

信号与系统作业:

3.6 (4) 解:

① 零输入响应应满足方程:

$$y_{zi}(k) + 3y_{zi}(k-1) + 2y_{zi}(k-2) = 0$$

特征根为 $-1, -2$. 齐次解为 $y_{zi}(k) = C_1(-1)^k + C_2(-2)^k$

$$\text{代入初始条件得: } \begin{cases} y_{zi}(-1) = -C_1 - \frac{1}{2}C_2 = 1 \\ y_{zi}(-2) = C_1 + \frac{1}{4}C_2 = 0 \end{cases} \Rightarrow \begin{cases} C_1 = 1 \\ C_2 = 4 \end{cases}$$

故零输入响应为: $y_{zi}(k) = (-1)^k - 4(-2)^k, k \geq 0$

② 零状态应满足方程:

$$y_{zs}(k) + 3y_{zs}(k-1) + 2y_{zs}(k-2) = 1, k \geq 0.$$

$$y_{zs}(-1) = y_{zs}(-2) = 0.$$

由 $y_{zs}(0) + 3y_{zs}(-1) + 2y_{zs}(-2) = 1$, 可得 $y_{zs}(0) = 1$

由 $y_{zs}(1) + 3y_{zs}(0) + 2y_{zs}(-1) = 1$, $\Rightarrow y_{zs}(1) = -2$.

$$\text{方程的解为: } y_{zs}(k) = y_{zsh}(k) + y_{zsp}(k) \\ = C_3(-1)^k + C_4(-2)^k + \frac{1}{6}, k \geq 0$$

代入初始条件得: $y_{zs}(0) = 1, y_{zs}(1) = -2$

故零状态响应为: $y_{zs}(k) = -\frac{1}{2}(-1)^k + \frac{4}{3}(-2)^k + \frac{1}{6}, k \geq 0$

全响应为: $y(k) = y_{zi}(k) + y_{zs}(k) = \frac{1}{2}(-1)^k - \frac{8}{3}(-2)^k + \frac{1}{6}, k \geq 0$

3.10 (b) 解:

设左端加法器的输出为 $x(k)$, 则右端延迟单元的输入为 $x(k-1), x(k-2)$.

由左端加法器的输出得: $x(k) = f(k) + x(k-1) - 0.24x(k-2)$

即 $x(k) - x(k-1) + 0.24x(k-2) = f(k)$.

由右端加法器的输出得: $y(k) = x(k) - \frac{1}{2}x(k-1)$

设 $h_1(k)$ 满足: $y(k) = x(k) - \frac{1}{2}x(k-1)$

又由 $h_1(-1) = h_1(-2) = 0$ 可得: $h_1(0) = h_1(1) = 1$.

方程的齐次解为: $h_1(k) = C_1\left(\frac{2}{5}\right)^k + C_2\left(\frac{3}{5}\right)^k, k \geq 0$.

代入初始条件得: $C_1 = -2, C_2 = 3$

故 $h_1(k) = \left[-2\left(\frac{2}{5}\right)^k + 3\left(\frac{3}{5}\right)^k\right] \varepsilon(k)$.

系统单位序列响应为:

$$\begin{aligned} h(k) &= h_1(k) - \frac{1}{2}h_1(k-1) \\ &= \left[-2\left(\frac{2}{5}\right)^k + 3\left(\frac{3}{5}\right)^k\right] \varepsilon(k) - \frac{1}{2} \left[-2\left(\frac{2}{5}\right)^{k-1} + 3\left(\frac{3}{5}\right)^{k-1}\right] \varepsilon(k-1) \\ &= \frac{1}{2} \left[\left(\frac{2}{5}\right)^k + \left(\frac{3}{5}\right)^k\right] \varepsilon(k) \end{aligned}$$