(k) $f'(\theta) = \dfrac{\sec^2\theta}{(1+\tan\theta)^2}$

(l) *Worked Solution:*

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx}\left[\frac{\cos x}{1+\sin x}\right] \\
&= \frac{(1+\sin x)\cdot \frac{d}{dx}\cos x - \cos x \cdot \frac{d}{dx}(1+\sin x)}{(1+\sin x)^2} \\
&= \frac{-(1+\sin x)\cdot \sin x - \cos x \cdot \cos x)}{(1+\sin x)^2} \\
&= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2} \\
&= \frac{-\sin x - 1}{(1+\sin x)^2} \\
&= -\frac{1}{1+\sin x}
\end{aligned}$$

(m) $y' = -\dfrac{\sec x + \sec^2 x}{\tan^2 x}$

(n) $\frac{dy}{dx} = x\cos(x)\cot(x)[2 + x\cot(x)]\ \ x\cos x[2\cot x - x - x\csc^2 x]$

66. *Worked Solution:*

$$\begin{aligned}
\frac{d}{dx}\cos x &= \lim_{h\to 0}\frac{\cos(x+h)-\cos(x)}{h} \\
&= \lim_{h\to 0}\frac{\cos x\cos h - \sin x \sin h - \cos x}{h} \\
&= \lim_{h\to 0}\frac{\cos x(\cos h - 1) - \sin x\sin h}{h} \\
&= \lim_{h\to 0}\left[\cos x\frac{\cos h - 1}{h} - \sin x\frac{\sin h}{h}\right] \\
&= \cos x\cdot \lim_{h\to 0}\frac{\cos h - 1}{h} - \sin x\cdot\lim_{h\to 0}\frac{\sin h}{h} \\
&= \cos x \cdot 0 - \sin x\cdot 1 \\
&= -\sin x.
\end{aligned}$$

67. *Worked Solution for* $\sec x$*:*

$$\frac{d}{dx}\sec x = \frac{d}{dx}\frac{1}{\cos x} = -\frac{\frac{d}{dx}\cos x}{\cos^2 x} = -\frac{-\sin x}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x} = \tan x\cdot\sec x$$

68. **(a)** $y = \frac{2}{3}x + (\frac{2}{\sqrt{3}} - \frac{\pi}{9})$

(b) $y = x$

(c) *Worked Solution:* Since $y = x + \sec x$, we have $\frac{dy}{dx} = \frac{d}{dx}[x + \sec x] = 1 + \sec x \tan x$, so when $x = \pi$ we have $\frac{dy}{dx} = 1 + \sec\pi\tan\pi = 1 + -1 \cdot 0 = 1$. Thus the equation of the tangent line is $y - (\pi - 1) = 1(x - \pi)$, or in other words $y = x - 1$.

69. *Worked Solution:*

Since $45 = 4 \times 11 + 1$, we have $\frac{d^{45}}{dx^{45}} \sin x = \frac{d}{dx} \sin x = \cos x$.

For the second part, we work out that $\frac{d^4}{dx^4}[x\cos x] = \frac{d^3}{dx^3}[-x\sin x + \cos x] = \frac{d^2}{dx^2}[-x\cos x - 2\sin x] = \frac{d}{dx}[x\sin x - 3\cos x] = x\cos x + 4\sin x$. Thus $\frac{d^8}{dx^8}[x\cos x] = \frac{d^4}{dx^4}[x\cos x + 4\sin x] = x\cos x + 4\sin x + 4\sin x = x\cos x + 8\sin x$. Proceeding in this way, we find that $\frac{d^{16}}{dx^{16}}[x\cos x] = x\cos x + 16\sin x$.

70. It has horizontal tangent at the points of the form $(\frac{2\pi}{3} + 2k\pi, \frac{1}{\sqrt{3}})$ and $(-\frac{2\pi}{3} + 2k\pi, -\frac{1}{\sqrt{3}})$.

71. **(a)** *Worked Solution:* $y = \cos(3x) = \cos(g(x)) = f(g(x))$ where $g(x) = 3x$ and $f(u) = \cos(u)$.

Consequently $\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$, and since $f'(u) = -\sin(u)$ and $g'(x) = 3$, we have

$$\frac{dy}{dx} = -\sin(g(x)) \cdot 3 = -3\sin(3x).$$

(b) $g(x) = 4 + 3x$, $f(u) = \sqrt{u}$, $\frac{dy}{dx} = \frac{3}{2\sqrt{4+3x}}$

(c) $g(x) = 1 - x^3$, $f(u) = u^5$, $\frac{dy}{dx} = -15x^2(1 - x^3)^4$.

(d) *Worked Solution:* $y = \sqrt[3]{\sin(x)} = \sqrt[3]{g(x)} = f(g(x))$ where $g(x) = \sin(x)$ and $f(u) = \sqrt[3]{u}$.

So $\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$. Now $f'(u) = \frac{1}{3u^{2/3}}$ and $g'(x) = \cos(x)$, so that

$$\frac{dy}{dx} = f'(g(x)) \cdot g'(x) = \frac{1}{3(g(x))^{2/3}} \cdot \cos(x) = \frac{\cos(x)}{3(\sin(x))^{2/3}}.$$

72. **(a)** $F'(x) = 16x(x^4 - 2x^2 + 2)^3(x^2 - 1)$

(b) $F'(x) = \dfrac{1 - x^2}{(1 + 3x - x^3)^{2/3}}$

(c) $g'(t) = -\dfrac{12t^2}{(t^3 + 1)^5}$.

(d) $\frac{dy}{dx} = 4x^3\cos(a^4 + x^4)$

(e) $\frac{dy}{dx} = 4\sin^3 x \cdot \cos x$

(f) $y' = x\csc(kx)[2 - kx\cot(kx)]$.

(g) $f'(x) = (3x-2)^3(x^4-x-1)^4[72x^4 - 40x^3 - 27x - 2]$

(h) $h'(t) = [34t^2 - 32t - 1]\dfrac{(2t^2-1)^3}{2(t-1)^{1/2}}$

(i) $y' = 12x\dfrac{(x^2-1)^2}{(x^2+1)^4}$

(j) $y' = -\sin(x\sin x)(x\cos x + \sin x)$

(k) $F'(z) = -\dfrac{1}{(z+1)^{1/2}(z-1)^{3/2}}$

(l) $y' = \dfrac{1}{(1-r^2)^{3/2}}$

(m) $y' = \dfrac{x\sin\sqrt{1-x^2}}{\sqrt{1-x^2}}$

(n) $y' = 2\cos(2x)\cdot cos^{-2}(\sin 2x)$

(o) $y' = -16\cos 2x\dfrac{(1-\sin 2x)^3}{(1+\sin 2x)^5}$

(p) $y' = \dfrac{2x^{1/2}-1}{4x^{1/2}(x-x^{1/2})^{1/2}}$

73. **(a)** *Worked Solution:* The first derivative is

$$\frac{dy}{dx} = \cos(x^2)\cdot 2x = 2x\cos(x^2)$$

and the second derivative is

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{d}{dx}(2x)\cdot\cos(x^2) + 2x\cdot\frac{d}{dx}\cos(x^2)\\ &= 2\cos(x^2) + 2x\cdot 2x\cdot(-\sin(x^2))\\ &= 2\cos(x^2) - 4x^2\sin(x^2).\end{aligned}$$

(b) $\frac{dy}{dx} = 2\sin(x)\cos(x)$ and $\frac{d^2y}{dx^2} = 2(\cos^2(x) - \sin^2(x))$.

(c) $K'(t) = 5\sec^2(5t)$ and $K''(t) = 50\sec^2(5t)\tan(5t)$

74. *Worked Solution:* The product rule tells us that

$$f'(x) = \frac{d}{dx}(x)\cdot g(x^3) + x\cdot\frac{d}{dx}(g(x^3)) = g(x^3) + x\frac{d}{dx}(g(x^3)),$$

and the chain rule tells us that $\frac{d}{dx}(g(x^3)) = g'(x^3)\cdot 3x^2$. So

$$f'(x) = g(x^3) + 3x^3g'(x^3).$$

Similarly,

$$f''(x) = 3x^2g'(x^3) + 9x^2g'(x^3) + 3x^3\cdot 3x^2\cdot g''(x^3) = 12x^2g'(x) + 9x^5g''(x^3).$$

75. **(a)** $y' = -\dfrac{x^4}{y^4}$

(b) *Worked Solution:* Differentiating both sides of the equation $4\sqrt{x} - 4\sqrt{y} = 1$ yields

$$4\frac{d}{dx}\sqrt{x} - 4\frac{d}{dx}\sqrt{y} = \frac{d}{dx}1$$

i.e.

$$4\frac{1}{\sqrt{x}} - 4\frac{1}{\sqrt{y}}\frac{d}{dx}y = 0$$

i.e.

$$\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{y}}y' = 0.$$

Rearranging this gives

$$y' = \frac{\sqrt{y}}{\sqrt{x}}.$$

(c) $y' = -\dfrac{2x+y}{x+2y}$

(d) $y' = -\dfrac{3x^2+4xy-2y^3}{2x^2-6xy^2}$

(e) *Worked Solution:* The given equation $x^3(x+y) = y^3(2x-y)$ can be written as $x^4 + x^3y = 2xy^3 - y^4$. Differentiating both sides with respect to x gives

$$4x^3 + (3x^2y + x^3y') = (2y^3 + 2x \cdot 3y^2y') - 4y^3y'.$$

Rearranging,

$$4x^3 + 3x^2y + x^3y' = 2y^3 + 6xy^2y' - 4y^3y'.$$

$$4x^3 + 3x^2y - 2y^3 = 6xy^2y' - 4y^3y' - x^3y'$$

$$4x^3 + 3x^2y - 2y^3 = y'(6xy^2 - 4y^3 - x^3)$$

so that

$$y' = \frac{4x^3 + 3x^2y - 2y^3}{6xy^2 - 4y^3 - x^3}.$$

(f) $y' = -\dfrac{y^2 + 2x\sin y}{2xy + x^2\cos y}$

(g) $y' = \dfrac{1 - y^2\sin(xy^2)}{2xy\sin(xy^2)}$

(h) $y' = -\dfrac{\tan x}{\tan y}$

(i) $y' = \dfrac{\cos y^2 + 2xy\sin x^2}{\cos x^2 + 2xy\sin y^2}$

(j) $y' = \dfrac{y(\cos(x/y) - y^2)}{x(y^2 + \cos(x/y))}$

(k) *Worked Solution:* Differentiating both sides of the equation $\sqrt{x-y} = 1 - x^2y^2$ with respect to x gives

$$\frac{1}{2\sqrt{x-y}}\frac{d}{dx}(x-y) = -\frac{d}{dx}(x^2y^2)$$

or in other words

$$\frac{1}{2\sqrt{x-y}}(1-y') = -(2x\cdot y^2 + x^2\cdot 2yy').$$

Rearranging gives the following:

$$\frac{1}{2\sqrt{x-y}} - \frac{y'}{2\sqrt{x-y}} = -2xy^2 - 2x^2yy'$$

$$\frac{1}{2\sqrt{x-y}} + 2xy^2 = \frac{y'}{2\sqrt{x-y}} - 2x^2yy'$$

$$\frac{1}{2\sqrt{x-y}} + 2xy^2 = y'\left(\frac{1}{2\sqrt{x-y}} - 2x^2y\right)$$

Thus

$$y' = \frac{\frac{1}{2\sqrt{x-y}} + 2xy^2}{\frac{1}{2\sqrt{x-y}} - 2x^2y} = \frac{1+4xy^2\sqrt{x-y}}{1-4x^2y\sqrt{x-y}}.$$

(l) $y' = \dfrac{y - 2y^{5/2}x^{1/2}}{4x^{3/2}y^{3/2} - x}$

(m) $y' = \dfrac{\cos y + y\sin x}{\cos x + x\sin y}$

76. **(a)** $f'(1) = 2/3$

(b) *Worked Solution:* Differentiating both sides of the equation $g(x) + x\cos(g(x)) = x^3$ gives $g'(x) + \cos(g(x)) - x\sin(g(x))g'(x) = 3x^2$, and setting $x = 0$ gives $g'(0) + \cos(g(0)) = 0$. We can compute $g(0)$ by taking the original equation $g(x)+x\cos(g(x)) = x^3$ and setting $x = 0$ to give $g(0) = 0$, so that $\cos(g(0)) = 1$. It follows that $g'(0) = -1$.

77. **(a)** *Worked Solution:* Differentiating both sides of the equation $y\cos(2x) = x\sin(2y)$ gives

$$y'\cos(2x) - 2y\sin(2x) = \sin(2y) + 2x\cos(2y)y'.$$

Setting $x = \pi/4$ and $y = \pi/2$ gives

$$y'\cos(\pi/2) - \pi\sin(\pi/2) = \sin(\pi) + (\pi/2)\cos(\pi)y'$$

or in other words

$$-\pi = (-\pi/2)y'$$

so that $y' = 2$. Thus the gradient of the tangent line to the curve through the point $(\pi/4, \pi/2)$ is 2, and so the gradient of the tangent is

$$y - \pi/2 = 2(x - \pi/4)$$

or in other words $y = 2x$.

(b) $y = -3x + \pi$

(c) $y = x - 2$

(d) $y = -\sqrt{3}x + 4\sqrt{3}$

(e) $y = \frac{5\sqrt{5}}{8}x - \frac{9}{8}$

78. **(a)** *Worked Solution:* Differentiating both sides of the equation

$$x^2 + 4y^2 = 4$$

and rearranging gives

$$y' = -\frac{x}{4y}.$$

Differentiating this expression then gives

$$y'' = \frac{-4y + 4xy'}{16y^2}.$$

Substituting our expression for y' now gives

$$y'' = \frac{-4y + 4x\frac{-x}{4y}}{16y^2} = \frac{-4y^2 - x^2}{16y^3}.$$

We may now use the original equation to obtain

$$y'' = \frac{-1}{4y^3}.$$

(b) $y'' = \frac{\sqrt{xy} - y}{2x\sqrt{xy}}$

(c) *Worked Solution:* Differentiating the equation

$$x^5 + y^5 = 1$$

gives

$$5x^4 + 5y^4y' = 0$$

which rearranges to give

$$y' = -\frac{x^4}{y^4}.$$

Differentiating this expression gives

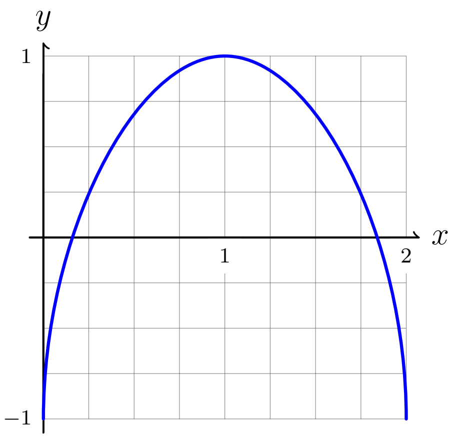
$$\begin{aligned} y'' &= -\frac{y^4 \cdot 4x^3 - x^4 \cdot 4y^3 y'}{y^8} \\ &= -\frac{y^4 \cdot 4x^3 - x^4 \cdot 4y^3 \cdot (-x^4/y^4)}{y^8} \\ &= -\frac{4x^3y^4 + 4x^8y^{-1}}{y^8} \\ &= -4\frac{x^3y^5 + x^8}{y^9} \\ &= -4\frac{x^3}{y^9}(y^5 + x^5). \end{aligned}$$

Now using the original equation we find that

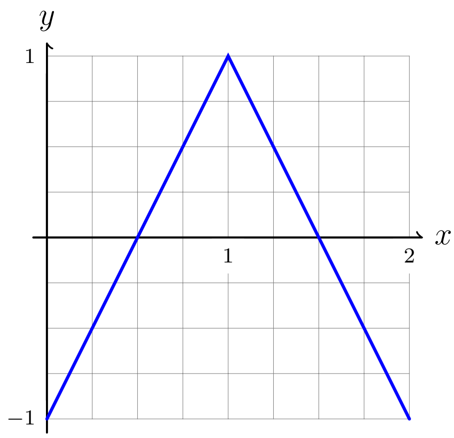
$$y'' = -4\frac{x^3}{y^9}.$$

(d) $y'' = \dfrac{-2xy}{(3y^2 - x)^3}$

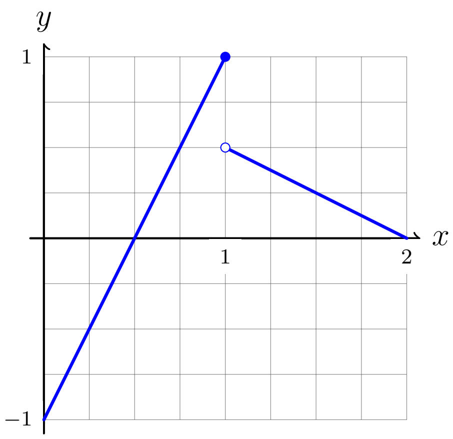
79. **(a)**



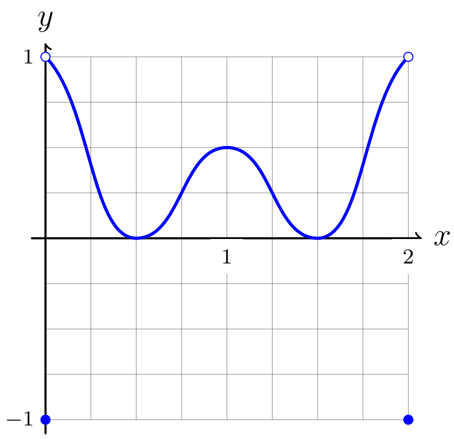
(b)



(c)



(d)



80. **(a)** $-1/4$

(b) 1

(c) *Worked Solution:* The function is continuous and its domain is $\mathbb{R}$.

For $t > 3/4$ we have $4t - 3 > 0$, and so $g(t) = 4t - 3$ and consequently $g'(t) = 4 \neq 0$. So $g'(t)$ exists and is nonzero for all $t > 3/4$, and consequently there are no critical numbers in this range.

For $t < 3/4$ we find, similarly, that there are no critical numbers.

For $t = 3/4$ we find that $g'(t)$ does not exist, since (as can be checked) $\lim_{h\to 0^+} \frac{g(3/4+h)-g(3/4)}{h}$ and $\lim_{h\to 0^-} \frac{g(3/4+h)-g(3/4)}{h}$ exist but are not equal. So $t = 3/4$ is a critical number.

The only critical number is $3/4$.

(d) 0, -2.

(e) *Worked Solution:* The domain of h is $[0, \infty)$. Now $h'(t) = \frac{t^{-3/4}}{4}(6t^{1/2} - 1)$, and in particular it exists for all t in the domain of h, except for $t = 0$. Thus the critical numbers are those t such that $h'(t) = 0$, i.e. $t = 1/36$, and 0.

(f) The critical numbers are 0, where $F'(x)$ does not exist, and 3 and 1/2, where $F'(x) = 0$.

81. **(a)** The absolute maximum is $f(3) = 9$ and the absolute minimum is $f(1) = 1$.

(b) The absolute maximum is $f(0) = 2$. and the absolute minimum is $f(2) = -14$.

(c) The absolute maximum is $f(2) = 187$ and the absolute minimum is $f(0) = -5$.

(d) *Worked Solution:* We will use the closed interval method. This can be applied because the function f is a product of a polynomial with a square root of a polynomial, and so is continuous on the whole domain $[-1, 3]$.

0. $f'(t) = \sqrt{9 - t^2} + \frac{t}{2\sqrt{9-t^2}}(-2t) = \frac{1}{9-t^2}(9 - 2t^2)$. This is defined for all $t \in [-1, 3)$, but not for $t = 3$. So the critical numbers are $t = 3$ and those $t \in [-1, 3)$ for which $f'(t) = 0$, i.e. $3/\sqrt{2}$.

1. The only critical number in the interior $(-1, 3)$ is $3/\sqrt{2}$, and $f(3/\sqrt{2}) = 3/\sqrt{2}\sqrt{9 - 9/2} = 9/2$.

2. $f(-1) = -2\sqrt{2}$ and $f(3) = 0$.

So the absolute maximum and minimum are the largest and smallest numbers from parts 1 and 2. In other words the absolute maximum is $f(3/\sqrt{2}) = 9/2$ and the absolute minimum is $f(-1) = -2\sqrt{2}$.

(e) The absolute maximum is $f(\pi/2) = 3$ and the absolute minimum is $f(7\pi/6) = f(11\pi/6) = -1.5/$

82. **(a)** The only value satisfying the conclusion is $c = 1$.

(b) *Worked Solution:* f is continuous on the interval $[0, 2]$ because $x \geqslant 0$ for all $x \in [0, 2]$, and f is differentiable on this interval because $x > 0$ for all $x \in (0, 2)$. So the hypotheses of the mean value theorem hold.

Next, for the conclusions of the theorem to hold for $c \in (0, 2)$ means that

$$f'(c) = \frac{f(2) - f(0)}{2 - 0} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}.$$

Since $f'(x) = \frac{1}{2\sqrt{x}}$, we see that the conclusions hold for c if $\frac{1}{2\sqrt{c}} = \frac{1}{\sqrt{2}}$, so the conclusions hold if and only if $c = \frac{1}{2}$.

83. *Worked Solution:* Let f be the function defined by $f(x) = 2x + \sin(x)$. Then $f(2\pi) = 4\pi + 0 = 4\pi > 0$ and $f(-2\pi) = -4\pi + 0 = -4\pi$. Thus $f(2\pi) > 0 > f(-2\pi)$, and f is continuous on the interval $[-2\pi, 2\pi]$, so by the intermediate value theorem there is $x_0 \in (-2\pi, 2\pi)$ such that $f(x_0) = 0$. In other words the equation has a root x_0.

Suppose that the equation has a second root x_1. Then since $f(x_0) = f(x_1) = 0$ and f is differentiable everywhere, Rolle's theorem shows that there is $c \in (x_0, x_1)$ such that $f'(c) = 0$. However $f'(x) = 2 + \cos(x) \geqslant 2 - 1 = 1 > 0$ so that no such c exists, and so no second root could have existed.

We have shown that there is at least one root, but there cannot be two. So there is exactly one root as required.

84. **(a)** *Worked Solution:* We will show that it is impossible for the equation to have two roots in $[-2, 2]$. Suppose it does, call them x_0 and x_1, and (by swapping them if necessary) assume that $-2 \leqslant x_0 < x_1 \leqslant 2$.

Let f be the function defined by $f(x) = x^3 - 14x + 5$. Then f is a polynomial, so is continuous on $[x_0, x_1]$ and is differentiable on (x_0, x_1). (In fact it is continuous and differentiable everywhere.) Then $f(x_0) = f(x_1) = 0$, so that by Rolle's Theorem there is $c \in (x_0, x_1)$ such that $f'(c) = 0$. Now by computing f' we see that $f'(c) = 3c^2 - 14$. However, since $c \in (-2, 2)$, we have $f'(c) = 3c^2 - 14 < 3 \times 4 - 14 = -2$, so that $f'(c) = 0$ is impossible.

This contradiction means that there could not have been two roots in the first place.

85. **(a)** *Worked Solution:* Since f is differentiable, it satisfies the assumptions of the Mean Value Theorem for the interval $[-1, 2]$. Thus there is $c \in (-1, 2)$ for which

$$\frac{f(2) - f(-1)}{2 - (-1)} = f'(c)$$

or in other words $f(2) - f(-1) = 3f'(c)$. Since $2 \leqslant f'(x) \leqslant 4$, it follows that $6 \leqslant f(2) - f(-1) \leqslant 12$.

86. Define h by $h(x) = g(x) - f(x)$. The given inequalities show that $h(a) = g(a) - f(a) \geqslant 0$ and that $h'(x) = g'(x) - f'(x) \geqslant 0$ for $x \in (a, b)$. And we want to show that $h(b) \geqslant 0$ since then $g(b) - f(b) \geqslant 0$ so that $g(b) \geqslant f(b)$.

Since f and g are continuous on $[a, b]$ and differentiable on (a, b), the same is true for the difference h. So we may apply the Mean Value Theorem: there is $c \in (a, b)$ such that

$$\frac{h(b) - h(a)}{b - a} = h'(c)$$

and consequently $h(b) = (b-a)h'(c)+h(a)$. Since $a < b$ we have $(b-a) > 0$, and since $c \in (a, b)$ we have $h'(c) \geqslant 0$, so that $(b-a)h'(c) \geqslant 0$. And we know that $h(a) \geqslant 0$, so that $(b-a)h'(c) + h(a) \geqslant 0$. Thus $h(b) \geqslant 0$ as required.

87. **(a)** *Worked Solution:* We will use the fact that the local maxima or minima of f are critical points of f.

The function is continuous and its derivative exists for all x, so its critical points are the points where $f'(x) = 0$. Now $f'(x) = 3x^2 - 3$, which is zero for $x = 1$ and $x = -1$. So the possible local maxima and minima occur at $x = \pm 1$.

To determine if they are local maxima or minima, we use the second derivative test, which applies since $f''(x)$ exists and is continuous for all x, and is given by $f''(x) = 6x$.

Since $f''(1) = 6 > 0$, f has a local minimum at 1.

Since $f''(-1) = -6 < 0$, f has a local maximum at -1.

(b) *Worked Solution:* We will use the fact that the local maxima and minima are all critical points. The domain of the function is $\mathbb{R}$ and it is continuous everywhere, so that the critical points are the points where $f'(x) = 0$ or where $f'(x)$ does not exist.

The derivative is $f'(x) = 1 + \frac{1}{3}x^{-2/3}$, which exists for all $x \in \mathbb{R}$ except for $x = 0$. So $x = 0$ is a critical number. There are no other critical numbers, since $f'(x) > 0$ for all $x \neq 0$. So the only possible local maximum or minimum is at $x = 0$. Since $f'(x)$ does not exist there, we cannot use the second derivative test. But the first derivative test applies, and indeed, since $f'(x) > 0$ for all $x \neq 0$, it tells us that 0 is neither a local maximum nor a local minimum.

So f has no local maxima or minima.

(c) f has a local minimum at 0 and a local maximum at -2.

88. **(a)** Increasing on $(-\infty, -2)$ and $(3, \infty)$, decreasing on $(-2, 3)$, local maximum at $(-2, 44)$, local minimum at $(3, -81)$.

(b) *Worked Solution:* First,

$$f'(x) = 12x^2 - 18x + 6 = 6(2x^2 - 3x + 1) = 6(2x - 1)(x - 1).$$

Thus $f'(x) = 0$ if and only if $x = 1/2$ or $x = 1$. The sign of $f'(x)$ can be computed as follows:

- For $x < 1/2$ we have $(2x - 1) < 0$ and $(x - 1) < 0$, so that $f'(x) > 0$.
- For $1/2 < x < 1$ we have $(2x - 1) > 0$ and $(x - 1) < 0$, so that $f'(x) < 0$.
- For $1 < x$ we have $(2x - 1) > 0$ and $(x - 1) > 0$ so that $f'(x) > 0$.

So f is increasing on $(-\infty, 1/2)$ and $(1, \infty)$ and it is decreasing on $(1/2, 1)$.

As above, the critical numbers of f are $x = 1/2$ and $x = 1$. Now $f(1/2) = 9/4$ and $f(1) = 2$ so that the critical points are $(1/2, 9/4)$ and $(1, 2)$. And we can compute

$$f''(x) = 24x - 18$$

so that $f''(1/2) = -6 < 0$ and $f''(1) = 6 > 0$ and consequently $(1/2, 9/4)$ is a local maximum and $(1, 2)$ is a local minimum.

(c) f is decreasing on $(-\infty, -1)$ and $(0, 1)$, it is increasing on $(-1, 0)$ and $(1, \infty)$, there is a local maximum at $(0, -3)$ and there are local minima at $(-1, -4)$ and $(1, -4)$.

(d) f is increasing on $(-\infty, -\sqrt{2})$ and on $(-\sqrt{2}, 0)$. f is decreasing on $(0, \sqrt{2})$ and on $(\sqrt{2}, \infty)$, and there is a local maximum at $(0, 0)$.

(e) *Worked Solution:* First we compute $f'(x) = 1 - 2\sin(x)$, so that $f'(x) = 0$ when $\sin(x) = 1/2$, or in other words when $x = \pi/6$ and $x = 5\pi/6$. Now by considering the definition of $\sin(x)$ we see that:

- For $0 \leqslant x < \pi/6$ we have $f'(x) > 0$;
- for $\pi/6 < x < 5\pi/6$ we have $f'(x) < 0$;
- for $5\pi/5 < x \leqslant 2\pi$ we have $f'(x) > 0$.

So f is increasing on $[0, \pi/6)$ and $(5\pi/6, 2\pi]$, and it is decreasing on $(\pi/6, 5\pi/6)$.

Next,

$$f''(x) = -2\cos(x),$$

so that $f''(\pi/6) = -2\cos(\pi/6) = -\sqrt{3} < 0$ and $f''(5\pi/6) = -2\cos(5\pi/6) = \sqrt{3} > 0$ so that there is a local maximum at $(\pi/6, \pi/6 + \sqrt{3})$ and a local minimum at $(5\pi/6, 5\pi/6 - \sqrt{3})$.

(f) f is increasing on $(4, \infty)$ and it is decreasing on $[0, 4)$. It has a local minimum at $(4, -4)$.

89. **(a)** *Worked Solution:* The domain of $f(x) = \frac{1-e^{x^2}}{1-e^{4-x^2}}$ is the set of x such that $1 - e^{4-x^2} \neq 0$, or equivalently $e^{4-x^2} \neq 1$, or equivalently $4 - x^2 \neq 0$, or equivalently $x^2 \neq 4$, or equivalently $x \neq \pm 2$. So the domain is

$$\{x \mid x \neq \pm 2\}$$

or, written another way,

$$(-\infty, -2) \cup (-2, 2) \cup (2, \infty).$$

(b) $\mathbb{R}$

(c) $\{x \mid x \neq k\pi,\ k \in \mathbb{Z}\}$

(d) $(-\infty, 0]$

90. **(a)** 0

(b) $e^s - es^{e-1}$

(c) $(x^4 + 4x^3 - 2x - 2)e^x$

(d) $y' = \frac{d}{dx}(e^{bx^4}) = e^{bx^4}\frac{d}{dx}(bx^4) = e^{bx^4} \cdot 4bx^3 = 4bx^3e^{bx^4}$.

(e) *Worked Solution:*

$$\begin{aligned} y' &= \frac{d}{dt}(e^{2t}\sin(4t)) \\ &= \frac{d}{dt}(e^{2t})\sin(4t) + e^{2t}\frac{d}{dt}\sin(4t) \\ &= 2e^{2t}\sin(4t) + e^{2t} \cdot 4\cos(4t) \\ &= e^{2t}(2\sin(4t) + 4\cos(4t)) \end{aligned}$$

(f) $-e^{x-e^x}$

(g) $\dfrac{(ad - bc)e^{-x}}{(c + de^{-x})^2}$

(h) $\dfrac{-e^{2x}(1 + 2x)}{2\sqrt{1 - xe^{2x}}}$

(i) $4\sin(e^{\cos^2 t})\cos(e^{\cos^2 t})e^{\cos^2 t}\sin t\cos t$

91. The line has equation $y = \dfrac{e}{1 - e}(x - 1)$.

92. **(a)** The domain of f is $[0, \infty)$ and $f^{-1}(x) = -\ln(1 - x^2)$.

(b) *Worked Solution:* The domain of ln is $(0, \infty)$. So the domain of $f(x) = \ln(3 - \ln x)$ is the set of x such that $x > 0$ and $3 - \ln x > 0$, or in other words $x > 0$ and $\ln x < 3$, or in other words $x > 0$ and $x < e^3$, or in other words the set $(0, e^3)$.

To find an expression for f^{-1} we first set $y = f(x)$, i.e. $y = \ln(3 - \ln x)$. Taking exponentials of both sides, we find $e^y = 3 - \ln x$, so that $\ln x = 3 - e^y$, so that $x = e^{3-e^y}$. Next we interchange x and y to find $y = e^{3-e^x}$, so that

$$f^{-1}(x) = e^{3-e^x}.$$

(c) The domain is $(0, \infty)$, and $f^{-1}(x) = \ln(e^x + 1)$.

93. **(a)** $2 + \ln x$

(b) $-\sin(\ln x)/x$

(c) *Worked Solution:* $f'(x) = \dfrac{d}{dx}(\ln(\cos x)) = \dfrac{1}{\cos x}\dfrac{d}{dx}(\cos x) = \dfrac{1}{\cos x}(-\sin x) = -\tan x$.

(d) $\dfrac{1}{4x(\ln x)^{3/4}}$

(e) *Worked Solution:* Since $\frac{d}{dx}\log_{10}(x) = \frac{1}{x\ln 10}$, the chain rule gives us

$$\frac{d}{dx}\log_{10}(x^2+1) = \frac{1}{(x^2+1)\ln 10}\cdot\frac{d}{dx}(x^2+1) = \frac{1}{(x^2+1)\ln 10}\cdot 2x = \frac{2x}{(x^2+1)\ln 10}.$$

(f) $4x^3 + \ln(4)\cdot 4^x$

(g) $\frac{-1}{x(x^2-1)}$

(h) $\dfrac{-1}{\sqrt{x^2-1}}$

(i) $\dfrac{x}{x-1}$

95. **(a)** One-to-one.

(b) *Worked Solution:* The function is not one-to-one because $f(0) = 1 = f(4)$ but $1 \neq 4$.

(c) Not one-to-one.

(d) *Worked Solution:* The function is one-to-one because if $g(x_1) = g(x_2)$ then $\sqrt[4]{x_1} = \sqrt[4]{x_2}$, and by taking fourth powers we find that $x_1 = x_2$.

(e) Not one-to-one.

(f) *Worked Solution:* The function is one-to-one because no horizontal line crosses its graph more than once.

96. **(a)** $f^{-1}(3) = 1$ and $f^{-1}(-1) = -1$.

(b) $h^{-1}(10) = 8$ and $h^{-1}(2) = 1$.

98. **(a)** $f^{-1}(x) = \frac{4-x}{3}$

(b) $f^{-1}(x) = \frac{x^2-2x}{2}$

(c) $g^{-1}(x) = 1 + \sqrt{1+x}$

(d) *Worked Solution:* First we solve the equation $h(x) = y$ to express x in terms of y. If $\frac{1-2\sqrt{x}}{1+2\sqrt{x}} = y$ then $y + 2y\sqrt{x} = 1 - 2\sqrt{x}$, so that $\sqrt{x}(2y+2) = 1-y$ and consequently $\sqrt{x} = \frac{1-y}{2y+2}$ so that $x = \left(\frac{1-y}{2y+2}\right)^2$. Next we replace x with y and *vice versa* to obtain $y = \left(\frac{1-x}{2x+2}\right)^2$. Finally, this equation is $y = h^{-1}(x)$, so that

$$h^{-1}(x) = \left(\frac{1-x}{2x+2}\right)^2.$$

(e) *Worked Solution:* First we solve the equation $k(t) = y$ to express t in terms of y. If $y = 4t^2 - 2t$ then $4t^2 - 2t - y = 0$, so by the quadratic formula $t = \dfrac{2 \pm \sqrt{4+16y}}{8} = \dfrac{2 \pm 2\sqrt{1+4y}}{8} = \dfrac{1}{4} \pm \dfrac{1}{4}\sqrt{1+4y}$. Now since $t \geqslant 1/4$, we must have $t = \frac{1}{4} + \frac{1}{4}\sqrt{1+4y}$.

Next, we interchange y and t to obtain $y = \frac{1}{4} + \frac{1}{4}\sqrt{1+4t}$. Finally, this is the equation $y = k^{-1}(t)$, so that

$$k^{-1}(t) = \frac{1}{4} + \frac{1}{4}\sqrt{1+4t}.$$

(f) $f^{-1}(x) = \dfrac{1+x}{2-4x}$.

99. **(a)** *Worked Solution:* We know that $(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))}$. Since $f(0) = 3$, we have $f^{-1}(3) = 0$, so that $(f^{-1})'(3) = \frac{1}{f'(0)}$. Now $f'(x) = 9x^2 + 4x + 8$, so that $f'(0) = 8$, and $(f^{-1})'(3) = 1/8$.

(b) $(f^{-1})'(3) = 1/2$

(c) $(f^{-1})'(4) = 1/2$

(d) $(f^{-1})'(1) = 12/13$

100. Using the quotient rule gives us

$$G'(x) = \frac{f^{-1}(x) \cdot \frac{d}{dx}(f(x)^2) - f(x)^2 (f^{-1})'(x)}{f^{-1}(x)^2} = \frac{2f(x)f'(x)f^{-1}(x) - f(x)^2/f'(f^{-1}(x))}{f^{-1}(x)^2}$$

and in particular

$$G'(2) = \frac{2f(2)f'(2)f^{-1}(2) - f(2)^2/f'(f^{-1}(2))}{f^{-1}(2)^2}.$$

Now, since $f(3) = 2$, we have $f^{-1}(2) = 3$, and so

$$G'(2) = \frac{6f(2)f'(2) - f(2)^2/f'(3)}{3^2} = \frac{6 \cdot 1 \cdot 3 - 1^2/4}{3^2} = 71/36.$$

101. (a): Expanding gives

$$\begin{aligned}\int (2t-7)^3\,dt &= \int (2t)^3 + 3\cdot(-7)(2t)^2 + 3\cdot(-7)^2(2t) + (-7)^3\,dt \\ &= \int 8t^3 - 84t^2 + 294t - 343\,dt \\ &= 8\tfrac{t^4}{4} - 84\tfrac{t^3}{3} + 294\tfrac{t^2}{2} - 343t + C \\ &= 2t^4 - 28t^3 + 147t^2 - 343t + C.\end{aligned}$$

(b): Use

$$u = 2t - 7, \quad du = 2dt$$

to get

$$\begin{aligned}\int (2t-7)^3\,dt &= \int u^3\,\frac{dt}{2} \\ &= \frac{u^4}{8} \\ &= \frac{(2t-7)^4}{8} + C.\end{aligned}$$