第一次作业

1.5(2)

判断下列各序列是否为周期性的，若是，确定其周期

$$f_2(k)=\cos\left(\frac{3\pi}{4}k+\frac{\pi}{4}\right)+\cos\left(\frac{\pi}{3}k+\frac{\pi}{6}\right)$$

解：$\cos(\frac{3\pi}{4}k+\frac{\pi}{4})$的周期为 $2\pi\cdot\frac{4}{3\pi}=\frac{T}{N}$，N为一整数，T为满足等式关系的最小正整数，故$T_1=8$.

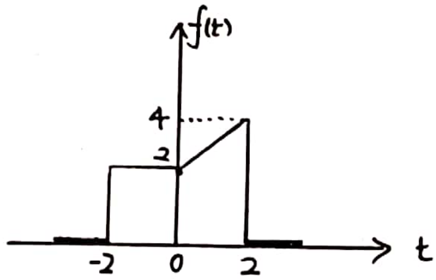
同理，对 $\cos(\frac{\pi}{3}k+\frac{\pi}{6})$的周期 $2\pi\cdot\frac{3}{\pi}=\frac{T}{N}$，$T_2=6$.

$f_2(k)$周期应为T_1、T_2的最小公倍数 即 $T=24$.

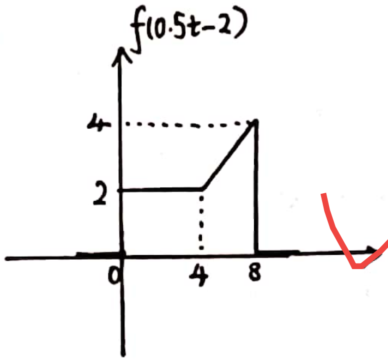
1.6(6)(8) 已知信号波形如下图，画出下列各波形函数

(6) $f(0.5t-2)$

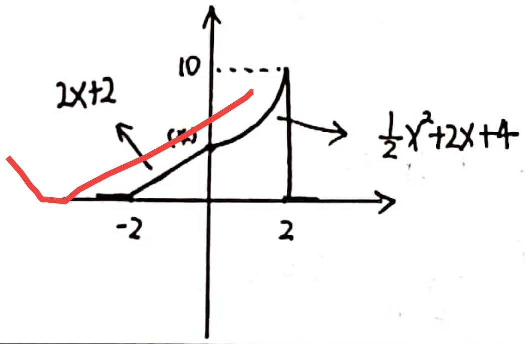
(8) $\int_{-\infty}^{t} f(x)dx$



解：(6)

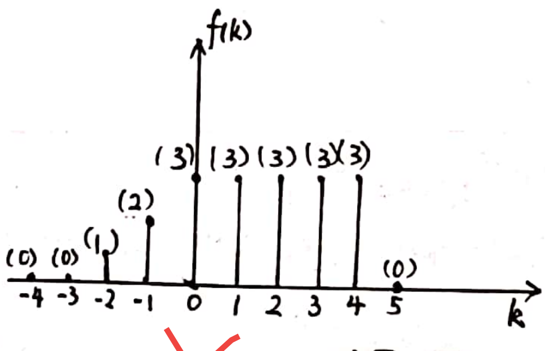


(8)



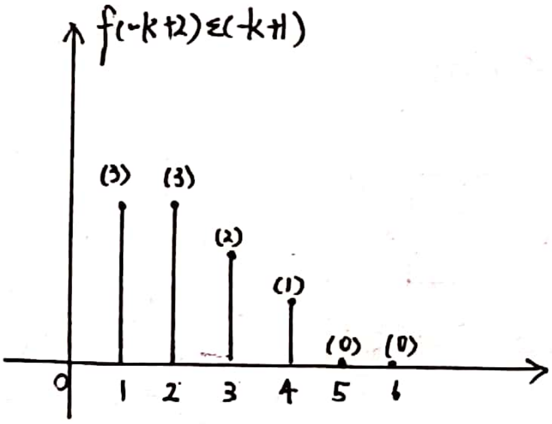
1.7 已知序列f(k)图形如下，画出下列序列图形

(5) $f(-k+2)\varepsilon(-k+1)$

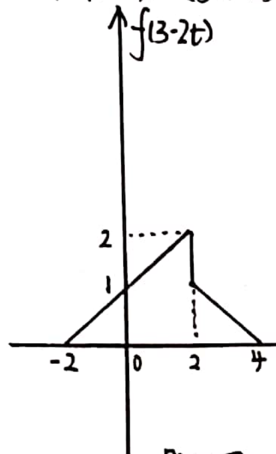


题图

解：

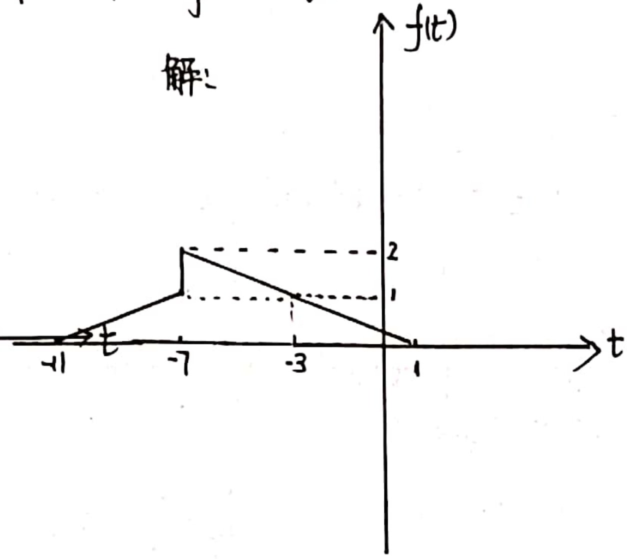


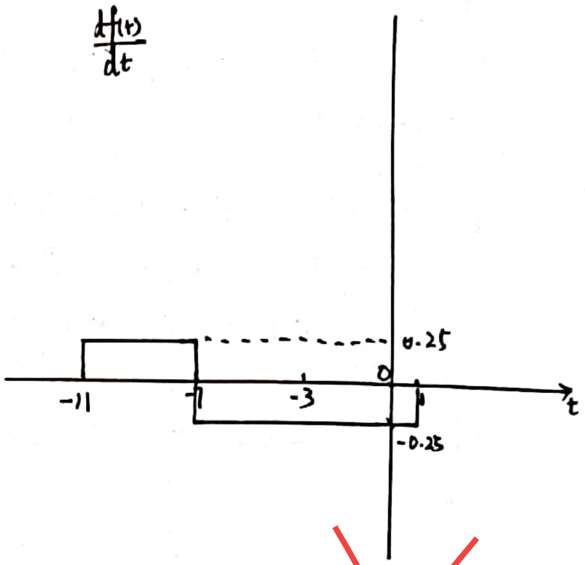
1.9. 已知信号波形如下，分别画出 $f(t)$ 和 $\frac{df(t)}{dt}$ 的波形.



题1.9图

解:





1.10 (3)(7) 计算

(3) $\int_{-\infty}^{\infty} \frac{\sin(\pi t)}{t}\delta(t)dt$　(7) $\int_{-\infty}^{\infty}(t^3+2t^2-2t+1)\delta'(t-1)dt$

解: (3) $\int_{-\infty}^{\infty} \frac{\sin(\pi t)}{t}\delta(t)dt$

$$= \lim_{t\to 0}\frac{\sin(\pi t)}{t} = \lim_{t\to 0}\frac{\pi\cos(\pi t)}{1} = \pi$$

(7) $\int_{-\infty}^{\infty}(t^3+2t^2-2t+1)\delta'(t-1)dt$

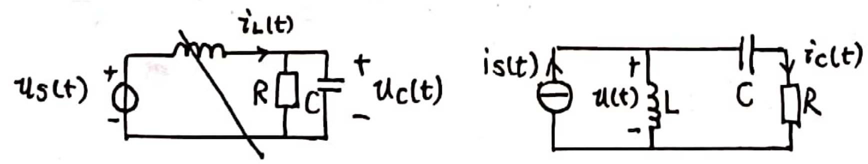
$$= -(t^3+2t^2-2t+1)'\big|_{t=1}$$

$$= -(3t^2+4t-2)\big|_{t=1}$$

$$= -5$$

如图1.13所示电路，写出

1.13 (1)以 $u(t)$ 为响应的微分方程 (2)以 $i_C(t)$ 为响应的微分方程



解:(1) 设电容C电压为 $u_C(t)$. 电感电流 $i_L(t)$.

$u(t)=u_C(t)+i_C(t)R$,

$u'(t)=u_C'(t)+i_C'(t)R$

$\because i_C(t)=Cu_C'(t)$

$\therefore u_C'(t)=\frac{i_C(t)}{C}$ ①

又 $\because i_C(t)=i_S(t)-i_L(t)$

$u(t)=Li_L'(T)$

$\therefore i_C'(t)=i_S'(t)-i_L'(t)$

$= i_S'(t)-\frac{u(t)}{L}$ ②

①、②式代入 $u'(t)$ 表达式

$$u'(t)=\frac{i_C(t)}{C}+i_S'(T)R-\frac{u(t)}{L}R$$

两端求导

得 $u''(t)=\frac{1}{C}i_C'(t)+i_S''(t)R-\frac{u'(t)}{L}R$

即 $u''(t)=\frac{1}{C}i_S'(t)-\frac{u(t)}{CL}+i_S''(t)R-\frac{u'(t)}{L}R$

∴以$u(t)$为响应的微分方程为

$$u''(t)+\frac{R}{L}u'(t)+\frac{1}{CL}u(t)=\frac{1}{C}i_S'(t)+Ri_S''(t)$$

(2) 以$i_C(t)$为响应的微分方程

$$i_C(t)=i_S(t)-i_L(t)$$

$$i_C'(t)=i_S'(t)-i_L'(t)$$

∵ $i_L'(t)=\dfrac{u(t)}{L}$，

∴ $i_C'(t)=i_S'(t)-\dfrac{u(t)}{L}$

对两端求导有 $i_C''(t)=i_S''(t)-\dfrac{u'(t)}{L}$

∵ $u'(t)=u_C'(t)+i_C'(t)R$

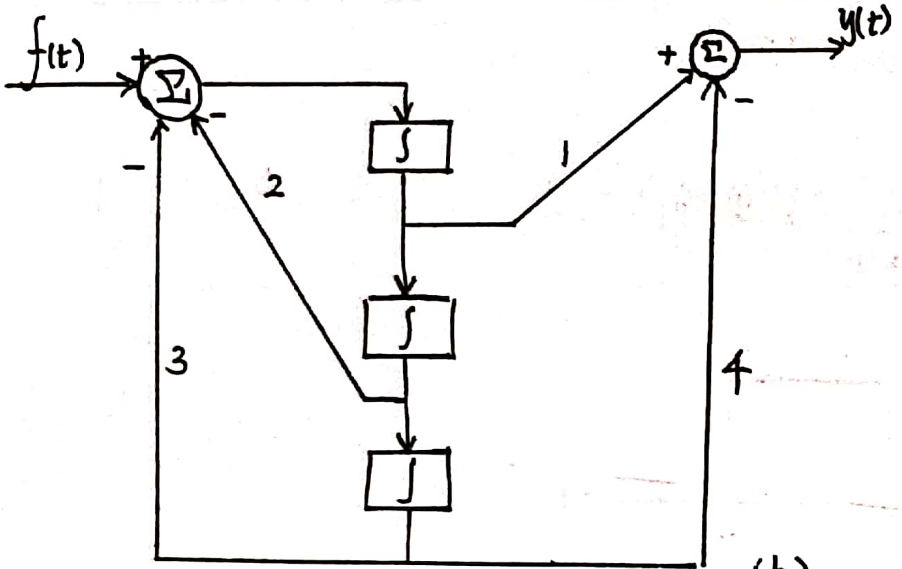
且 $u_C'(t)=\dfrac{i_C(t)}{C}$，∴ $u'(t)=\dfrac{i_C(t)}{C}+i_C'(t)R$

∴ $i_C''(t)=i_S''(t)-\dfrac{1}{LC}i_C(t)-\dfrac{R}{L}i_C'(t)$

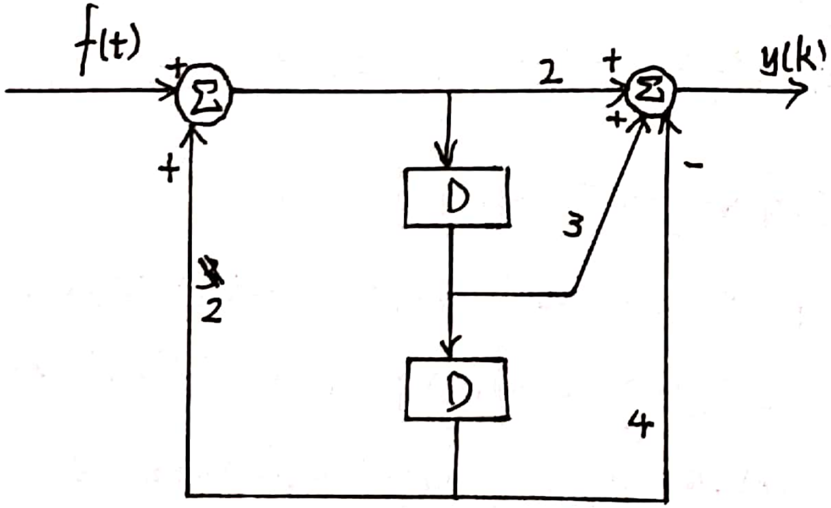
故以$i_C(t)$为响应的微分方程为

$$i_C''(t)+\frac{R}{L}i_C'(t)+\frac{1}{LC}i_C(t)=i_S''(t)$$

1.20 (b)(d) 写出下面图中各系统的微分和差分方程



(b)



解:

(b) 设最下方积分器输出为 $x(t)$、

左端加法器输出 $x^{(3)}(t)=f(t)-2x^{(1)}(t)-3x(t)$

$f(t)=x^{(3)}(t)+2x^{(1)}(t)+3x(t)$

右端加法器输出 $y(t)=x^{(2)}(t)-4x(t)$

$$\begin{cases} y^{(3)}(t) \cancel{+2y^{(1)}(t)} = [x^{(2)}(t)]''-4x^{(3)}(t) \\ 2y^{(1)}(t)=[2x^{(1)}(t)]''-4[2x^{(1)}(t)] \\ 3y(t)=[3x(t)]''-4[3x(t)] \end{cases}$$

$y^{(3)}(t)+2y^{(1)}(t)+3y(t)=[x^{(3)}(t)+2x^{(1)}(t)+3x(t)]''-$

$4[x^{(3)}(t)+2x'(t)+3x(t)]$

即 $y^{(3)}(t)+2y^{(1)}(t)+3y(t)=f^{(2)}(t)-4f(t)$

此为系统的微分方程.

(d). 设最上方延迟单元输入为 $x(k)$.

左方加法器输出 $x(k) = f(t) + 2x(k-2)$

即 $f(k) = x(k) - 2x(k-2)$.

右方加法器输出

$$y(k) = 2x(k) + 3x(k-1) - 4x(k-2)$$

$$-2y(k-2) = 2[-2x(k-2)] + 3[-2x(k-3)] - 4[-2x(k-4)]$$

相加得

$$y(k) - 2y(k-2) = 2[x(k) - x(k-2)] + 3[x(k-1) - 2x(k-3)] - 4[x(k-2) - 2x(k-4)]$$

即

$$y(k) - 2y(k-2) = 2f(k) + 3f(k-1) - 4f(k-2)$$

1.27 某LTI连续系统，其初始状态一定，已知当激励为$f(t)$时，其全响应为

$$y_1(t)=e^{-t}+\cos(\pi t),\ t\geq 0$$

若初始状态不变，激励为$2f(t)$时，其全响应为

$$y_2(t)=2\cos(\pi t),\ t\geq 0$$

求初始状态不变而激励为$3f(t)$时系统的全响应。

解：令系统零状态响应为$y_{zs}(t)$，零输入响应为$y_{zi}(t)$

激励为$f(t)$时，

$$y_{zi}(t)+y_{zs}(t)=e^{-t}+\cos(\pi t),\ t\geq 0$$

∵LTI系统具有齐次性

$$2y_{zs}(t)=T[2f(t)],\ 3y_{zs}(t)=T[3f(t)]$$

初始状态不变，激励变为$2f(t)$时，全响应

$$y_{zi}(t)+2y_{zs}(t)=2\cos(\pi t),\ t\geq 0$$

解得

$$y_{zi}(t)=2e^{-t}$$

$$y_{zs}(t)=-e^{-t}+\cos(\pi t),\ t\geq 0$$

初始状态不变，激励为$3f(t)$时系统的全响应为

$$y_{zi}(t)+3y_{zs}(t)=-e^{-t}+3\cos(\pi t),\ t\geq 0$$

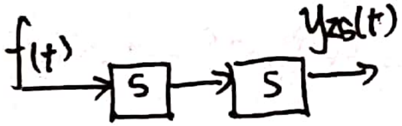
即激励为$3f(t)$时全响应

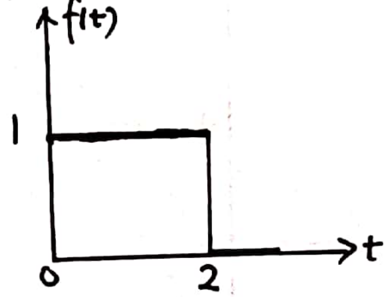
$$y_3(t)=-e^{-t}+3\cos(\pi t),\ t\geq 0$$

1.31 如有LTI连续系统S，已知当激励为阶跃函数$\varepsilon(t)$时，其零状态响应为

$$\varepsilon(t)-2\varepsilon(t-1)+\varepsilon(t-2)$$

现将两个完全相同的系统相级联，如1.31图(a)示，当这个复合系统的输入为题1.31(b)所示信号$f(t)$时，求该系统的零状态响应.

(a)


(b)


解：对系统S输入信号$\varepsilon(t)$，输出响应

$$y_{fs}(t)=\varepsilon(t)-2\varepsilon(t-1)+\varepsilon(t-2)$$

令~~ε(t)~~ $y_{fs}(t)$作为输入信号，得系统S输出为

$$y(t)=y_{fs}(t)-2y_{fs}(t-1)+y_{fs}(t-2)$$

$$=\varepsilon(t)-4\varepsilon(t-1)+6\varepsilon(t-2)-4\varepsilon(t-3)+\varepsilon(t-4)$$

当复合系统输入信号$\varepsilon(t)$时，其输出响应

$$y(t)=\varepsilon(t)-4\varepsilon(t-1)+6\varepsilon(t-2)-4\varepsilon(t-3)+\varepsilon(t-4)$$

1.31(b)的输入信号为 $\varepsilon(t)-\varepsilon(t-2)$

以LTI系统具有齐次性、可加性与时不变性，

得复合系统的零状态响应

$$y_f(t)=[\varepsilon(t)-4\varepsilon(t-1)+6\varepsilon(t-2)-4\varepsilon(t-3)+\varepsilon(t-4)]$$
$$-[\varepsilon(t-2)-4\varepsilon(t-3)+6\varepsilon(t-4)-4\varepsilon(t-5)+\varepsilon(t-6)]$$

$$=\varepsilon(t)-4\varepsilon(t-1)+5\varepsilon(t-2)-5\varepsilon(t-4)+4\varepsilon(t-5)-\varepsilon(t-6)$$