UNIVERSITY OF ABERDEEN **SESSION 2016–17**

DEGREE EXAMINATION

MA1005 Calculus I

December 13th 2016 (09:00 - 11:00)

Calculators are not permitted in this examination.

Marks may be deducted for answers that do not show clearly how the solution is reached.

Attempt ALL FOUR questions from SECTION A and TWO questions from SECTION B.

All questions are worth 10 marks, except for question 4, which is worth 20 marks.

SECTION A – answer all questions

1. **(2 marks per part.)** Compute the following limits.

(a) $\lim_{x\to 1} \dfrac{x^2+4x-5}{x^2+3x-4}$ **(b)** $\lim_{x\to 1} \dfrac{x^2-2x+1}{x^2-1}$ **(c)** $\lim_{t\to 0} \dfrac{\sqrt{3+t}-\sqrt{3-t}}{t}$

(d) $\lim_{x\to 2} \left[\dfrac{1}{x-2} - \dfrac{5}{x^2+x-6}\right]$ **(e)** $\lim_{\theta\to 0} \dfrac{\theta-\theta^2}{\sin(\theta)}$

2. **(a) (8 marks.)** Differentiate the functions f, g, h and k defined as follows.

$$f(x) = \frac{x^3}{x^2+1} \qquad g(x) = x^2\sin^2(x) \qquad h(x) = \frac{1}{\sqrt{3\sin(x)-1}} \qquad k(t) = \sqrt[3]{t^4} + \sqrt[4]{t^3}$$

(b) (2 marks.) Given that $\sin(y) + xy^2 = 4$, use implicit differentiation to find y'. Your solution should express y' in terms of x and y.

3. **(a) (5 marks.)** Let f be the function defined by $f(x) = \dfrac{1}{x^2+1}$. Use the definition of the derivative to show that $f'(x) = \dfrac{-2x}{(x^2+1)^2}$.

(b) (5 marks.) Use the precise definition of the limit to show that $\lim_{x\to 2}(x^2+2x+3) = 11$.

4. Let f be the function defined by $f(x) = \dfrac{x}{x^2+1}$.

(a) (6 marks.) Show that $f'(x) = \dfrac{1-x^2}{(x^2+1)^2}$ and $f''(x) = \dfrac{2x^3-6x}{(x^2+1)^3}$.

(b) (3 marks.) What are the critical numbers of f? Are they local maxima or minima?

(c) (3 marks.) On what intervals is f increasing? On what intervals is f decreasing?

(d) (4 marks.) What are the absolute maximum and minimum values of f on the interval $[1/2, 3]$? (In your answer you must state the method you are using and you must explain why it applies in this situation.)

(e) (4 marks.) Use the intermediate value theorem to show that there is a number c in the interval $(-1, 1)$ for which $f(c) = 1/3$. (In your answer you must explain why the intermediate value theorem applies.)

SECTION B – answer two questions

5. **(a) (5 marks.)** Let p and q be functions satisfying $q(4) = 2$, $q'(4) = 5$, and $p'(2) = 3$. Let s and t be the functions defined by $s(x) = p(q(x^2))$ and $t(x) = x/q(x)$. Find $s'(2)$ and $t'(4)$.

(b) (5 marks.) Let k be the function defined as follows.

$$k(x) = \begin{cases} x \cdot |x| \cdot \sin(\frac{1}{x}) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

Use the definition of the derivative to show that $k'(0) = 0$. (You may need to make use of the squeeze theorem to answer this question.)

6. **(a) (5 marks.)** Use the precise definition of the limit to show that $\lim_{x \to 4} \frac{x+5}{x+8} = \frac{3}{4}$.

(b) (5 marks.) Let k be the function defined by $k(x) = x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f$. Find the values of a, b, c, d, e and f for which k has **all three** of the following properties.

- k is an even function.
- The critical numbers of k are 0, 1, -1, 2 and -2.
- $k(0) = 1$.

7. **(a) (6 marks.)** Consider the function f defined as follows.

$$f(x) = \begin{cases} 4x^2 + x + 1 & \text{if } x \leqslant a \\ x + 2 & \text{if } x > a \end{cases}$$

For what positive value of a is f continuous at $x = a$? (Your answer must demonstrate that f is indeed continuous for this value of a.) For that particular value of a, use the definition of the derivative to show that f is not differentiable at a.

(b) (4 marks.) Sketch the graph of **one** function f that satisfies **all four** of the following properties:

- f is continuous at x for all $x \in (-\infty, \infty)$.
- $f(x) \leqslant f(y)$ for $x \leqslant y$.
- f is differentiable at x if x is not an integer.
- f is not differentiable at x if x is an integer.

Ensure that your graph gives enough detail for the reader to see that the required properties hold.