

Material implication may refer to:

a. Material conditional, a logical connective

b. Material implication (rule of inference), a rule of replacement for some propositional logic

a. The material conditional (also known as material implication, material consequence, or simply implication, implies, or conditional) is a logical connective (or a binary operator) that is often symbolized by a forward arrow " \rightarrow ". The material conditional is used to form statements of the form $p \rightarrow q$ (termed a *conditional statement*) which is read as "if p then q ". Unlike the English construction "if...then...", the material conditional statement $p \rightarrow q$ does not specify a causal relationship between p and q . It is merely to be understood to mean "if p is true, then q is also true" such that the statement $p \rightarrow q$ is false only when p is true and q is false. The material conditional only states that q is true when (but not necessarily only when) p is true, and makes no claim that p causes q .

The material conditional is also symbolized using:

1. $p \supset q$ (Although this symbol may be used for the superset symbol in set theory.);

2. $p \Rightarrow q$ (Although this symbol is often used for logical consequence (*i.e.*, logical implication) rather than for material conditional.)

3. Cpq (using Łukasiewicz notation)

With respect to the material conditionals above:

- p is termed the antecedent of the conditional, and
- q is termed the consequent of the conditional.

Conditional statements may be nested such that either or both of the antecedent or the consequent may themselves be conditional statements.

In the example " $(p \rightarrow q) \rightarrow (r \rightarrow s)$ ", meaning "if the truth of p implies the truth of q , then the truth of r implies the truth of s), both the antecedent and the consequent are conditional statements.

In classical logic $p \rightarrow q$ is logically equivalent to $\neg p \vee q$ by De Morgan's Law. Whereas, in minimal logic (and therefore also intuitionistic logic) only $\neg p \vee q$ logically entails $p \rightarrow q$; and in intuitionistic logic (but not minimal logic) $p \rightarrow q$ entails $\neg p \vee q$.

b. In propositional logic, material implication is a valid rule of replacement that allows for a conditional statement to be replaced by a disjunction in which the antecedent is negated. The rule states that $P \rightarrow Q$

implies Q is logically equivalent to not- P or Q and can replace each other in logical proofs.

Where \vdash is a metalogical symbol representing "can be replaced in a proof with."

The material implication rule may be written in sequent notation:

Where \vdash is a metalogical symbol meaning that is a syntactic consequence of in some logical system; or in rule form:

$$\frac{}{P \supset Q} \quad \text{at wherever an instance of } "P \supset Q" \text{ appears on a line of}$$
 a proof, it can be replaced with \supset ; or as the statement of a truth-functional tautology or theorem of propositional logic:

$$(P \supset Q) \supset (P \supset Q)$$

Where P and Q are propositions expressed in some formal system.