume of *Principia Mathematica*.

I shall return to these matters briefly at the end, where we may glimpse in the far distance the faint shadow of Kant's ghost. But now I want to turn back to a comparison of Hilbert's and Russell's axiomatizations. Consider how Russell presents his axioms. There are three classes of objects – the big A's, the little a's, and the α's – and the axioms are all about how pairs and triples of objects in one class determine objects in another. The statement is entirely formal. To understand the system

19 Russell's adoption of the name 'descriptive geometry' is a carry-over from Euclidean geometry, where theorems which did not involve quantitative notions were often called 'descriptive' (cf. Cremona [1893], §61).

as a geometry at all, you have to provide an *interpretation*: the big *A*'s are points, the little *a*'s lines, and the α's planes. Or, equivalently, since points and planes are duals in projective geometry, the big *A*'s are planes, the little *a*'s lines, and the α's points (Russell [1899], p. 404). The relation 'determine' is to be understood as an incidence relation, thus (Axiom I): two points determine a line, and three points determine a plane, provided no two pairs of them determine the same line (i.e. provided the three points are not collinear). By duality, two planes will determine a line and three a point.

Russell seems to take abstraction a stage further by presenting the axioms in what he calls an algebraic form, using numerals to represent the big *A*'s (i.e. the points, or equivalently, the planes) and treating the objects determined by them as their product (represented by either concatenation or a period, '.'). Thus we have the following (I omit the second axiom which is long and complex and not germane to my purpose):

Axiom I. (α) 12 = 21
(β) 123 = 132 = 231 = … = 1.23 = …
Axiom iii. 123.124 = 12
Axiom iv. 123.124.125 is impossible.
123.145.167 = 1 unless 123.145 = 123.167.
Axiom v. 123.145 = 1.(23.145).
Axiom vi. (12.345)(12.678) = 12.

It is this formal system that he actually uses in his proofs. The notation is a curious one. It is important to realize that Russell is not here presenting an arithmetical model of projective geometry. The single numerals which occur in the axioms and proofs are in fact variables ranging over points (or planes)[20] – and I am not quite sure what he would have done had any of his constructions required more than ten points. The notation emerges in his unpublished notes on geometry written at about the same time out of a more conventional notation where points are represented by capital letters with numerical subscripts, P_1, P_2, P_3, etc. (Russell [1899a]). In the notes, however, a different notation is introduced in which the members of the class of big *A*'s are represented by lower case *e*'s with numerical subscripts. This notation, in turn, appears briefly towards the end of his reply to Poincaré, but not in the formal development of projective geometry.[21] The numeral notation in 'The Axioms of Geometry' seems to have arisen from the *e*-notation by simply dropping the *e*'s for reasons which were never explained.

20 To put it this way, however, is anachronistic, since Russell at this time did not have the modern notion of a variable.

21 The *e*-notation is adapted from Whitehead's treatment of positional manifolds in Whitehead [1898], Book III.

It is always dangerous to apply labels in the history of philosophy. Different philosophers mean different things by them and the only way to avoid ambiguity is by precise definition. But precise definition makes the labels useless as a classificatory tool, since no two philosophers will fit under the same label. So I am not going to ask whether either Hilbert in *The Foundations of Geometry* or Russell in 'The Axioms of Geometry' was a formalist. But I will say that of the two works, Russell's treatment is more formalist than Hilbert's. Hilbert's allegiance to Kant may only have been skin deep, but his geometry was founded upon an analysis of the simple facts of intuition, and his axioms reflect this: they are formulated in terms of points, lines, and planes, points 'lie' on lines and lines 'pass through' points – no room is left for tables, chairs, and beer mugs. The system is a fully interpreted geometry. Russell's system, on the other hand, is wholly abstract. There are three classes of object and the axioms specify how couples and triples of elements in one class 'determine' elements in another. The system acquires geometrical meaning only by means of an interpretation which is given alongside and independently of the axioms.

It is not hard to see where this pre-logicist, formalist turn in Russell's thinking came from. It came from Whitehead's *Universal Algebra* [1898], which Russell had read in proof in March 1898, a little over a year before he wrote 'The Axioms of Geometry'. In the *Universal Algebra*, Whitehead defines mathematics as the 'development of all types of formal, necessary, deductive reasoning'. 'The reasoning', he continues, 'is formal in the sense that the meaning of propositions forms no part of the investigation' (Whitehead [1898], p. vi). The various algebras, or mathematical calculi – 'systems of Symbolic Reasoning' as he calls them (p. v) – are systems of what he calls 'substitutive signs': signs which 'in thought… [take] the place of that for which [they are] substituted' (p. 3). He compares them to counters in a game (p. 3).[22]

22 The term 'substitutive sign' comes from Stout ([1891], p. 187; reprinted as Bk. II, Ch. 10 of Stout [1896]) as a contrast to 'expressive signs'. An expressive sign, e.g. a word, focuses attention on its meaning (i.e. on what it is about); a substitutive sign, on the other hand, 'is a counter which takes the place of its meaning' – Stout cites the symbols of algebra and formal logic as examples. 'The counters', he says, 'are manipulated according to certain rules of operation, until a certain result is reached, which is then interpreted. The operator may be actually unable to interpret the intermediate steps.... It is possible to use signs of this kind whenever fixed and definite rules of operation can be derived from the nature of the things symbolised, so as to be applied in manipulating the signs without further reference to their signification. A word is an instrument for thinking about the meaning which it expresses; a substitute sign is a means of not thinking about the meaning which it symbolises'. This is perhaps as close as we get in Russell and Whitehead at this time to the notion of a schematic variable.

> In order that reasoning may be conducted by means of substitutive signs, it is necessary that rules be given for the manipulation of the signs. The rules should be such that the final state of the signs after a series of operations according to rule denotes, when the signs are interpreted in terms of the things for which they are substituted, a proposition true of the things represented by the signs (p. 4).

The signs can be chosen and interpreted arbitrarily and we can manipulate them 'according to any rules we choose to assign', though 'in general such occupations must be frivolous' unless the signs are signs for 'things and the relations of things' and the rules chosen appropriately (p. 4). Mathematics is concerned only with 'the inference of proposition from proposition' (p. vi). The justification of the rules of inference is a matter for the philosopher, not the mathematician: 'The business of mathematics is simply to follow the rule' (p. vi).

In the *Universal Algebra*, Whitehead conceives of algebra as 'an independent science dealing with the relations of certain marks conditioned by the observance of certain conventional laws' (p. 11). Its importance for the other sciences resides in the fact that if two sciences share the same rules of inference but differ in interpretation, then any truth derivable in the one will be true, when interpretable, in the other (p. 11). His aim is to develop the algebras he deals with as systems of symbolism. But he also holds that they can all be interpreted on a 'generalized conception of space' which he calls a 'positional manifold' (pp. v, ix). He thus also conceives of the algebras as 'engines for the investigation of the possibilities of thought and reasoning connected with the abstract general idea of space' (p. v). This is the approach which underlies Russell's treatment in 'The Axioms of Geometry' and Whitehead's distinction between the calculus, which is mathematics, and its interpretation, which is 'every province of thought, or of external experience, in which the succession of thoughts, or of events can be definitely ascertained and precisely stated' (p. viii), is quite clear. It is reproduced in Russell's treatment of projective geometry, and the projective space on which he models the formal system is a species of positional manifold.

With the advantages of hindsight, it is natural to see Russell's surprisingly formalist account of projective geometry in 1899 as a step on the way to logicism. As already noted, however, until he discovered Peano, Russell did not have a logic powerful enough to make logicism feasible, and, of course, he didn't know he would ever find one. Until then, a formal, algebraic approach was the best option he had. The account of pure mathematics that would naturally accompany such an approach – though Russell himself said much less about this than Whitehead did – would be that it was the study of abstract, uninterpreted systems of symbolic reasoning. This I take to be a kind of formalism. Not, indeed, in the later, proof-theoretic sense of Hilbert's program – that would be

wildly anachronistic – but in the sense in which, as Russell famously said a couple of years later, mathematics is the subject in which 'we never know what we are talking about, nor whether what we are saying is true' (Russell [1901], p. 366) – a view which Russell took to be 'a fatal blow to the Kantian philosophy' (*ibid.*, p. 379). 'The whole doctrine of a priori intuitions, by which Kant explained the possibility of pure mathematics, is wholly inapplicable to mathematics in its present form' (*ibid.*). Not surprisingly, the logicism gets stronger as Russell's account of projective geometry develops, and in *The Principles of Mathematics* we have a wholly logicist account. The difference between pure projective geometry and descriptive geometry stems entirely from the fundamental relation used in each. In pure projective geometry, the basic relation is a symmetrical relation: if a line is incident on A and B, it is equally incident on B and A. In descriptive geometry, it is an asymmetrical, transitive relation, fixing the line as an ordered sequence of points from A to B and permitting the identification of points that are invariantly between A and B. The line in descriptive geometry is thus a ray; it has direction. Now, relations are paradigmatically logical items, and the differences between symmetrical and asymmetrical, transitive relations are logical differences. We have, thus, pure projective geometry founded upon symmetrical, incidence relations and descriptive geometry founded upon asymmetrical, transitive, ordering relations: a beautiful example of what Gandon [2012] has called Russell and Whitehead's 'topic specific logicism'.

Nonetheless, if we look back to 1898 and 1899 and think of Whitehead's 'abstract general idea of space', which – to put it a bit contentiously – underlies all his algebras; or of 'every province of thought, or of external experience, in which the succession of thoughts, or of events can be definitely ascertained and precisely stated' on which they can be interpreted; or of Russell's projective space as an incidence structure, perhaps here we have some vestige of Kant's objects of intuition that Hilbert, much to my surprise, was happy to emphasize. In this we may see – though Russell will turn in his grave to hear me say it – a faint reflection of the fading glow that used to be Kant.[23]

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23 I'm grateful to Richard Arthur for help with German translations and to Albert Lewis for crucial information about Hilbert.

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