2.4 (1)

$y''(t)+4y'(t)+3y(t)=f(t)$ , $y(0_-)=y'(0_-)=1$ , $f(t)=\varepsilon(t)$

零输入:

$\because$ 微分方程求解可得: $\lambda^2+4\lambda+3=0$ $\therefore \lambda_1=-3$ $\lambda_2=-1$

$\therefore y_{zi}(t)=C_{zi1}e^{-3t}+C_{zi2}e^{-1t}$

$\because y(0_-)=y_{zi}(0_-)+y_{zs}(0_-)$

$\therefore y'(0_-)=y'_{zi}(0_-)+y'_{zs}(0_-)$

$\therefore y_{zi}(0_-)=1$ , $y'_{zi}(0_-)=1$ $\therefore C_{zi1}=-3$ $C_{zi2}=-t$

$\therefore y_{zi}(t)=(-e^{-3t}+2e^{-t})\varepsilon(t)$

零状态: $y''_{zs}(t)+4y'_{zs}(t)+3y_{zs}(t)=\varepsilon(t)$

$\because y_{zs}(0_-)=y'_{zs}(0_-)=0$

$\therefore \int_{0_-}^{0_+}y''_{zs}(t)+4\int_{0_-}^{0_+}y'_{zs}(t)+3\int_{0_-}^{0_+}y_{zs}(t)=\int_{0_-}^{0_+}\varepsilon(t)\,dt$

$\therefore y'_{zs}(0_+)-y'_{zs}(0_-)+4[y_{zs}(0_+)-y_{zs}(0_-)]=0$

$\therefore y'_{zs}(0_+)-y'_{zs}(0_-)=0$ $\therefore y'_{zs}(0_+)=y'_{zs}(0_-)=0$

$t>0$ 时 $y''_{zs}(t)+4y_{zs}(t)+3y_{zs}(t)=1$

$\therefore y_{zs}(t)=C_{zs1}e^{-t}+C_{zs2}e^{-3t}+P_0$ $\therefore P=\frac{1}{3}$

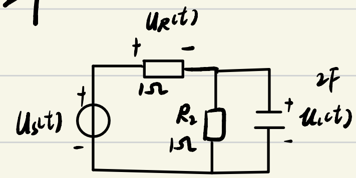
$\therefore y'_{zs}(t)=-C_{zs1}e^{-t}-3C_{zs2}e^{-3t}$

$\therefore$ 代入得 $C_{zs1} = -\frac{1}{2}$ $C_{zs2} = \frac{1}{6}$

$$\therefore y_{zs}(t) = (-\frac{1}{2}e^{-t} + \frac{1}{6}e^{-3t} + \frac{1}{3})\varepsilon(t)$$

$$\therefore y(t) = (-\frac{5}{6}e^{-3t} + \frac{3}{2}e^{-t} + \frac{1}{3})\varepsilon(t)$$

2.9




$\therefore$ $f(t) = u_s(t)$ $y(t) = u_c(t)$

$$\begin{cases} u_R(t) + u_c(t) = u_s(t) \\ u_c(t) + Cu_c'(t) = u_R(t) \end{cases}$$

$\therefore$ 消去得 $2u_c(t) + 2u_c'(t) = u_s(t)$

$\therefore$ 微分方程为 $2u_c(t) + 2\frac{du_c(t)}{dt} = u_s(t)$

冲激响应：

$$\left.\begin{aligned} 2u_c(t) + 2u_c'(t) = \delta(t) \\ u_c'(0_-) = u_c(0_-) = 0 \end{aligned}\right\}$$

$\because$ 微分方程解得

$\lambda = -1$

$\therefore$ $= C_1 e^{-t}$

$$\int_{0_-}^{0_+} u_c(t)\,dt + \int_{0_-}^{0_+} u_c'(t)\,dt = \frac{1}{2}$$

$u_c'(t) = a\delta(t) + r_0(t)$ $a\int_{0_-}^{0_+} \delta(t)\,dt = u_c(0_+) - u_c(0_-) = a$

$\therefore a = \frac{1}{2}$

$\because u_c(0_+) - u_c(0_-) = a$ $\therefore u_c(0_+) = a + u_c(0_-) = \frac{1}{2}$

$t>0$ 时

$\because h(t) = Ce^{-t}$ 代入得 $C = 0.5$

$\therefore h(t) = 0.5e^{-t}\varepsilon(t)$

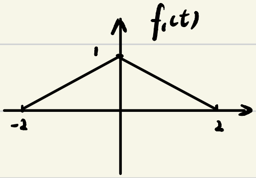
阶跃响应 $g(t) = \int_{-\infty}^{t} h(t)\,dt = -0.5(e^{-t} - 1)\varepsilon(t)$

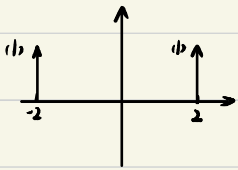
∴冲激响应 $h(t) = 0.5e^{-t}\varepsilon(t)$

阶跃响应 $g(t) = 0.5(1-e^{-t})\varepsilon(t)$

2.16

(1) $f_1(t) * f_2(t)$

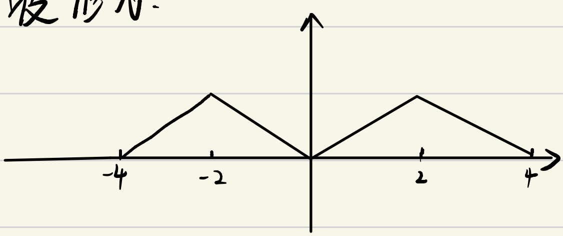


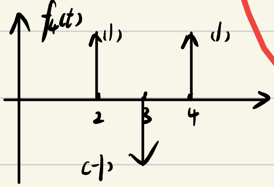

$$f(t) = f_1(t) * f_2(t) = f_1(t) * (\delta(t+2) + \delta(t-2))$$

$$= f(t+2) + f(t-2)$$

∴波形为:

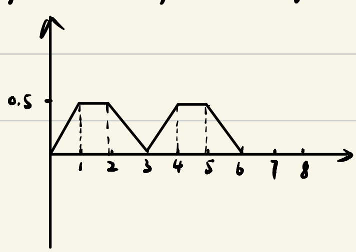



(3) $f_1(t) * f_4(t)$




$$f_1(t) * f_4(t) = f_1(t) * (\delta(t-2) - \delta(t-3) + \delta(t-4))$$

$$= f(t-2) - f(t-3) + f(t-4)$$



2.17

(7) $f_1(t) = \varepsilon(t) - \varepsilon(t-4)$ $\quad f_2 = \sin(\pi t)\varepsilon(t)$

$$\therefore f(t) = f_2^{(-1)} * f_1^{(1)} = (\delta(t) - \delta(t-4)) * \int_{-\infty}^{t} \sin(\pi t)\varepsilon(t)\,dt$$

$$= (\delta(t) - \delta(t-4)) * \left[\frac{1}{\pi}(1-\cos\pi t)\right]\varepsilon(t)$$

$$= \frac{1}{\pi}(1-\cos(\pi t))\varepsilon(t) - \frac{1}{\pi}(1-\cos\pi(t-4))\varepsilon(t-4)$$

$$= \frac{1}{\pi}(1-\cos(\pi t))\varepsilon(t) - \frac{1}{\pi}(1-\cos\pi t)\varepsilon(t-4)$$

$$= \frac{1}{\pi}(1-\cos\pi t)[\varepsilon(t) - \varepsilon(t-4)]$$

2.18

(1)

$$f(t) = h(t) * \varepsilon(t) = h^{(1)}(t) * \varepsilon^{(-1)}(t)$$

$$= (\varepsilon(t) - \varepsilon(t-2)) * \varepsilon(t)$$

$$= \varepsilon(t) * \varepsilon(t) - \varepsilon(t-2) * \varepsilon(t)$$

$$= t\varepsilon(t) - \delta(t-2) * (t\varepsilon(t))$$

$$= t\varepsilon(t) - (t-2)\varepsilon(t-2)$$

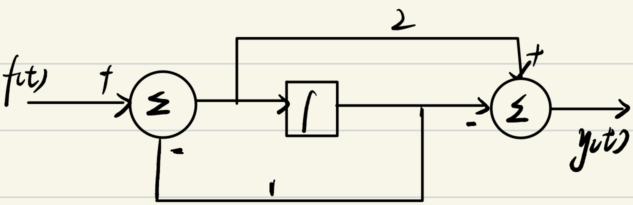
(2) $f_1(t) = \varepsilon(t-2) - \varepsilon(t-3)$

$$f(t) = h(t) * f_1(t) = [\varepsilon(t) - \varepsilon(t-2)] * [\varepsilon(t-2) - \varepsilon(t-3)]$$

$$= [\delta(t) - \delta(t-2)] * [(t-2)\varepsilon(t-2) - (t-3)\varepsilon(t-3)]$$

$= (t-2)\varepsilon(t-2) - (t-3)\varepsilon(t-3) - (t-4)\varepsilon(t-4) + (t-5)\varepsilon(t-5)$

2.26




$f(t) - x(t) = x'(t) \quad \therefore \quad f(t) = x'(t) + x(t)$

$2x'(t) - x(t) = y(t) \quad\quad y(t) = 2x'(t) - x(t)$

$y'(t) = 2x''(t) - x'(t) \quad \therefore \quad y'(t) + y(t) = 2(x'(t) + x(t))' - (x'(t) + x(t))$

$= 2f'(x) - f(x)$

$\therefore \quad y'(t) + y(t) = 2f'(x) - f(x)$

$\therefore \quad h'(t) + h(t) = 2\delta'(x) - \delta(x)$

$\because \quad h'(0_-) = h(0_-) = 0$

又$\because \quad \lambda + 1 = 0 \quad \lambda = -1 \quad\quad \therefore \quad h(t) = (C_1 e^{-t})\varepsilon(t)$

设 $h'(t) = a\delta'(x) + b\delta(x) + r_0(t)$

$\therefore \quad h(t) = \int_{0_-}^{0_+} a\delta'(x) + b\delta(x) + r_0(t)\, dt$

$= a\delta(x) + r_1(t)$

$\therefore h'(t) + h(t) = a\delta'(x) + (a+b)\delta(x) + r_0(t) + r_1(t)$

$\therefore a=2 \quad b=-3$

$\therefore \int_{0_-}^{0_+} h'(t)\,dt = \int_{0_-}^{0_+} 2\delta'(x) - 3\delta(x) + r_0(t)\,dt = -3$

$\therefore h(0_+) - h(0_-) = -3 \quad \therefore h(0_+) = -3$

$\because h(t) = (C_1 e^{-t})\varepsilon(t) \quad \therefore$ 代入得 $C = -3$

$\therefore h(t) = (-3e^{-t})\varepsilon(t) + 2\delta(t)$

2.30

$y(t) = f(t) * h_1(t) + f(t) * h_2(t) * h_1(t) * h_3(t)$

$= f(t) * (h_1(t) + h_2(t) * h_1(t) * h_3(t))$

$h_2(t) * h_1(t) * h_3(t) = \delta(t-1) * \varepsilon(t) * -\delta(t) = -\varepsilon(t-1)$

$h_1(t) = \varepsilon(t)$

$\therefore y(t) = f(t) * (\varepsilon(t) - \varepsilon(t-1))$

当 $f(t) = \delta(t)$ 时 $\quad h(t) = \varepsilon(t) - \varepsilon(t-1)$