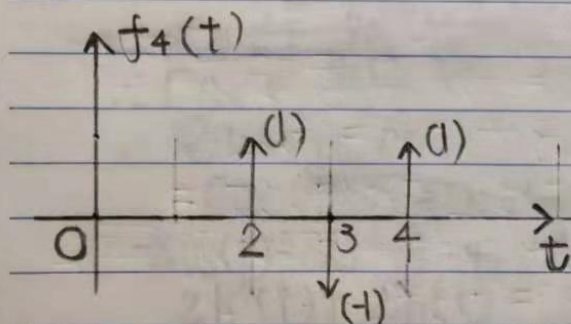
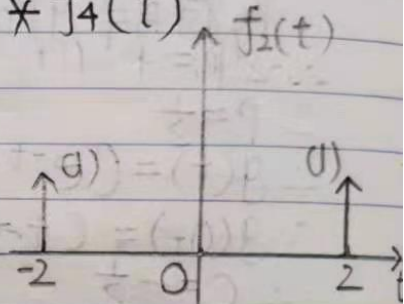
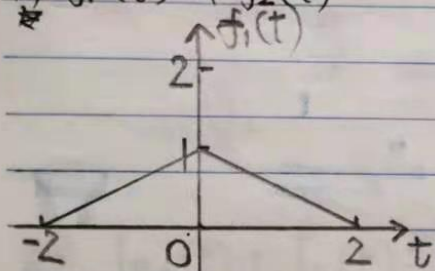


2.16 各函数波形如图所示, 图(c,d)为单位冲激函数, 试求下列卷积, 并画出波形图。

(1) $f_1(t) * f_2(t)$

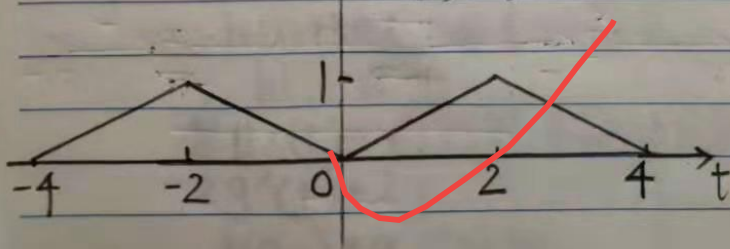
(3) $f_1(t) * f_4(t)$



(1) 解: $\because f_2(t) = \delta(t+2) + \delta(t-2)$

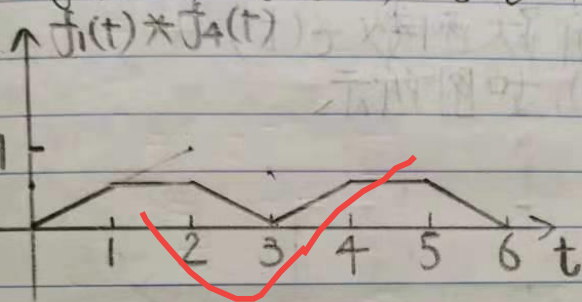
$$\therefore f_1(t) * f_2(t) = f_1(t) * [\delta(t+2) + \delta(t-2)] = f_1(t) * \delta(t+2) + f_1(t) * \delta(t-2) = f_1(t+2) + f_1(t-2)$$

\uparrow $f_1(t) * f_2(t)$



(3) 解: $f_4(t) = \delta(t-2) - \delta(t-3) + \delta(t-4)$

$\therefore f_1(t) * f_4(t) = f_1(t) * [\delta(t-2) - \delta(t-3) + \delta(t-4)]$
 $= f(t-2) - f(t-3) + f(t-4)$



2.17 求下列函数的卷积积分 $f_1(t) * f_2(t)$

(1) $f_1(t) = \varepsilon(t) - \varepsilon(t-4)$, $f_2(t) = \sin(\pi t) \varepsilon(t)$

解: $f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_2(\tau) * [\varepsilon(t) - \varepsilon(t-4)] d\tau =$

$f_2(t) * \varepsilon(t) - f_2(t) * \varepsilon(t-4)$

当 $t > 0$, $f_2(t) * \varepsilon(t) = \int_{-\infty}^{\infty} \sin \pi \tau \varepsilon(\tau) \varepsilon(t-\tau) d\tau$

$= \int_0^t \sin \pi \tau d\tau \varepsilon(t) = \frac{1}{\pi} (1 - \cos \pi t) \varepsilon(t)$

当 $t > 4$, $f_2(t) * \varepsilon(t-4) = \int_{-\infty}^{\infty} \sin \pi \tau \varepsilon(\tau) \varepsilon(t-4-\tau) d\tau$

$= \int_0^{t-4} \sin \pi \tau d\tau \varepsilon(t) = \frac{1}{\pi} (1 - \cos(t-4)\pi) \varepsilon(t) = \frac{1}{\pi} (1 - \cos \pi t) \varepsilon(t)$

\therefore 当 $t < 0$, $f_2(t) * \varepsilon(t) - f_2(t) * \varepsilon(t-4) = 0$;

当 $t > 4$, $f_2(t) * \varepsilon(t) - f_2(t) * \varepsilon(t-4) = \frac{1}{\pi} (1 - \cos \pi t) - \frac{1}{\pi} (1 - \cos \pi t) = 0$

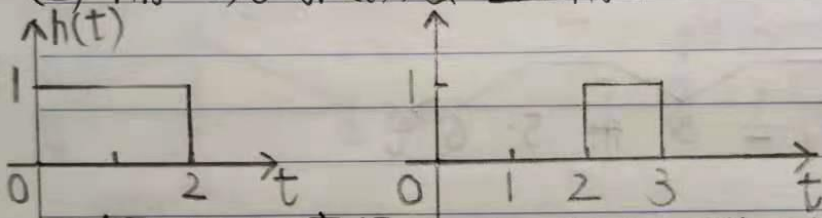
当 $0 < t < 4$, $f_2(t) * \varepsilon(t) - f_2(t) * \varepsilon(t-4) = \frac{1}{\pi} (1 - \cos \pi t)$

$\therefore f_1(t) * f_2(t) = \begin{cases} \frac{1}{\pi} (1 - \cos \pi t), & 0 < t < 4 \\ 0, & t < 0 \text{ 或 } t > 4 \end{cases}$

2.18 某 LTI 系统的冲激响应如图所示, 求输入为下列函数时的零状态响应, 或画出波形图)

(1) 输入为单位阶跃函数 $\varepsilon(t)$

(2) 输入为 $f_1(t)$, 如图所示



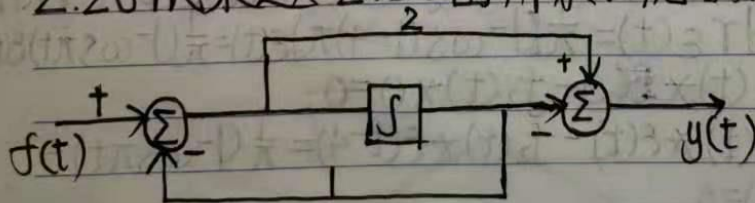
(1) 解: 依题意得: $h(t) = \varepsilon(t) - \varepsilon(t-2)$

$$\begin{aligned} \therefore y_{zs}(t) &= h(t) * \varepsilon(t) = [\varepsilon(t) - \varepsilon(t-2)] * \varepsilon(t) \\ &= t\varepsilon(t) + (t-2)\varepsilon(t-2) \end{aligned}$$

(2) 解: 依题意得: $h(t) = \varepsilon(t) - \varepsilon(t-2)$, $f_1(t) = \varepsilon(t-2) - \varepsilon(t-3)$

$$\begin{aligned} \therefore y_{zs}(t) &= h(t) * f_1(t) = [\varepsilon(t) - \varepsilon(t-2)] * [\varepsilon(t-2) - \varepsilon(t-3)] \\ &= (t-2)\varepsilon(t-2) - (t-3)\varepsilon(t-3) - (t-4)\varepsilon(t-4) + (t-5)\varepsilon(t-5) \end{aligned}$$

2.26 试求题 2.26 图所示系统的冲激响应



2.26

解：设积分器输出为 $x(t)$

∴ 左边加法器输出： $x'(t) = f(t) - x(t)$,

右加法器输出： $2x'(t) - x(t) = y(t)$

$$\therefore \begin{cases} x'(t) + x(t) = f(t) \\ 2x'(t) - x(t) = y(t) \end{cases}$$

$$\therefore \begin{cases} x'(t) + x(t) = f(t) \\ 2x'(t) - x(t) = y(t) \end{cases}$$

$$\therefore 2f'(t) - f(t) = y'(t) + y(t)$$

设冲激响应为 $h(t)$

$$\therefore 2h_1'(t) - h_1(t) = \delta(t)h(t)$$

$$\begin{cases} h'(0^-) = h(0^-) = 0 \\ \lambda = -1 \end{cases}$$

$$\therefore \lambda = -1$$

$$\therefore h_1(t) = c e^{-t} \varepsilon(t)$$

$$\therefore h_1'(t) + h_1(t) = \delta(t)$$

$$\varepsilon(t-3) \quad \therefore h_1(0^+) - h_1(0^-) = 1$$

$$3) \quad \therefore h_1(0^+) = 1$$

$$t=5) \quad \therefore c = 1$$

$$\therefore h_1(t) = e^{-t} \varepsilon(t)$$

$$\therefore h(t) = 2\delta(t) - 3e^{-t} \varepsilon(t)$$

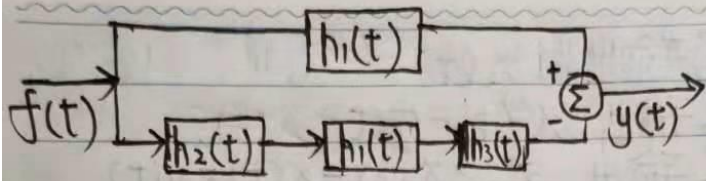
2.30 如题 2.30 图所示的系统，它由几个子系统所组成，各子系统的冲激响应为

$$h_1(t) = \varepsilon(t)$$

$$h_2(t) = \delta(t-1)$$

$$h_3(t) = -\delta(t)$$

求复合系统的冲激响应



解: $y(t) = f(t) * h_1(t) + f(t) * h_2(t) * h_1(t) * h_3(t)$
 $= \delta(t) * \epsilon(t) + \delta(t) * \epsilon(t) * \delta(t-1) * \delta(t)$
 $= \epsilon(t) + \epsilon(t-1)$

解: 由题可知, 系统是并联的, 且各子系统的冲激响应分别为 $h_1(t)$, $h_2(t)$, $h_3(t)$ 。根据卷积定理, 系统的总冲激响应为各子系统的冲激响应的卷积和。即 $y(t) = f(t) * h_1(t) + f(t) * h_2(t) * h_1(t) * h_3(t)$ 。代入 $f(t) = \delta(t)$, 可得 $y(t) = \delta(t) * \epsilon(t) + \delta(t) * \epsilon(t) * \delta(t-1) * \delta(t) = \epsilon(t) + \epsilon(t-1)$ 。